1. (a) $\ell_o^2 = (s_o + x)^2 + h^2$

$\ell_i^2 = (s_i - x)^2 + h^2$

$x = R - R\cos\theta = R(1-\cos\theta)$

$h = R\sin\theta$

therefore,

$$\ell_o^2 = \left(s_o + R(1-\cos\theta)\right)^2 + R^2\sin^2\theta$$

$$= \left(s_o^2 + 2Rs_o(1-\cos\theta) + R^2(1-\cos\theta)^2\right) + R^2\sin^2\theta$$

$$= s_o^2 + 2Rs_o - 2Rs_o\cos\theta + R^2(1 - 2\cos\theta + \cos^2\theta) + R^2\sin^2\theta$$

$$= s_o^2 + 2Rs_o + R^2 - 2R(s_o + R)\cos\theta$$

$$= (s_o + R)^2 - 2R(s_o + R)\cos\theta$$

$$\ell_i^2 = \left(s_i - R(1-\cos\theta)\right)^2 + R^2\sin^2\theta$$

$$= \left(s_i^2 - 2Rs_i(1-\cos\theta) + R^2(1-\cos\theta)^2\right) + R^2\sin^2\theta$$

$$= s_i^2 - 2Rs_i + 2Rs_i\cos\theta + R^2(1 - 2\cos\theta + \cos^2\theta) + R^2\sin^2\theta$$

$$= (s_i - R)^2 + 2R(s_i - R)\cos\theta$$

$$\frac{d\ell_o^2}{d\theta} = 2\ell_o\frac{d\ell_o}{d\theta} = 2R(s_o + R)\sin\theta$$

$$\frac{d\ell_i^2}{d\theta} = 2\ell_i\frac{d\ell_i}{d\theta} = -2R(s_i - R)\sin\theta$$

Fermat's Principle: $\frac{d\,OPL}{d\theta} = n_1\frac{d\ell_o}{d\theta} + n_2\frac{d\ell_i}{d\theta} = 0 = n_1\frac{2R(s_o+R)\sin\theta}{2\ell_o} - n_2\frac{2R(s_i-R)\sin\theta}{2\ell_i}$

$$\Rightarrow \quad n_1\cdot\frac{R(s_o+R)\sin\theta}{\ell_o} = n_2\cdot\frac{R(s_i-R)\sin\theta}{\ell_i}$$

$$\Rightarrow \quad \frac{n_1}{\ell_o}(s_o + R) = \frac{n_2}{\ell_i}(s_i - R)$$

$$\Rightarrow \quad R\left(\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i}\right) = \frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}$$

$$\Rightarrow \quad \boxed{\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right)}$$

(b) small $\Delta$ approx: $\ell_o \approx s_o$, $\ell_i = s_i$ Substitute into eqn:

$$\boxed{\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{1}{R}(n_2 - n_1)} \quad \text{when } \theta \to 0^\circ$$

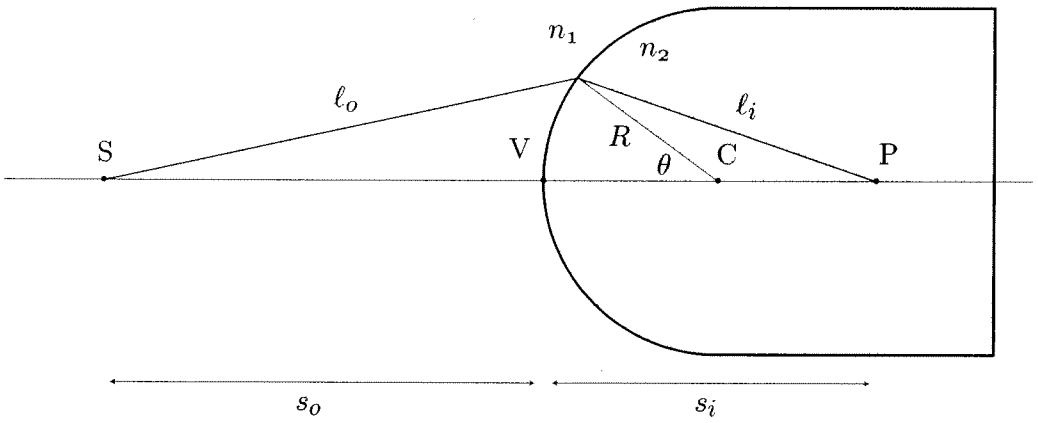



Figure 1: The figure shows light refracted by a single air-light interface.

