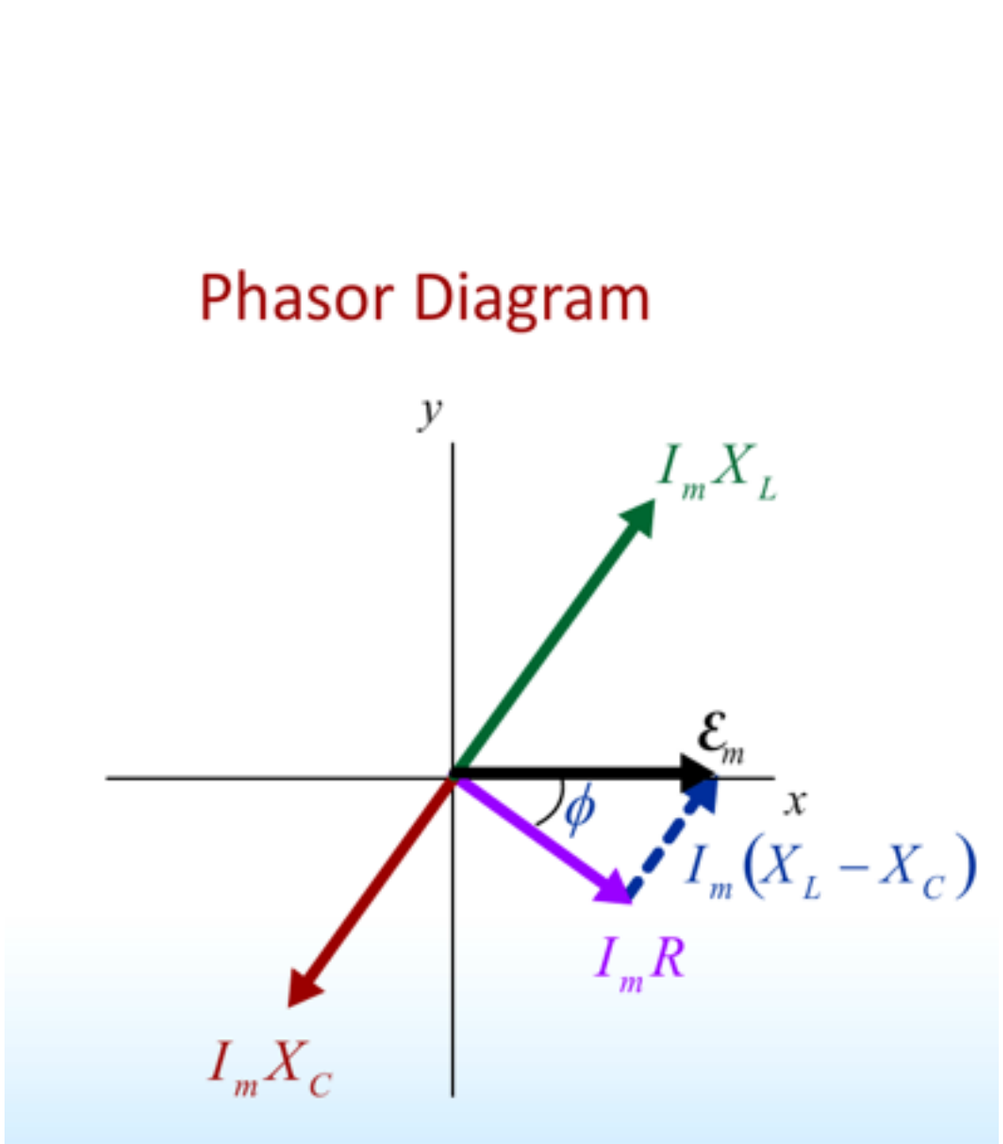
Resonance & Transformers

Lecture 27

Announcements

- Reading for Wednesday: 30-1 to 30-2
- Exam pickup: Today
- HW: dropped last three problems
- Instead: Please do course evaluations!

