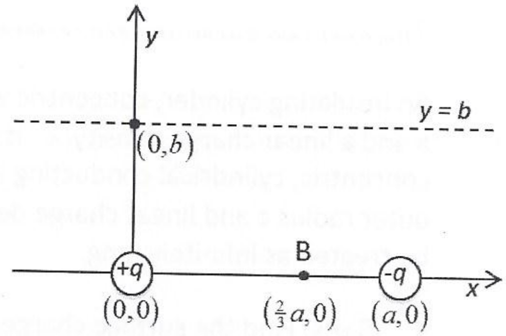
Name Faraday   Daniel   Student ID Number 4815162342
last   first

The first two questions pertain to the situation described below.

Two point charges are fixed in place as shown at the right. The magnitude of each charge is equal to $q = 5\ \mu C$. Both charges lie on the x-axis: one at the origin, and one at $x = a$ (= 5 cm). The signs of the charges are indicated in the drawing. The questions pertain to the electric field that these two charges produce.



1. (5 pts.) What is the magnitude of the electric field at point **B**, located at $(x,y) = (\frac{2}{3}a, 0)$?
   - A. -1.21×10^8 N/C
   - B. 2.02×10^8 N/C
   - C. 1.21×10^8 N/C
   - D. 4.05×10^6 N/C
   - E. 1.01×10^3 N/C

$$\vec{E}\left(\tfrac{2}{3}a,0\right) = \left(\frac{kq}{(\frac{2}{3}a)^2} + \frac{kq}{(\frac{1}{3}a)^2}\right)\hat{x} = \frac{45kq}{4a^2}\hat{x}$$

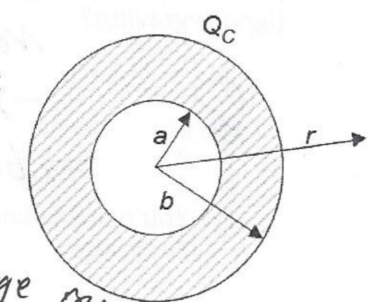
2. (3 pts) At what point on the line dashed line $y = b$ (= 4 cm) is the ***y* component** of the electric field zero? That is, for what value of x is $E_y = 0$?
   - A. 2.5 cm
   - B. -2.5 cm
   - C. Nowhere on the $y = b$ line.

$a = 5$ cm

So by symmetry, $E_y = 0$ midway between $+q$ & $-q$, i.e., $x = 2.5$ cm

$\frac{\vec{E}}{|\vec{E}|} = \hat{x}$

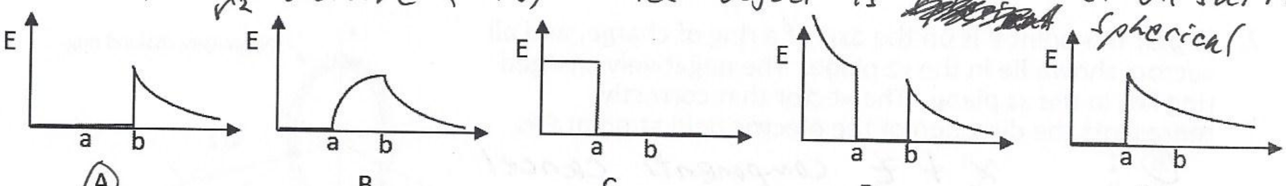
3. (5 pts) A conducting spherical shell of inner radius a and outer radius b has a net charge $Q = +6\ \mu C$. Which of the plots below best describes the radial component of the electric field as a function of distance from the center of symmetry for the above system?



$E_r = 0$ inside cavity $(r < a)$

$E_r = 0$ inside conductor $(a < r < b)$ — Charge resides on surface

$E_r \sim \frac{1}{r^2}$ outside $(r > b)$ since object is ~~[illegible]~~ spherical



A.   B.   C.   D.   E.

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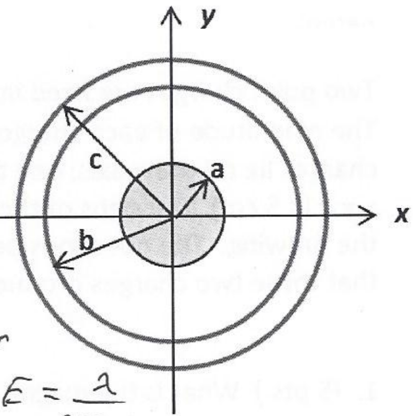
The next two questions are related.