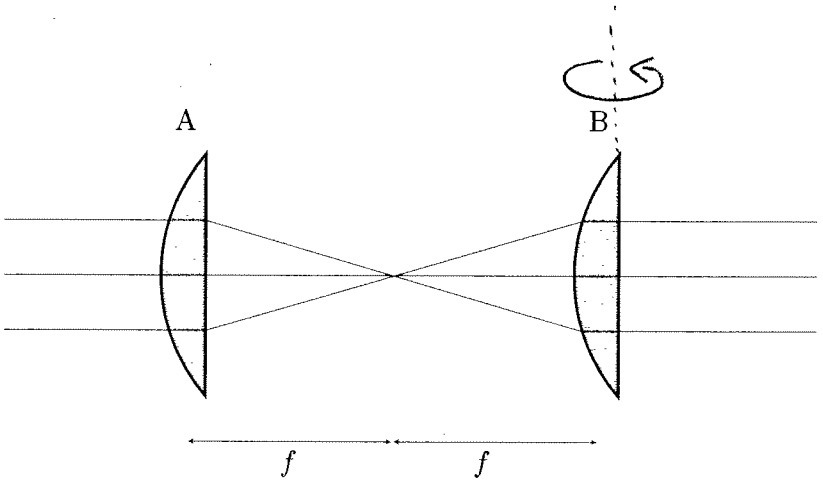
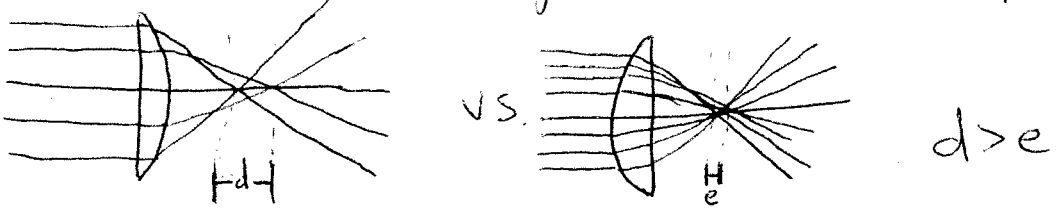
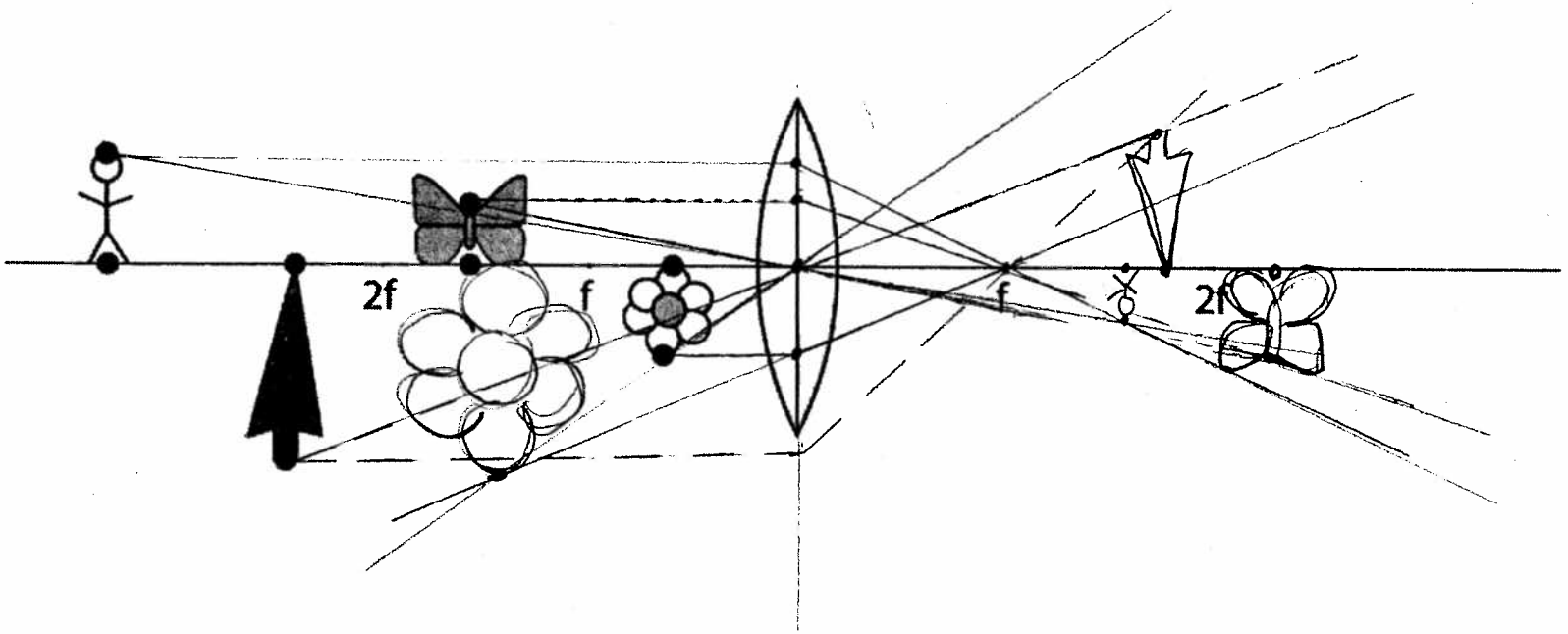



Figure 2: How could the configuration of the lenses be changed to reduce aberrations.

The lenses should be oriented with their planar surfaces facing each other. This then uses both ~~sides~~ air/glass interfaces to bend the light 2 ea minimizes spherical abberations.

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