Review: LRC driven circuits



- Unknowns:

$$I_{\max}, \quad \phi$$

- Choose unknowns such that:

$$\vec{V}_R + \vec{V}_C + \vec{V}_L = \vec{\mathcal{E}}$$

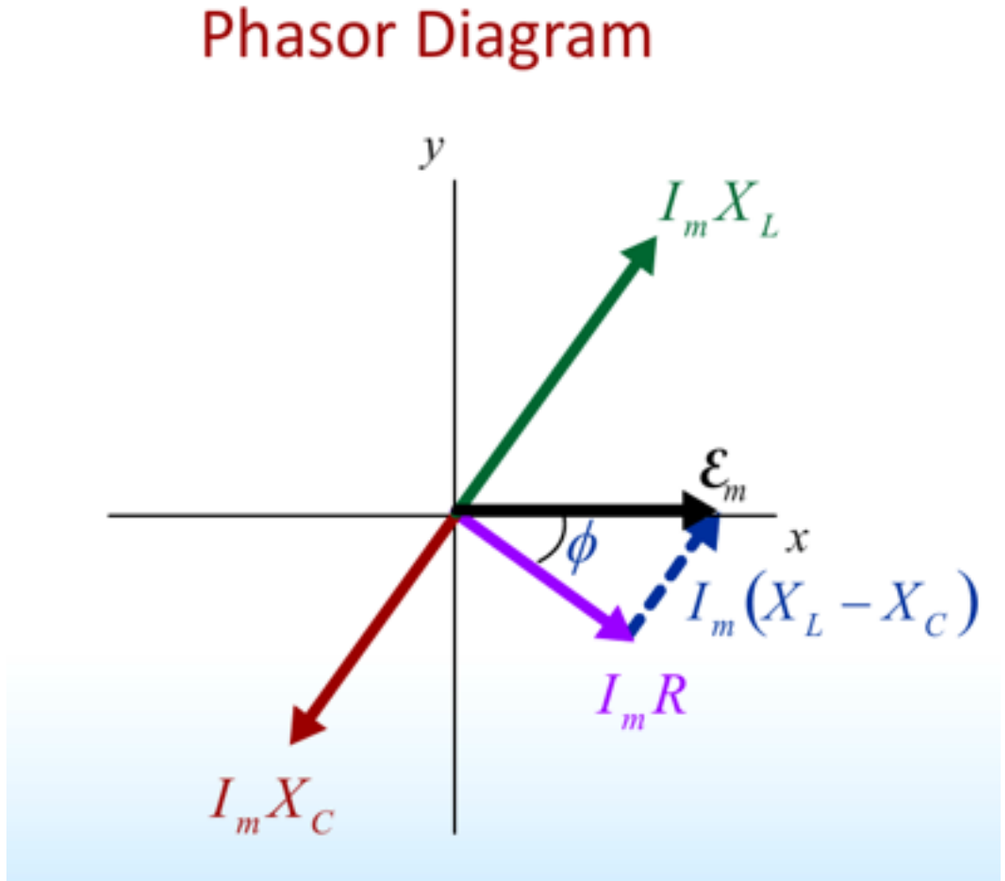
- Result:

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance

- Define Impedance:

