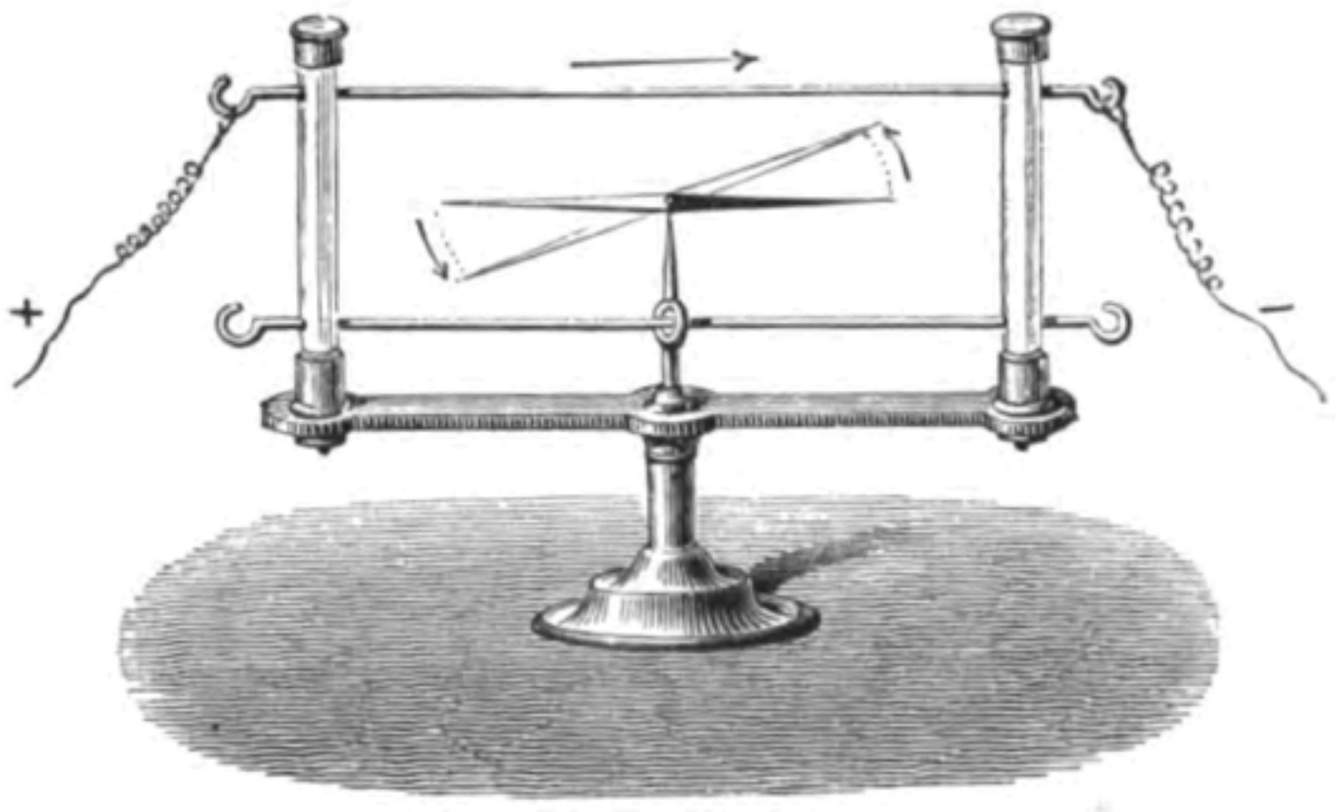
Special Lecture:
Special relativity & E&M

Not on exam!

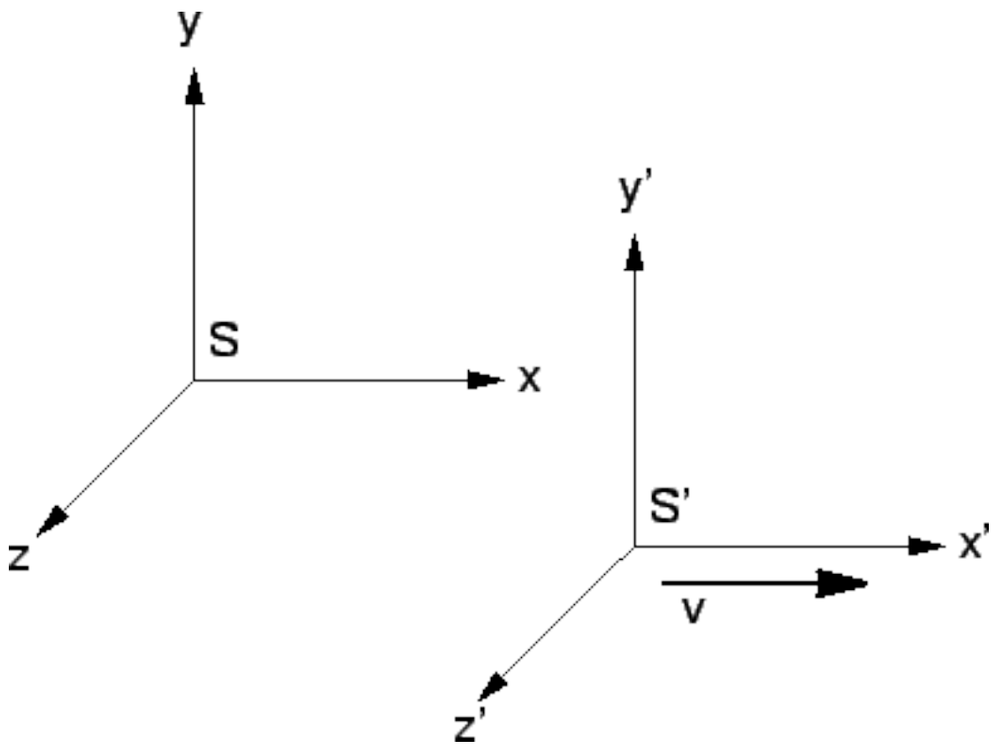
What is the B field?