An insulating cylinder, concentric with the z axis, has a radius **a** and a linear charge density λ. It is surrounded by a concentric, cylindrical conducting shell with inner radius **b**, outer radius **c** and linear charge density $-\lambda$. The cylinders will be treated as infinitely long.



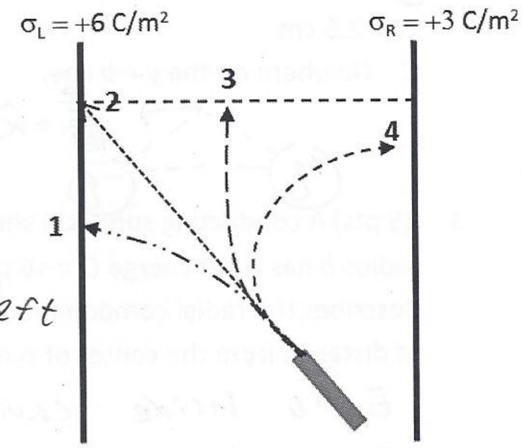
4. (5 pts) Find the <u>**surface**</u> charge density on the surface of the conducting shell at **r = b.** $E=0$ inside conductor

   A. $+\lambda/(2\pi b)$ By Gauss law, $a<r<b$, $E=\frac{\lambda}{2\pi r\epsilon_0}$
   B. $+2\lambda\pi b$
   C. 0 So, $b<r<c$, $E=\frac{\lambda_a+\lambda_b}{2\pi r\epsilon_0}=0 \Rightarrow \lambda_a=-\lambda_b$
   D. $-\lambda/(2\pi b)$ surface c.d. $\frac{q}{2\pi b l}=\frac{\lambda}{2\pi b}$ $\Rightarrow \lambda_b=-\lambda$
   E. $-2\lambda\pi b$
   $\rightarrow \lambda_b=\frac{-\lambda}{2\pi b}$

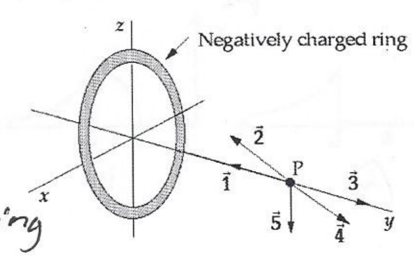
5. (3 pts) If you placed a positively charged particle on the x axis between r = b and r = c, which way would it move?
   A. +x direction
   B. −x direction $E=0$ inside conductor $\Rightarrow$ No force
   C. It would not move

6. (5 pts) In the figure on the right, two infinitely long charged planes have the indicated uniform charge densities. An electron "gun" emits electrons at the trajectory shown. What is the best choice for the path an electron might take in this configuration (ignore gravity)?

   Net $\vec{E}$ to right
   A. 1 $\rightarrow$ Force on e^- to left
   B. 2
   C. 3 $\Rightarrow$ best trajectory (1)
   D. 4
   E. You need to know the initial electron velocity to choose



7. (5 pts) The point P is on the axis of a ring of charge, and all vectors shown lie in the yz plane. The negatively charged ring lies in the xz plane. The vector that correctly represents the direction of the electric field at point P is
   A. 1 x + z components cancel
   B. 2 due to symmetry of ring
   C. 3
   D. 4 − charge on ring $\Rightarrow$ $\vec{E}$ points towards ring
   E. 5



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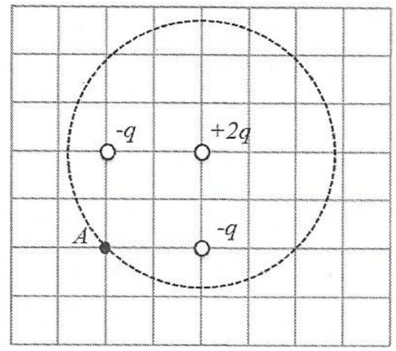
The next three questions pertain to the figures on the right.

A spherical Gaussian surface (dashed line) that passes through point **A** encloses the three point charges as shown.



8. (3 pts) What is the net flux through the Gaussian surface?

