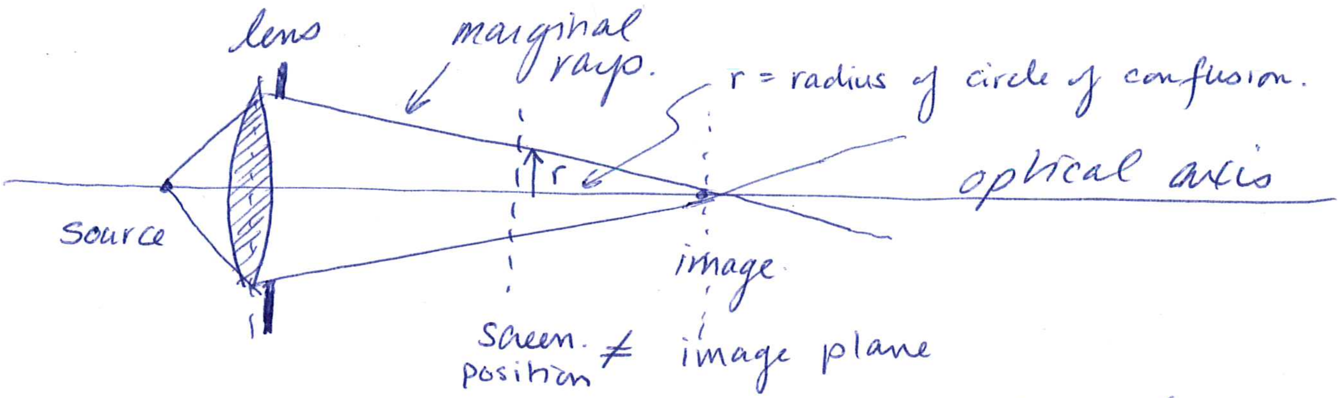
Homework #4

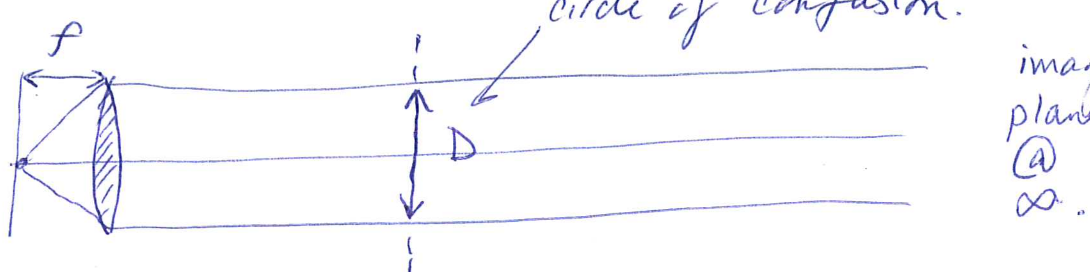
Problem 1

(a) We will see a blur on the image screen.

Explanation:
One way of thinking about what is happening is to compute the radius of the circle of confusion:



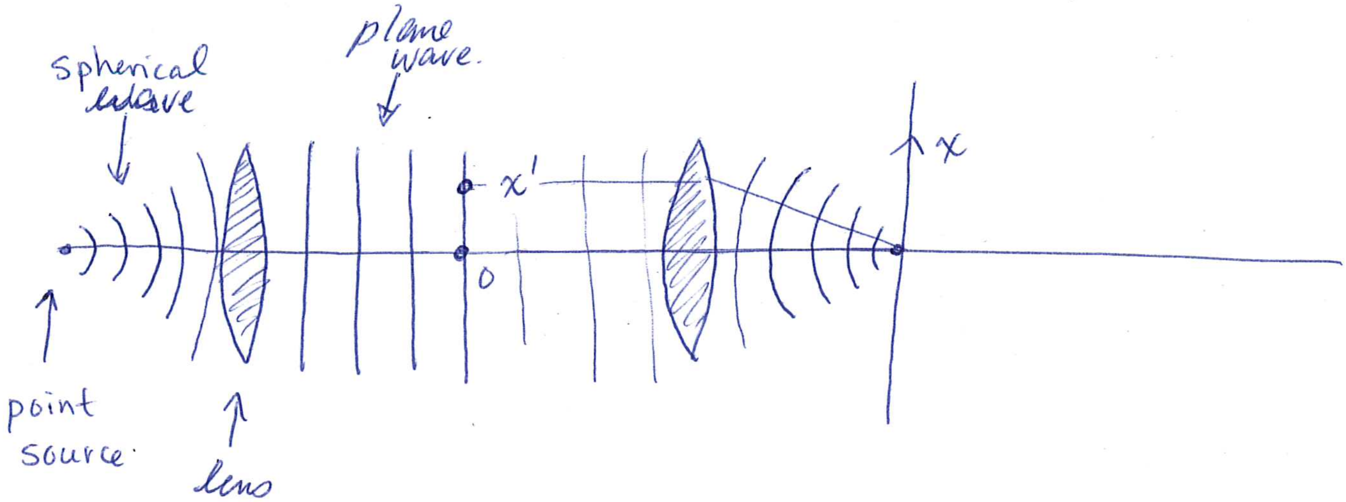
If a screen is placed away from the focus, (image plane) the point source projects onto a circle whose radius depends on the diameter of the lens (or NA). This circle is called the "circle of confusion." In the case of a image @ ∞, a screen at a finite position will
plane.
have point sources mapping to circle with the same diameter as the lens:



Problem 1 continued:

(b) Now that the light comes from a single point source, it is "coherent" meaning "in phase" ⟹ it will interfer.

(c) There is no phase difference between "red" + "green" sources.



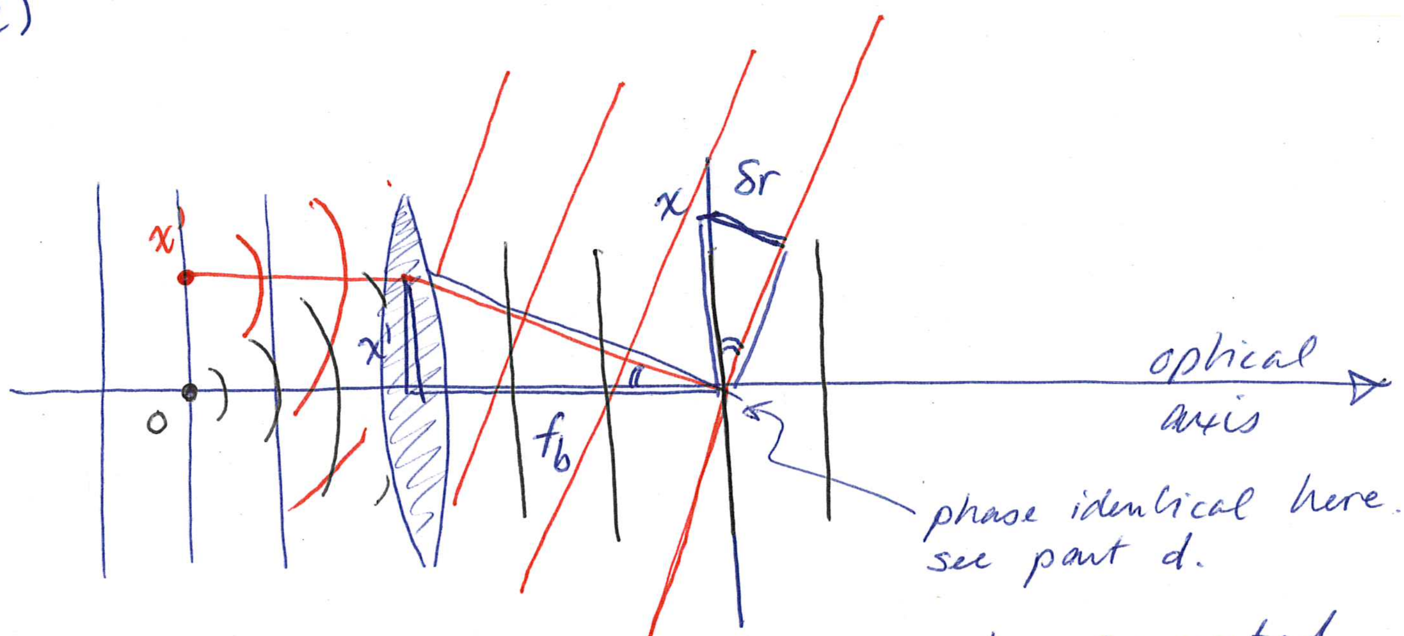
Clearly if point source is on optical axis, point @ x' + 0 have the same phase since they lie on the same wavefront.

(d) Clearly the phase difference is again 0.
Again, This is a consequence of what it means to focus light. Light is focused since the optical path

length is constant from any point on the wave front corresponding to the aperture.