- Phenomenology: Magnetism, lodestones, compass needles and the terrestrial magnetic field…
- Ørsted: Magnetism is a “hidden form” of electricity



Newtonian mechanics

- Newtonian physics is invariant to **galilean transformations**… *i.e.* working in moving reference frame



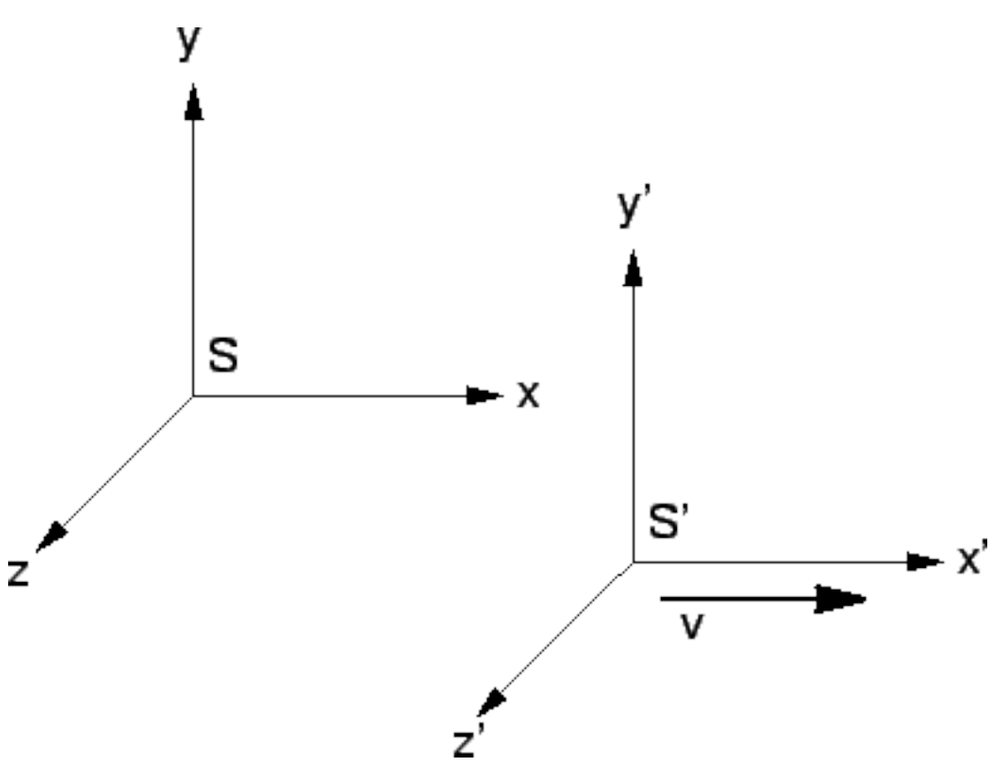
$$x' = x + vt,$$
$$t' = t,$$

$$F = ma,$$
$$F' = ma',$$

in an inertial frame, you can’t feel if you are moving…

E&M breaks this “rule”!

- Lorentz force law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- Assume E and B are uniform through all space…

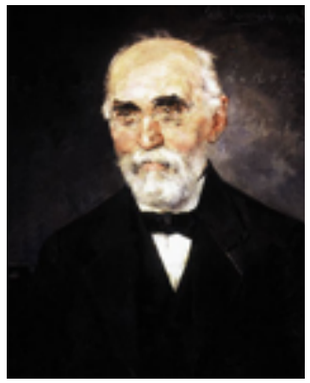


$$\vec{v}\,' \to 0$$

$$\vec{E}\,' \to \vec{E}$$

$$\vec{B}\,' \to \vec{B}$$

$$\vec{F}\,' \neq \vec{F}$$



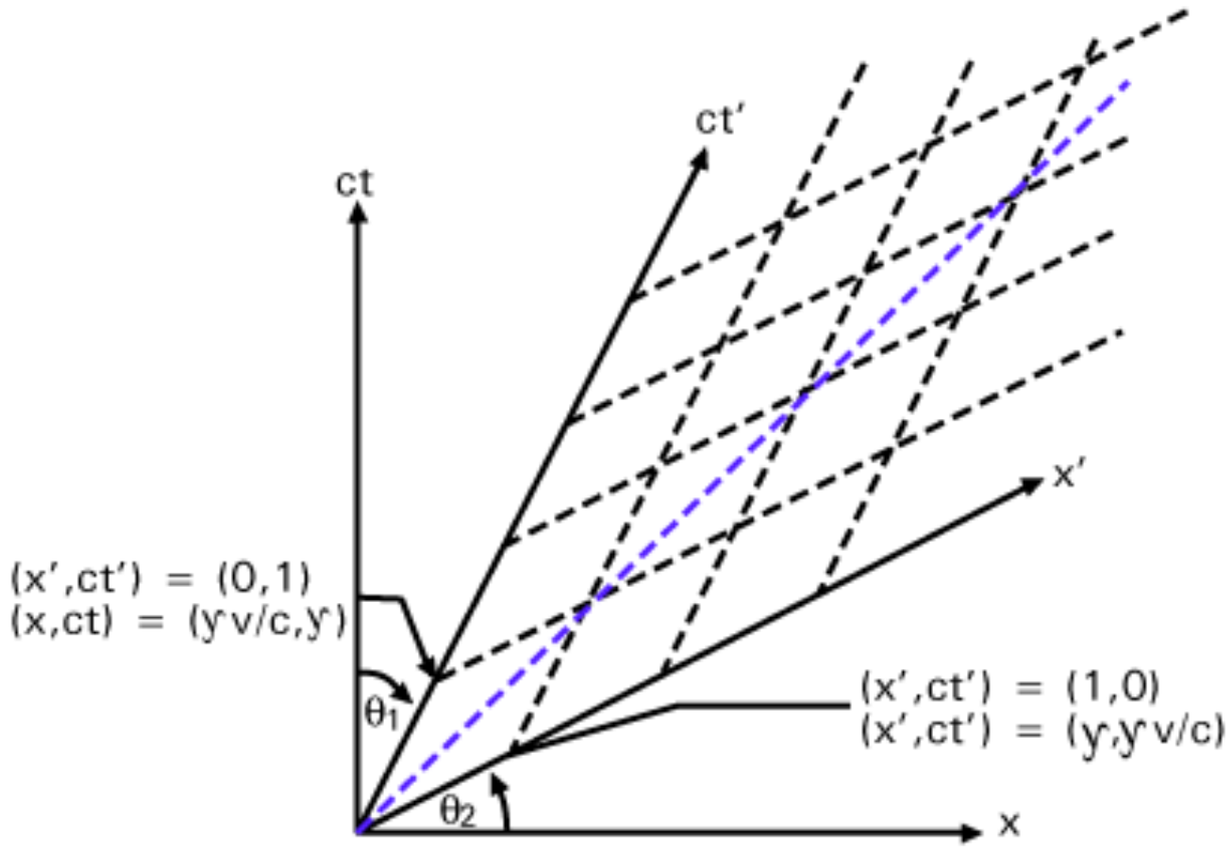
Symmetry of E&M: Lorentz transformation

- Lorentz Boost in x:

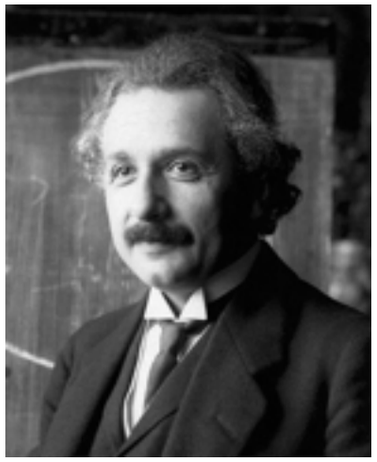
$$ct' = \gamma(ct - vx/c),$$
$$x' = \gamma(x - vt),$$
$$y' = y,$$
$$z' = z$$

- Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$



"Can't exceed the speed of light…"



Special relativity & Lorentz transformation

- Lorentz Boost in x:

$$ct' = \gamma(ct - vx/c),$$
$$x' = \gamma(x - vt),$$
$$y' = y,$$
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- Lorentz Factor

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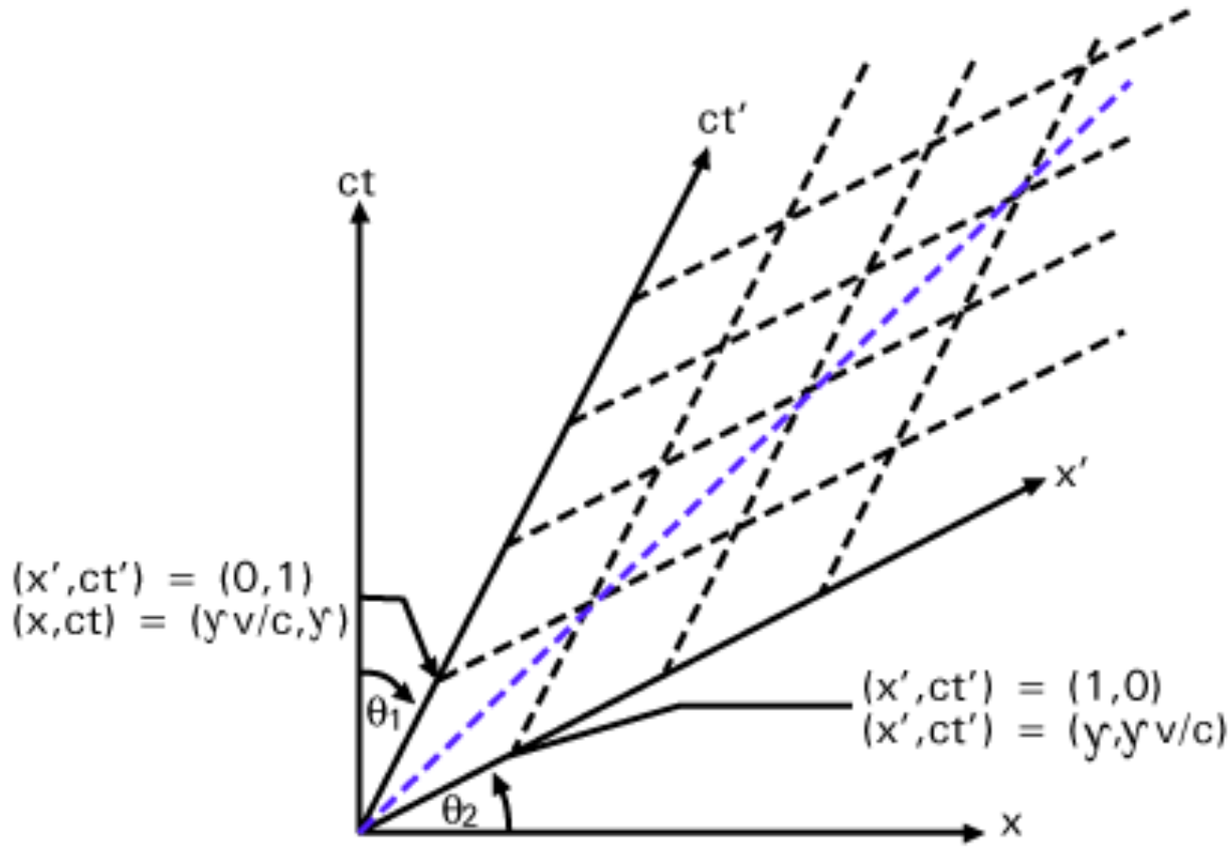
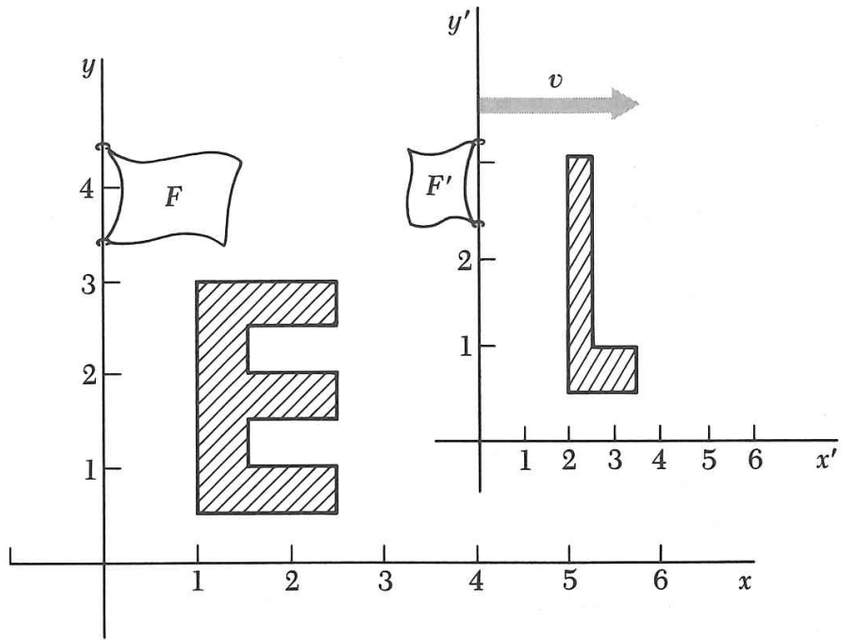


“Lorentz contraction”

(a)



$$
\begin{aligned}
ct' &= \gamma(ct - vx/c), \\
x' &= \gamma(x - vt), \\
y' &= y, \\
z' &= z
\end{aligned}
$$

- Consider the length of the L in frame F:
- Doc cam.
- Result:

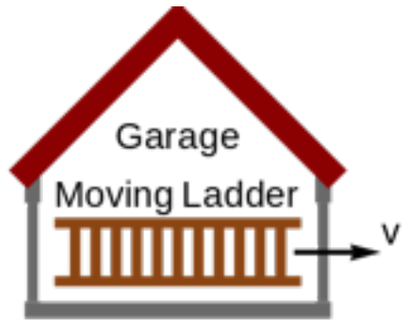
$$L = L_0/\gamma,$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

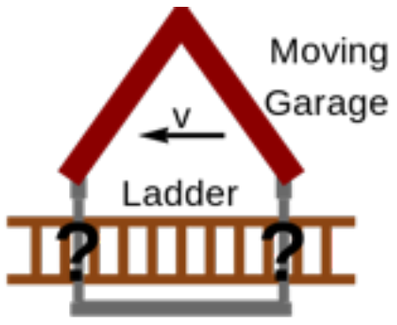
Ladder Paradox…

- **Problem:** the ladder won't fit in the garage!

- **Solution:** Just boost the ladder:

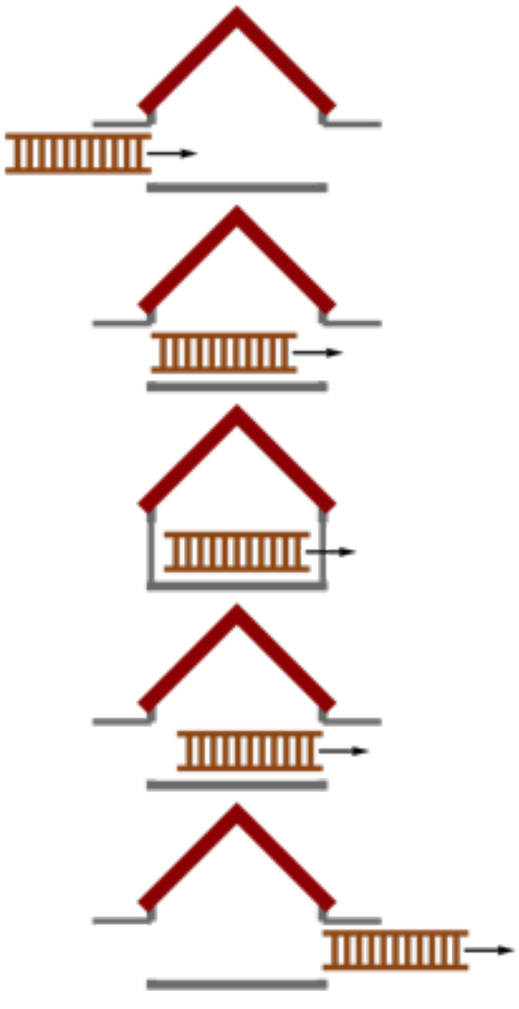


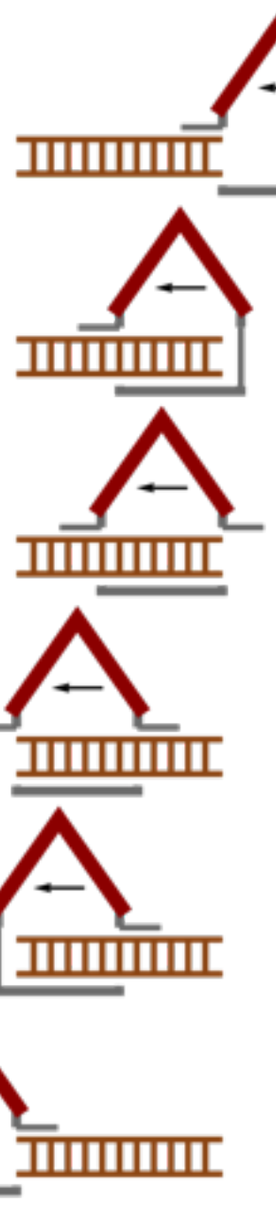
- **Paradox:** Consider the ladder frame?!?