   A. zero
   B. $4q/\varepsilon_0$
   C. You cannot determine it from this geometry

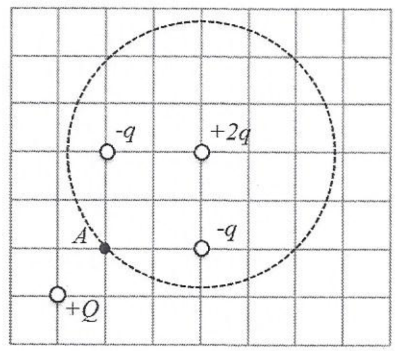
Gauss' law: $\Phi_E = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$

$q_{enc} = 0 \Rightarrow \Phi_E = 0$

9. (3 pts) A new point charge +Q, of unknown magnitude, is added as shown in the next figure. The magnitude of the flux through the Gaussian surface

   A. increases
   B. decreases
   C. remains the same

+Q not within Gaussian surface



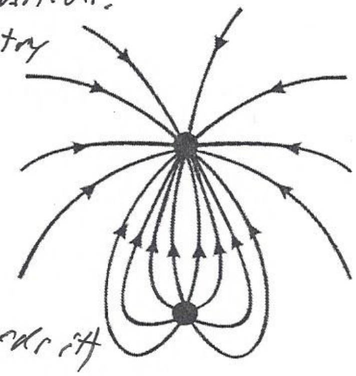
10. (4 pts) How does the electric field at point **A** change after the new charge +Q is added? It will ...

   A. change in direction only
   B. change in direction and magnitude
   C. change in magnitude only
   D. remain the same

Field from +Q changes E_{net} by superposition.
Since A + +Q are on axis of symmetry of system, $\vec{E}$ direction does not change

11. (4 pts) The figure shows the field lines for two charges. What is the ratio of the **top** charge to the **bottom** charge?

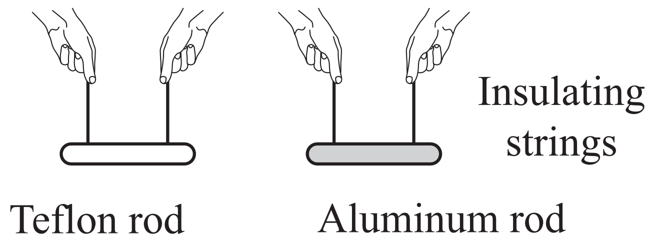
   A. 1 : 2
   B. -1 : 2
   C. 2 : 1
   D. -2 : 1
   E. 2 : -1



Process of elimination;
Top charge must be neg.
(field lines point towards it)
Bottom charge must be pos.
(field lines point away from it)

II. [10 pts total] Based on your experience in Lab.

Neutral Teflon (insulating) and neutral aluminum (conducting) rods are suspended by two insulating strings. Two people hold the strings as shown.



12. [3 pts.] The Teflon rod is then rubbed so it is negatively charged. The rods are again held near each other, but the rods never touch and no sparks jump between them.

    The two rods would

    a. attract.
    b. repel.
    c. not interact.
    d. There is not enough information to tell.

*The negative charge on the Teflon rod induces a separation of charge in the aluminum rod; the positive charge is attracted to the negative Teflon rod and the negative charge is repelled from the negative Teflon rod. Since the Teflon rod is closer to the positive than the negative charge in the aluminum rod, the attractive interaction is stronger than the repulsive interaction. Thus there is a net attraction between the two rods and the answer should be **(a).***

13. [4 pts] The rods are then made to briefly touch. They are then separated and held close to one another, but far enough apart that they no longer touch.

    The two rods would

    a. attract.
    b. repel.
    c. not interact.
    d. There is not enough information to tell.

*Negative charge is transferred from the contact point on the Teflon rod to the aluminum rod. The two rods then each have a net negative charge. If enough charge is transferred, the rods repel. If only a small amount of charge is transferred, the repulsive force that results may not be greater than the attractive force due to polarization (see above). Thus there may be attraction, or repulsion, or no net interaction and the answer should be **(d)**.*

14. [3 pts] The final net charge on the aluminum rod in *question 12:*

    a. has the same sign **<u>and</u>** the same charge distribution as the aluminum rod in *question 13*.
    b. has the same sign, **<u>but</u>** different charge distribution as the aluminum rod in *question 13*.
    c. has no net charge, it is neutral.
    d. There is not enough information to tell.