Equivalently, wave front focuses to a point @ $x = 0$. (Ignoring diffraction).

(e)



The phase difference @ x can be computed by computing the length of $\underline{\delta r}$.

$$\Delta\varphi = -k\,\delta r \quad \text{(by construction)}.$$

in the small angle limit:

$$\frac{\delta r}{x} = \frac{x'}{f_b} \Rightarrow \delta r = \frac{x' x}{f_b}$$

Problem 1 continued

(e) $\Delta\varphi = -\dfrac{kx'x}{f_b}$

In 2D: $\Delta\varphi(\vec{x},\vec{x}') = -\dfrac{k\,\vec{x}'\cdot\vec{x}}{f_b}$ See class notes.

Displacements in and out of page do not change phase.

(f) We need to sum over all sources now:

$$\psi(x) = \underbrace{\int d^2x'}_{\text{“}\sum\text{''}\ \text{all points } x' \text{ on aperture}} \psi(\overset{\text{amplitude @ } x}{x}\,|\,\overset{\text{emitted from } x'}{x'})$$

Transmission Coef.

$$= \text{const}\cdot\int_{\Sigma} d^2x'\, A(\vec{x}')\, e^{+i\Delta\varphi(\vec{x},\vec{x}')}$$

$$= \text{const}\ \overbrace{\int_{\Sigma} d^2x'\, A(\vec{x}')\, e^{-i\vec{Q}\cdot\vec{x}'}}^{\text{Fourier Transform of } A(\vec{x}')}$$

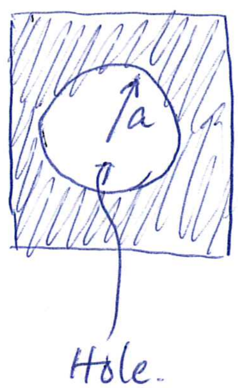
where $\vec{Q} \equiv \dfrac{k\vec{x}}{f_b}$.

This system takes the fourier transform of $A(\vec{x}')$.

Problem 2

(a) If we place a circular aperture @ position of the slide:

$$\mathcal{A} = \begin{cases} 1, & |\vec{x}'| < a \leftarrow \text{transmits} \\ 0, & |\vec{x}'| \geq a \leftarrow \text{blocks light} \end{cases}$$



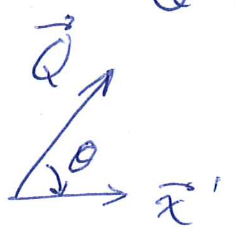
(b)
$$\psi(\vec{x},t) = \overbrace{\frac{\mathcal{E}_A \, e^{i(kR-\omega t)}}{R}}^{C_0} \int_{\substack{\text{All space} \\ \text{on aperture.}}} d^2x' \, \mathcal{A}(\vec{x}') \, e^{-i\vec{Q}\cdot\vec{x}'}$$

Go to POLAR COORDINATES:

$\mathcal{A}=0 \ \forall \ |\vec{x}'|>a.$

$$= C_0 \underbrace{\int_0^{a} dr \, r \int_0^{2\pi} d\theta}_{\int d^2x' \, \mathcal{A}(\vec{x}')} \, e^{-i\vec{Q}\cdot\vec{x}'}$$

Define θ: $\quad \vec{Q}\cdot\vec{x}\,' \equiv \underbrace{|\vec{Q}|}_{Q}\underbrace{|\vec{x}\,'|}_{r}\cos\theta$.



$$\psi = c_0 \int_0^a dr\, r \int_0^{2\pi} d\theta \; e^{-iQr\cos\theta}$$

$$\left\{ \nu \equiv \theta, \quad \mu \equiv -Qr \qquad d\mu = -Q\,dr. \right.$$

$$\psi = \frac{c_0}{Q^2} \int_0^{-Qa} d\mu\, \mu \int_0^{2\pi} d\nu \; e^{i\mu\cos\nu}$$

$$\left\{ J_0(\mu) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\nu \; e^{i\mu\cos\nu} \right.$$

$$\psi = \frac{c_0}{Q^2}\, 2\pi \int_0^{-Qa} d\mu\, \mu\, J_0(\mu).$$

$$\left\{ \frac{d}{d\mu}\left[\mu J_1(\mu)\right] = \mu J_0(\mu). \right.$$

$$\psi = \frac{c_0}{Q^2}\, 2\pi \int_0^{-Qa} d\mu \left[\frac{d}{d\mu}\mu J_1(\mu)\right] = \frac{c_0\, 2\pi}{Q^2}\left[J_1(\mu)\mu\right]_0^{-Qa}$$