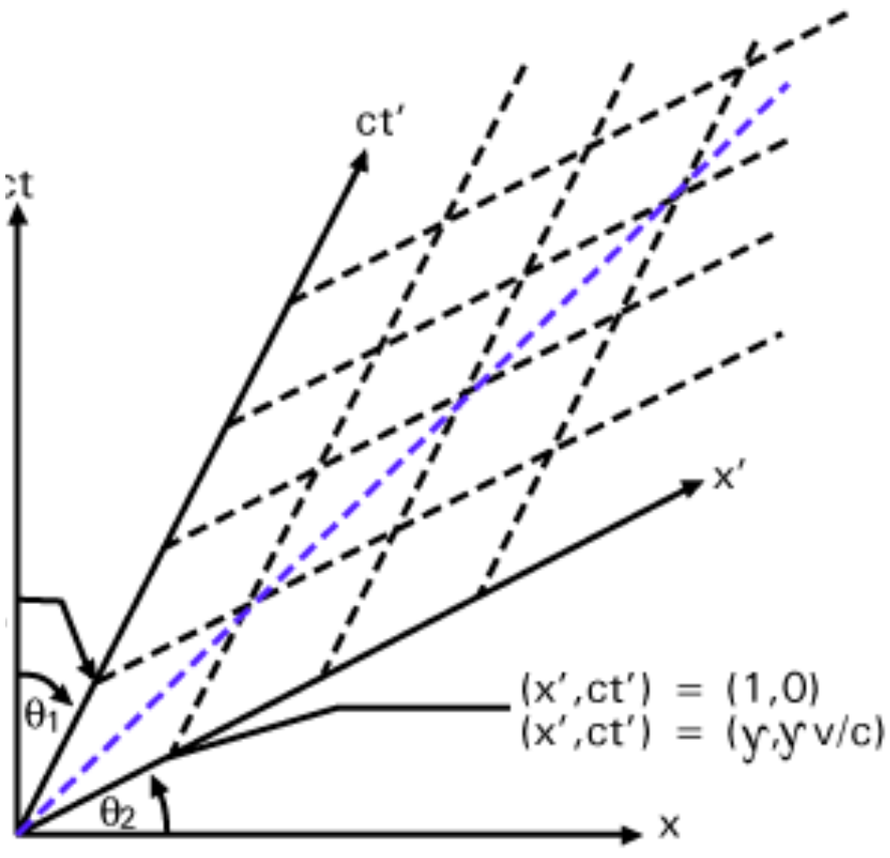


Simultaneous?
Ladder Paradox

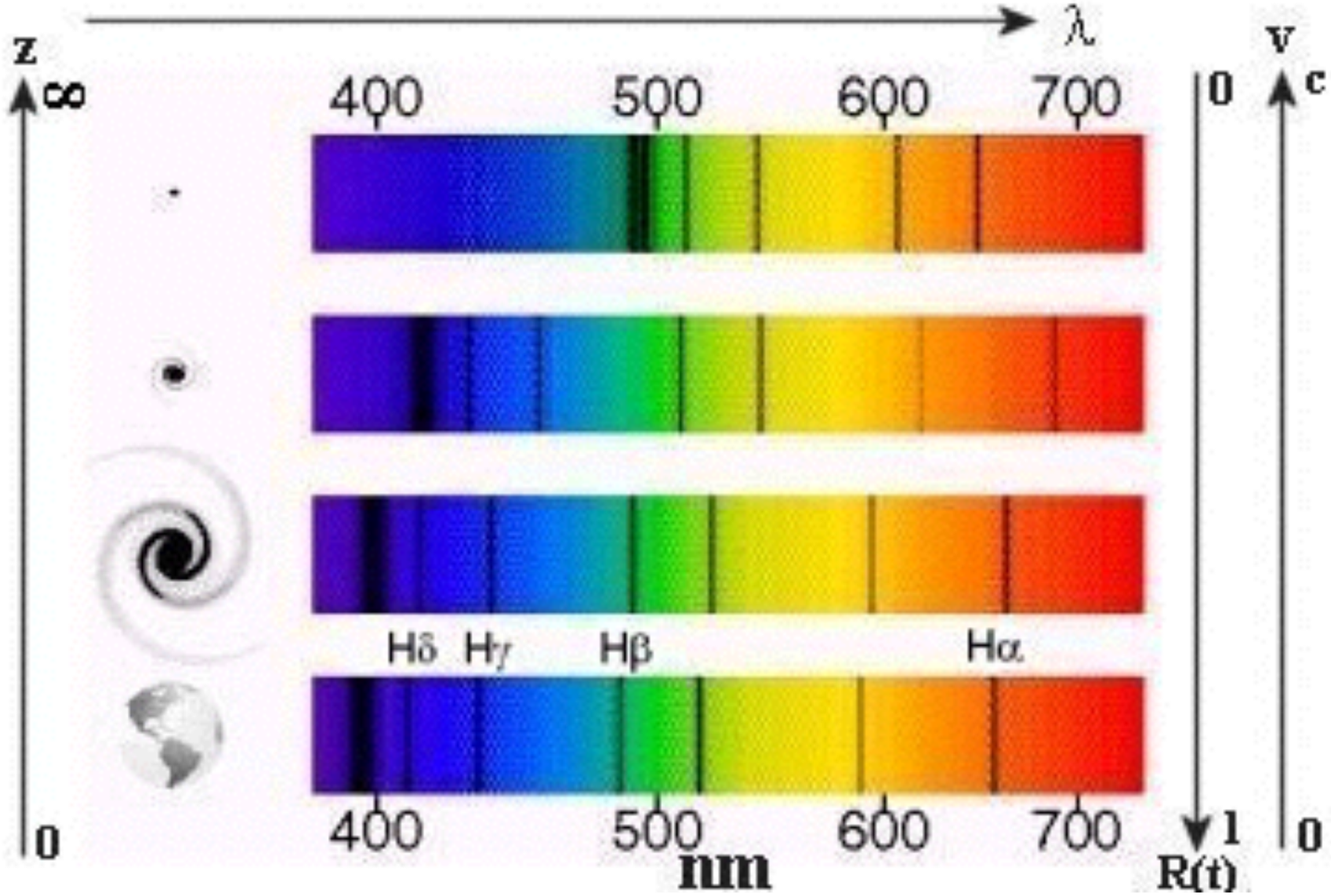
Garage Frame: Ladder Frame:







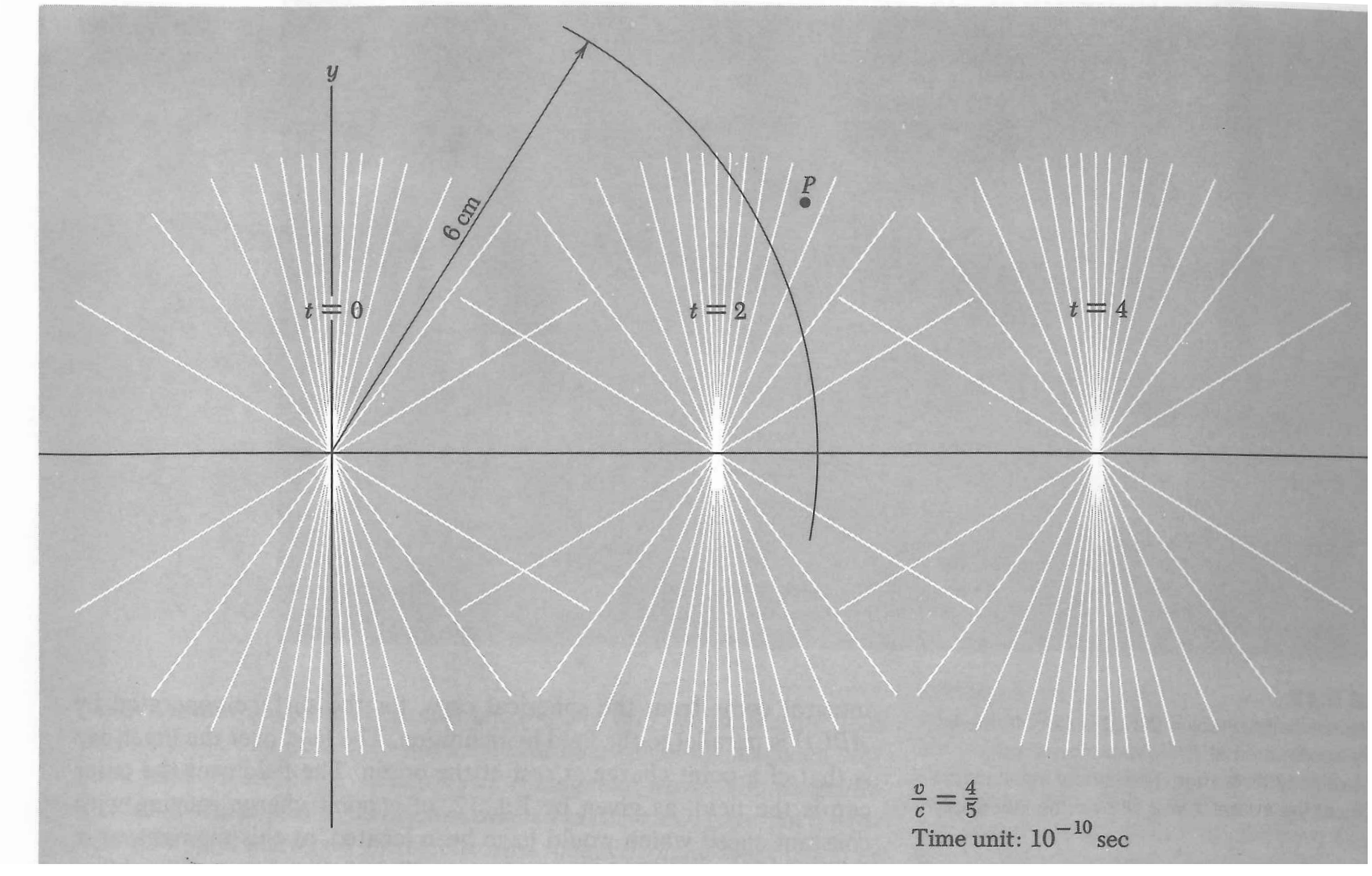
Doppler + Lorentz * = red shift (cosmology)