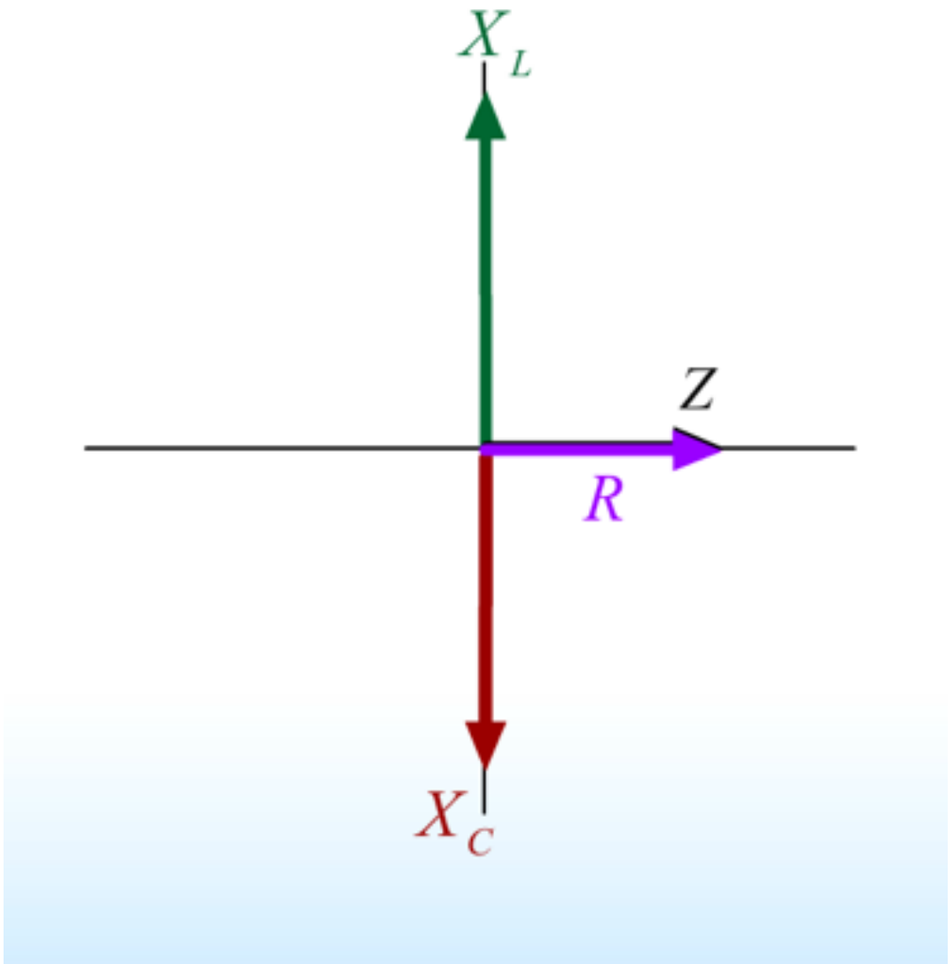


$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{Z}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

Resonance



- Maximize current (power):
- Minimize impedance:

$$I_{\max} = \frac{\mathcal{E}_{\max}}{Z}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

