
Contemporary Microscopy and Biophotonics

BioEn 498/599, Phys 427B

Problem Set #1

Due 4/13/2016

1. **The Fresnel Equations.** The Fresnel Equations give the ratio of the transmitted (and reflected) amplitude to the incident amplitude of the electromagnetic radiation from a flat interface between two dielectrics.

(a) (Optional) Please use the macroscopic Maxwell Equations to derive the following equations:

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}} \right)_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \quad (1)$$

$$t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}} \right)_{\perp} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} \quad (2)$$

$$r_{\parallel} \equiv \left(\frac{E_{0r}}{E_{0i}} \right)_{\parallel} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i} \quad (3)$$

$$t_{\parallel} \equiv \left(\frac{E_{0t}}{E_{0i}} \right)_{\parallel} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i} \quad (4)$$

when $\mu_i \approx \mu_t \approx \mu_0$. The subscripts \perp and \parallel describe the polarizations normal and parallel to the surfaces respectively, \vec{E}_{0i} , \vec{E}_{0r} , and \vec{E}_{0t} are the amplitudes of the incident, reflected, and transmitted waves respectively and θ_i and θ_t are the angles of the incident and transmitted waves respectively. (Hint: Any self-respecting book on optics or electricity and magnetism will present this derivation so no one has any excuse not to do this problem. If you use ones of these books as a reference, please cite it.)

- (b) Plot these amplitude coefficients as a function of incident angle for an air \rightarrow water and water \rightarrow air interface. (Hint: Remember to eliminate the angle of transmission using Snell's Law.)
- (c) What happens for large incident angles for a water \rightarrow air interface?
- (d) Please observe this effect for yourself and describe the experiment you performed.
- (e) Explain why polarizing sunglasses work.

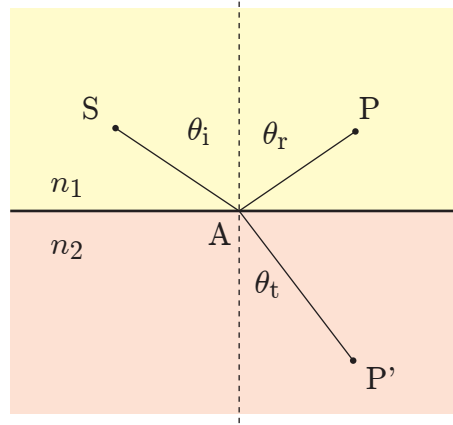


Figure 1: S is a source. Waves are both reflected and transmitted by the interface between media with refractive index n_1 and n_2 .

2. **Fermat's Principle** is an extremely useful tool for designing optical systems. In this problem, we will consider its consequences for the angles of reflection and refraction.
 - (a) Demonstrate that Snell's Law can be understood to be consequences of Fermat's Principle of least time. Consider a source S as shown in Fig 1. Vary the position of A in order to minimize the optical path length between S and P'. (Hint: Remember that the optical path length depends on the index of refraction of the media.)
 - (b) Please explain how your work above implies Snell's law for plane waves, even though your calculation used a point source. (Hint: Don't do anything hard.)
 - (c) Observe this effect and describe your experiment. (Hint: I usually use a coffee mug of water and a pen.)
 - (d) What is the relation between the incident and reflected angles. Vary the position of A to minimize the optical path length between S and P if the ray is reflected from the interface.