* slightly more complicated…

Back to E&M…

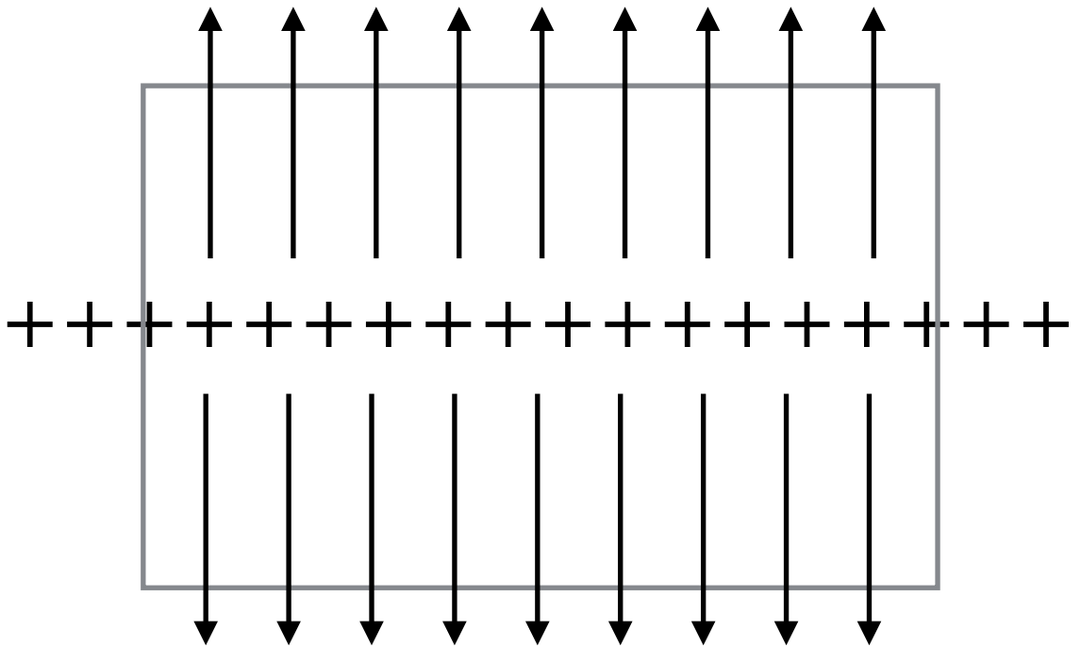
- Electric field lines from a moving charge…



How does the electric field transform?

- Gauss Law still holds (*i.e.* charge is invariant.)

- Doc cam derivation:

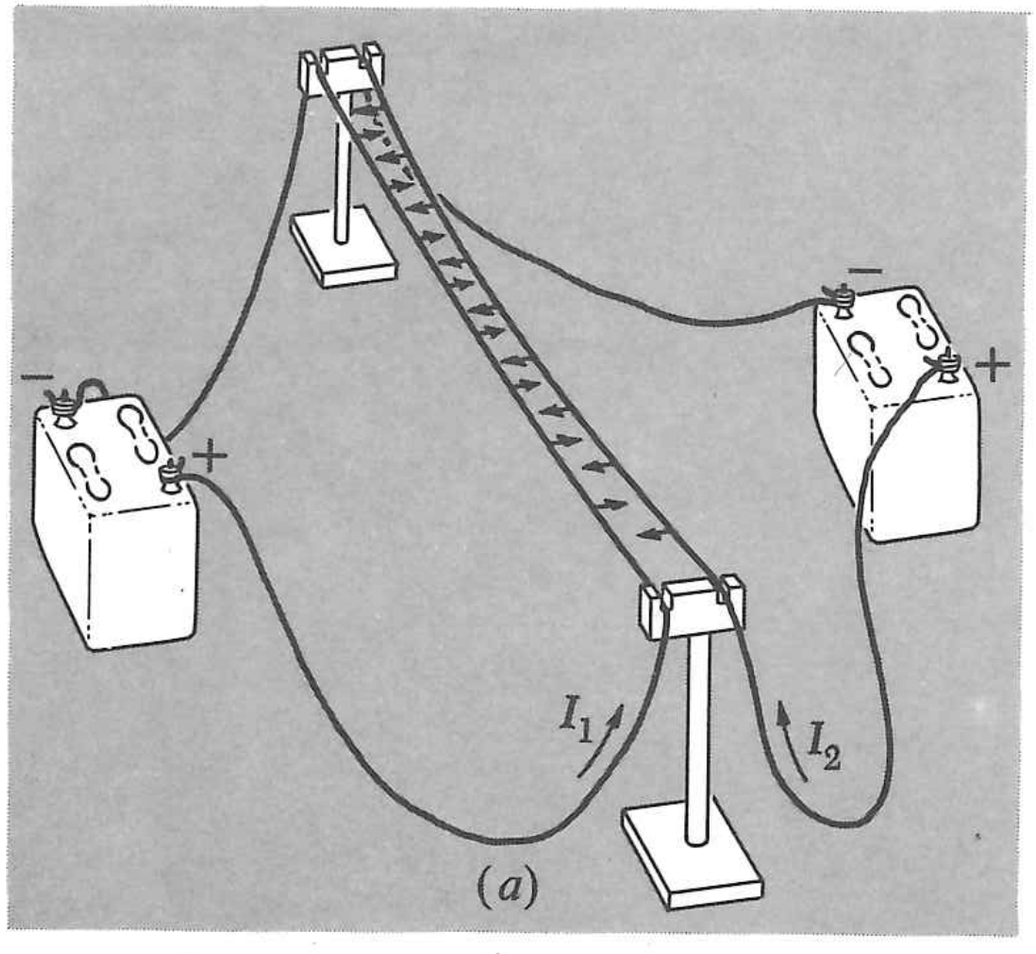


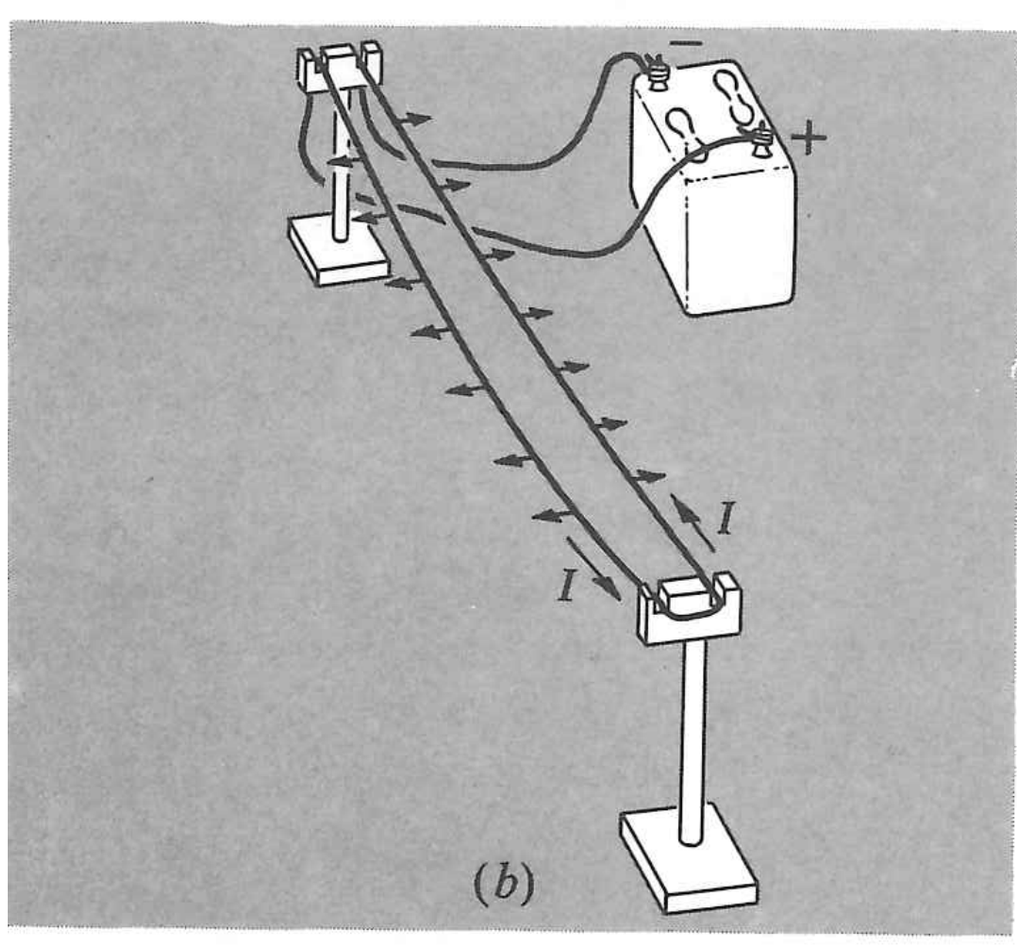
$$E'_{||} = E_{||},$$
$$E'_{\perp} = \gamma E_{||}$$

Analyze the wire problem… … without using B!

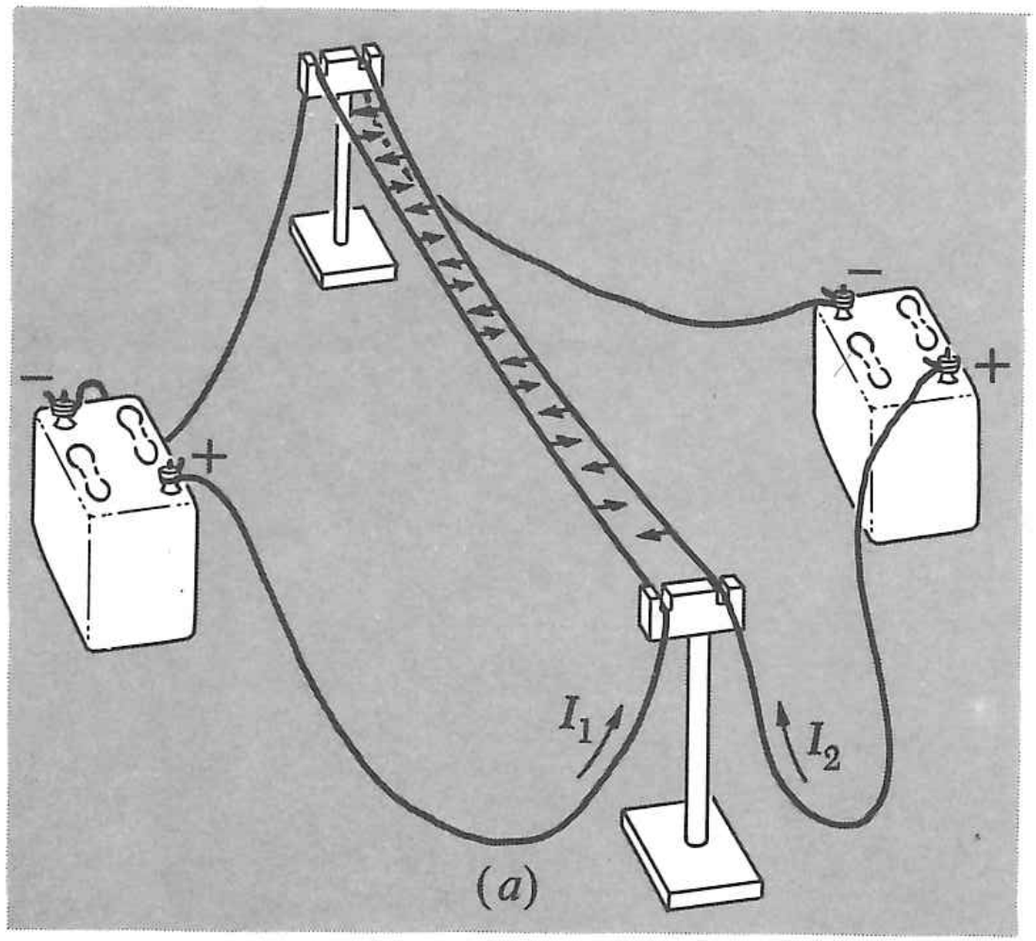
- Go to frame where v = 0 to compute F:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