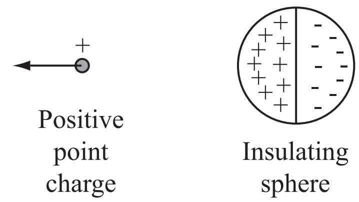
*The net charge of the aluminum rod in part i. is zero because there was no spark and nothing touched the aluminum rod, so there was no way for charge to be transferred to the initially neutral aluminum rod. In part ii., there was a transfer of negative charge from the Teflon rod to the aluminum rod. Thus the sign of the net charge on the aluminum is different in the two parts and the answer should be **(c).***

Name ______________________________________ **Student ID** _______________ **Score**________

last *first*

IV. [20 points total] This question is composed of two independent parts, A and B.

A. A positive point charge and insulating sphere are fixed in place as shown. The charges on the two halves of the insulating sphere are equal in magnitude, opposite in sign, and uniformly distributed throughout their respective halves.



i. [5 pts] On the diagram, indicate the direction of the net electric force on the point charge. If the force is zero, state so explicitly. Explain.

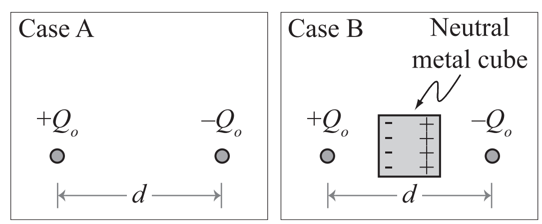
The positive point charge experiences a repulsive force (to the left) due to the left half of the sphere and an attractive force (to the right) due to the right side of the sphere. Although the magnitudes of the charges on both halves of the sphere are the same, the distances from these charges to the point charge are different. By Coulomb's law, the strength of the electric force increases with smaller distance, so the repulsive force (to the left) is stronger than the attractive force (to the right). ***Thus, the net electric force on the point charge is to the left.***

ii. [5 pts] The right half of the sphere is removed, as shown. The charge distribution on the remaining left half is unchanged.

After the right half of the sphere is removed, does the magnitude of the net electric force on the point charge *increase, decrease,* or *remain the same?* Explain.

As in part i above, the left half of the sphere causes a repulsive force to the left. In this case, there is not an attractive force that cancels part of this repulsive force. ***Thus, the magnitude of the electric force on the point charge increases.***

B. In case A, two charges $+Q_o$ and $-Q_o$ are separated by a distance d. In case B, a neutral metal cube has been inserted in between the two point charges.



i. [5 pts] On the diagram, draw the charge distribution for the metal cube in case B. Explain.

The $+Q_o$ charge attracts negative charges to the left side of the cube and repels positive charges to the right. The $-Q_o$ charge repels negative charges to the left side and attracts positive charges to the right. Thus, both charges tend to cause a collection of negative charges on the left and positive charges on the right, as shown.

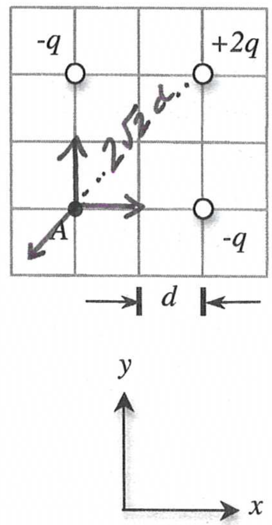
ii. [5 pts] Is the net electric force on the positive charge $+Q_o$ in case A *greater than, less than,* or *equal to* that in case B? Explain.

In case A, the $+Q_o$ charge experiences an attraction (to the right) due to the $-Q_o$ charge. In case B, this attraction is still present, but there is also a force due to each side of the metal cube. The effect of these two forces by each side of the cube is to cause a net attractive force on the $+Q_o$ charge (since the negative charges are closer than and equal in magnitude to the positive charges). This attractive force from the cube is in the same direction as the attractive force due to the original $-Q_o$ charge, so the net electric force on the $+Q_o$ charge with the cube present is stronger. ***Thus, the net electric force in case A is less than that in case B.***

Name ______________________________ **Student ID** ______________
last *first*

III. [25 pts]

A. [13 pts] Three point charges are fixed at the corners of a square of side $2d$ where $q = 5\mu C$ and $d = 15$ cm. Find the x and y components of the electric field at point A. Show your work.