$$\left\{ \begin{array}{l} \text{odd.} \quad J_1(\mu)\mu \Big|_{\mu=0} = 0 \\ \quad\quad J_1(-\mu)(-\mu) = J_1(\mu)\,\mu \end{array} \right.$$

$$\psi = C_0\, a^2\pi \left[\frac{2J_1(Qa)\,Qa}{(Qa)^2} \right]$$

$$\psi = C_0 \underbrace{a^2\pi}_{\text{area.}} \left[\frac{2J_1(Qa)}{Qa} \right]$$

$$= \pi a^2\, \mathcal{E}_A \frac{e^{i(kR-\omega t)}}{R} \left[\frac{2J_1(u)}{u} \right]_{u = Qa = \frac{kra}{f_b}}$$

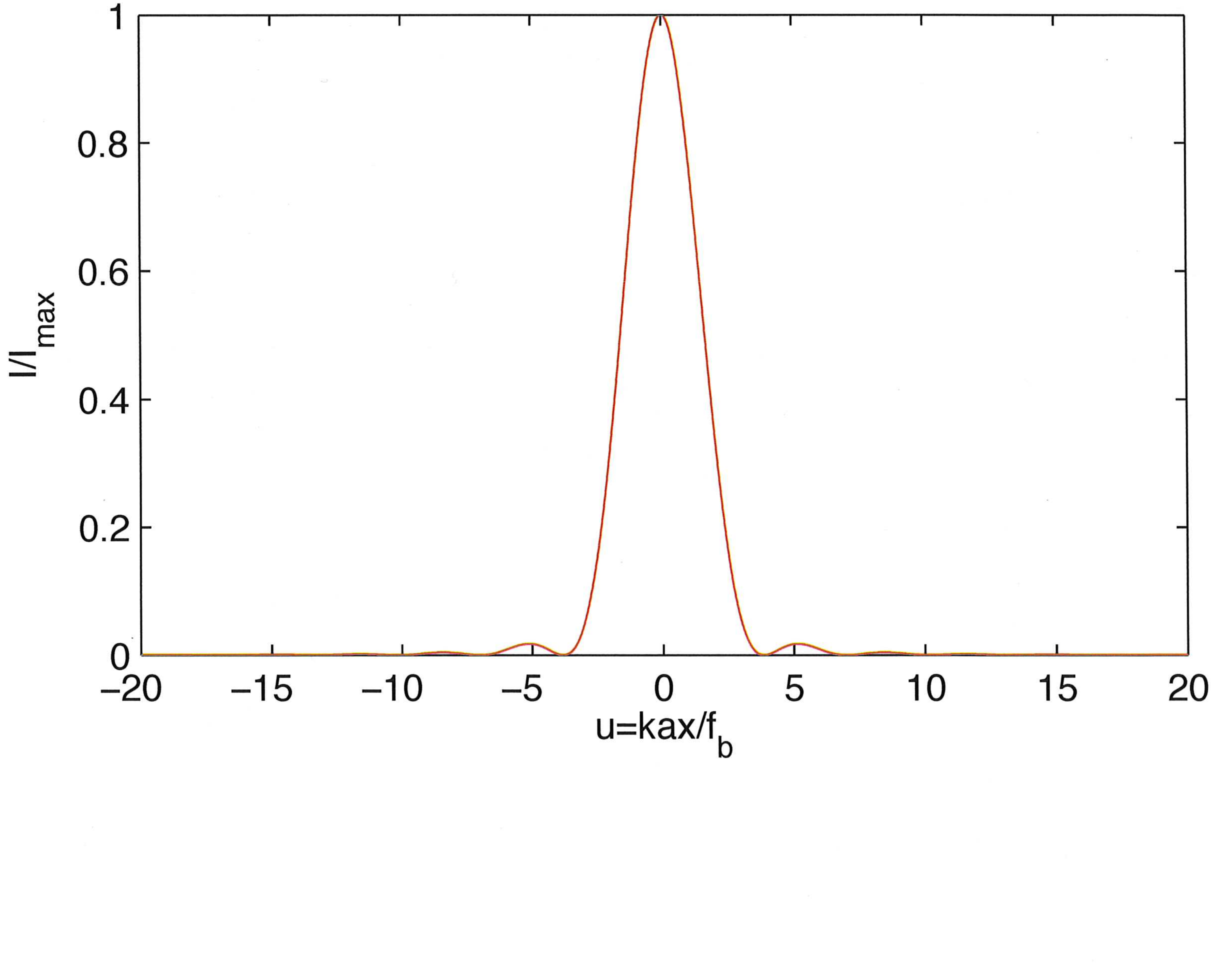
//.