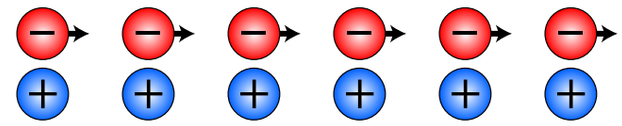


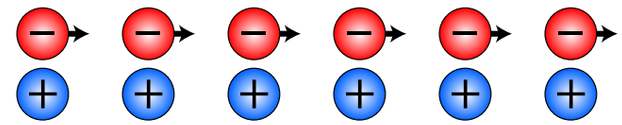


Same direction…

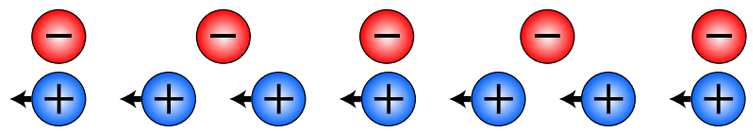


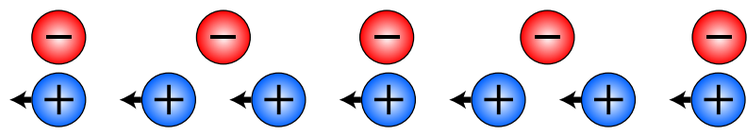
- Positive charge Frame:



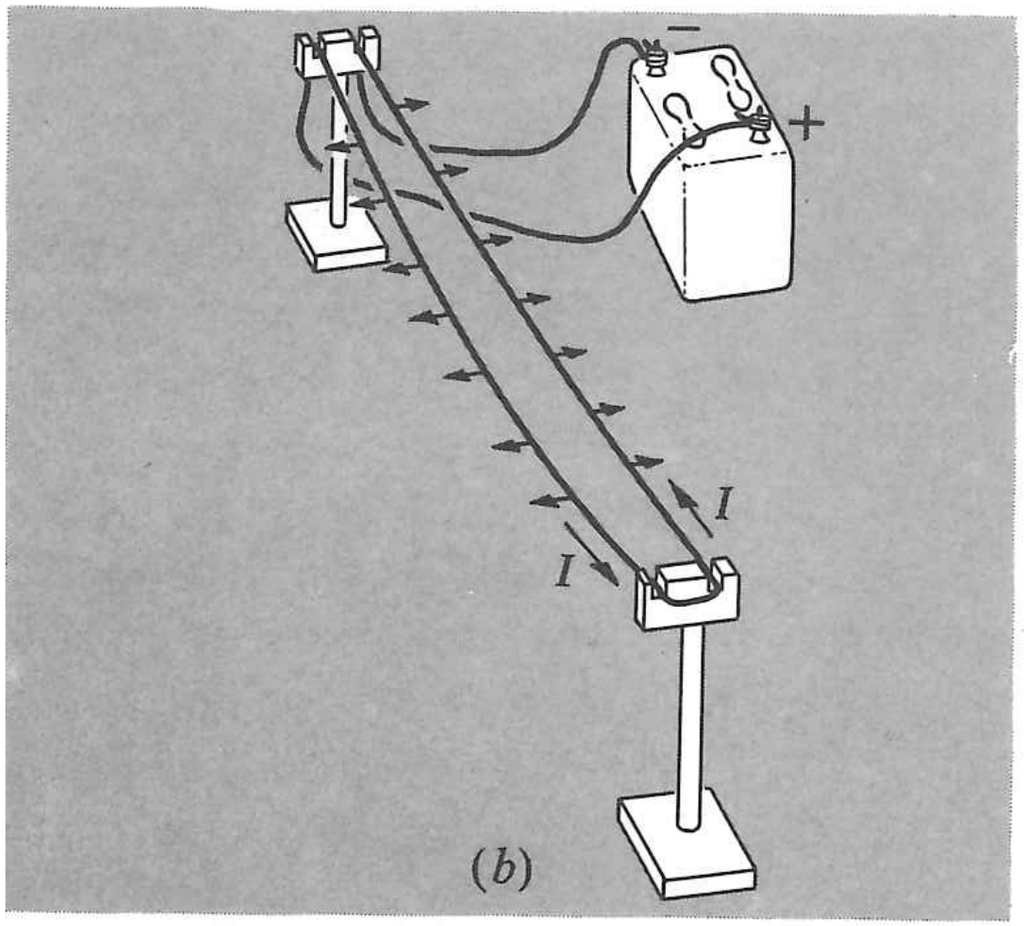


- Negative charge Frame

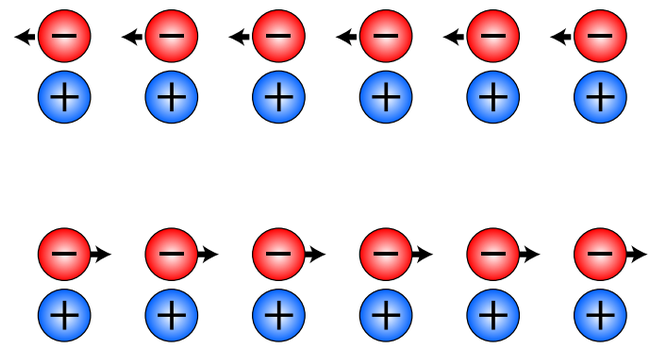




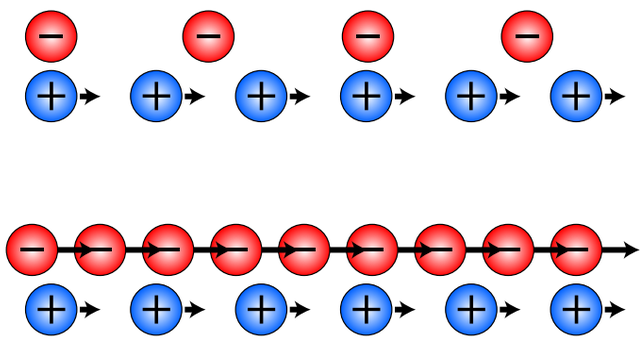
Opposite direction…



- Positive charge Frame:



- Negative charge Frame



Magnetic field B

- Relativistic correction to E field.
- Small correction when $v \ll c$
- … except when all net charges are zero.
- Natural geometric representation: E and B are both part of the field strength tensor:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$