Use superposition:

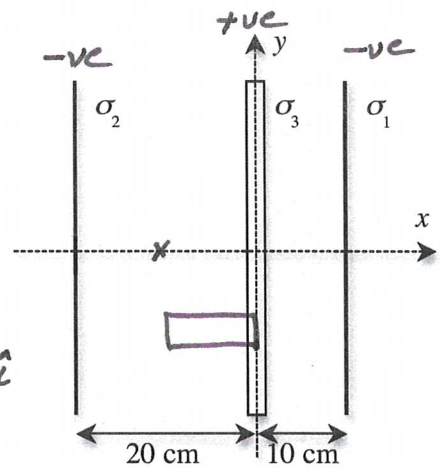
$$\vec{E}_x = \frac{Kq}{(2d)^2}\hat{i} - \frac{2Kq}{(2\sqrt{2}d)^2}\underbrace{\cos 45^\circ}_{\frac{1}{\sqrt{2}}}\hat{i}$$

$$= \left(\frac{Kq}{4d^2} - \frac{Kq}{4d^2}\frac{1}{\sqrt{2}}\right)\hat{i} = \frac{Kq}{4d^2}\left(1-\frac{1}{\sqrt{2}}\right)\hat{i}$$

$$|\vec{E}_x| = |\vec{E}_y|$$

E_x = 2.07x10^5 N/C	E_y = 2.07x10^5 N/C

B. Two infinite sheets with charge densities $\sigma_1 = -2\ \mu C/m^2$ and $\sigma_2 = -5\ \mu C/m^2$ are located at $x = +10$ cm and $x = -20$ cm respectively. An infinite conducting slab of thickness 2 cm is centered at the origin. The **net** charge density on the conducting slab is $\sigma_3 = +4\ \mu C/m^2$. The sheets and the slab are parallel to the yz plane.



i. [6 pts] What is the magnitude of the electric field E at $x = -10$ cm? Show your work.

Use superposition & formula for field due to infinite sheet

$$E(x=-10) = \left(\frac{|\sigma_1|}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0} - \frac{|\sigma_3|}{2\epsilon_0}\right)\hat{i} = -\frac{7\mu C/m^2}{2\epsilon_0}\hat{i}$$

E(x = -10cm) = -3.95x10^5 N/C

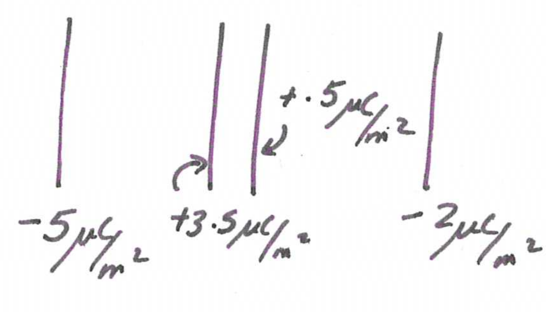
ii. [6 pts] What is the surface charge density on the left surface of the conducting slab (at $x = -1$ cm)? Show your work.

Use Gauss's Law for surface shown: $\vec{E}$ at left surface $= -\frac{7\mu C/m^2}{2\epsilon_0}\hat{i}$

and use $\vec{E} = 0$ inside conducting slab; $\vec{E}$ at right surface $= 0$

$$|E_{left}|A_{left} = \frac{\sigma_{left}A_{left}}{\epsilon_0} \Rightarrow +\frac{7\mu C/m^2}{2\epsilon_0} = \frac{\sigma_{left}}{\epsilon_0} \Rightarrow \sigma_{left} = +3.5\mu C/m^2$$

σ_{left} = +3.5 μC/m²



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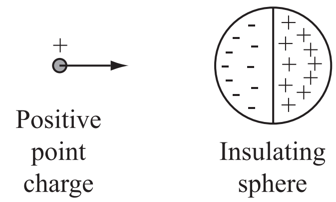

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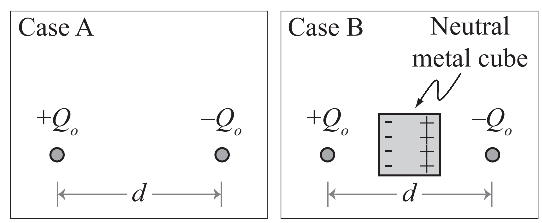

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