(c) Intensity:

See next page:

$$I \sim |\psi|^2 = I_0 \left[\frac{2J_1(u)}{u} \right]^2 \Bigg|_{u = \frac{kra}{f_b}}$$

max intensity.


1
0.8
0.6
0.4
0.2
0
I/I_max
−20
−15
−10
−5
0
5
10
15
20
u=kax/f_b

(d) since flux sources are not coherent, we can sum the intensity of each source independently.

Aside:

Proof: $\psi_{TOT} = \psi_1 + \psi_2$

$$= \psi_{01} e^{-i\omega_1(t-t_1)} + \psi_{02} e^{-i\omega_2(t-t_2)}$$

$$I_{TOT} = \langle |\psi_{TOT}|^2 \rangle = \underbrace{|\psi_{01}|^2}_{I_1} + \underbrace{|\psi_{02}|^2}_{I_2} +$$

$$\underbrace{\langle \text{const} \cdot e^{-i[\omega_1(t-t_1) - \omega_2(t-t_2)]} \rangle}$$

$$\langle + CC \rangle$$

$$\int dT\, e^{i(\omega_1-\omega_2)t} = 2\pi\,\delta(\omega_1-\omega_2) \to 0.$$

See Hecht Ch 12...

$$I_{TOT} = I_1 + I_2$$

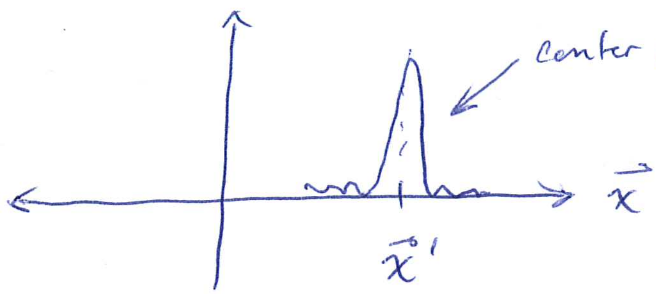
$$= \underbrace{I_{psf}(\vec{x}-\vec{x}_1)}_{\text{point spread function @ } \vec{x}_1} \cdot \overbrace{I_{10}}^{\text{intensity of source 1}} + \underbrace{I_{psf}(\vec{x}-\vec{x}_2)}_{\text{point spread function @ } \vec{x}_2} \cdot \overbrace{I_{02}}^{\text{intensity of source 2}}$$

For continuous sources:

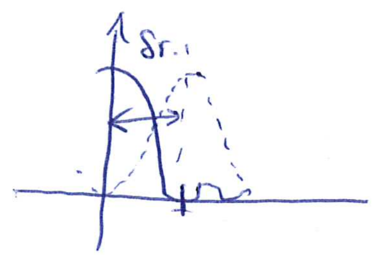
$$I_{TOT}(\vec{x}) = \overbrace{\int d^2\vec{x}'}^{\text{sum over all sources.}}\; I_{psf}(\vec{x}-\vec{x}')\; \underbrace{i_{object}(\vec{x}')}_{\text{image with infinit }\cancel{\text{intensity}}\text{ resolution. Intensity @ }\vec{x}'}$$

point spread function centered @ $\vec{x}'$

$I_{psf}(\vec{x},\vec{x}')$.



(e) Raleigh Limit of diffraction max of first source overlaps w/ zero of second source.



$$\delta u_1 \simeq \underbrace{3.83}_{\text{first zero of } J_1(u).} = \frac{ka\,\delta r}{f_b} = \frac{2\pi}{\lambda}\,\frac{a\,\delta r}{f_b}$$

$$\lambda = \frac{\lambda_0}{n}, \qquad \frac{a}{f_b} \simeq \sin\alpha_{max}$$

$$\delta r = \frac{\delta u_1}{2\pi}\,\frac{\lambda_0}{n\,\sin\alpha_{max}} = \frac{\delta u_1}{\pi}\,\frac{\lambda_0}{2NA} \simeq 1.22\,\frac{\lambda_0}{2NA}$$