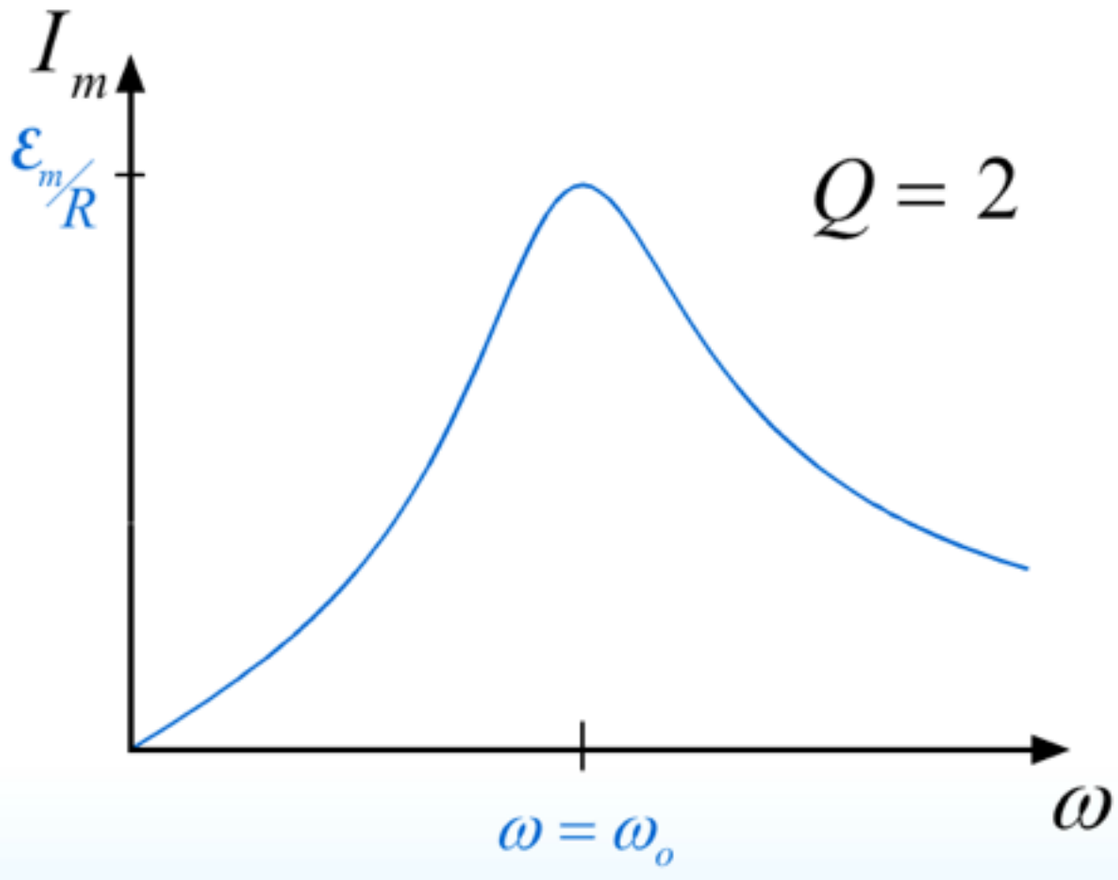
$$X_L = X_C$$

$$\omega = \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

Resonance

L

$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

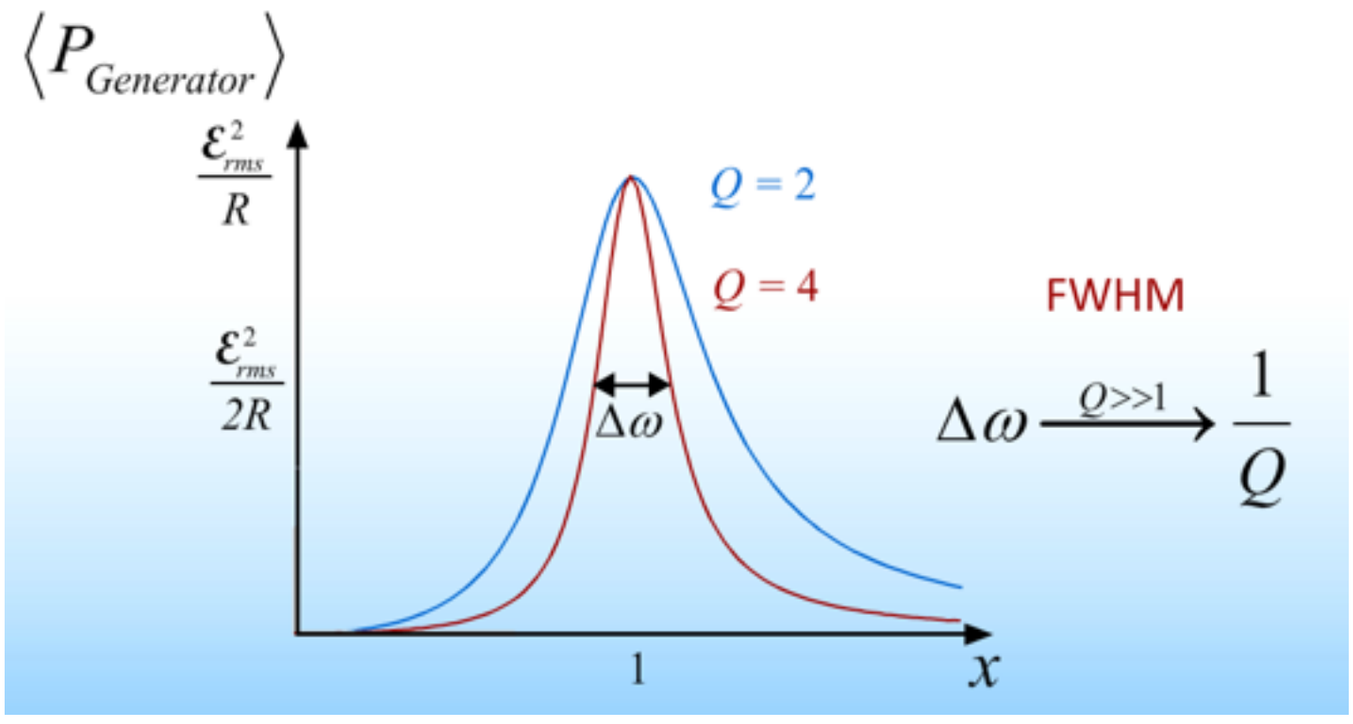


$$I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \dfrac{\left(x^2 - 1\right)^2}{x^2}}}$$

Resonance

Average Power per Cycle

$$\left\langle P_{Generator} \right\rangle = \frac{\mathcal{E}_{rms}^{2}}{R} \frac{x^2}{x^2 + Q^2\left(x^2 - 1\right)^2}$$



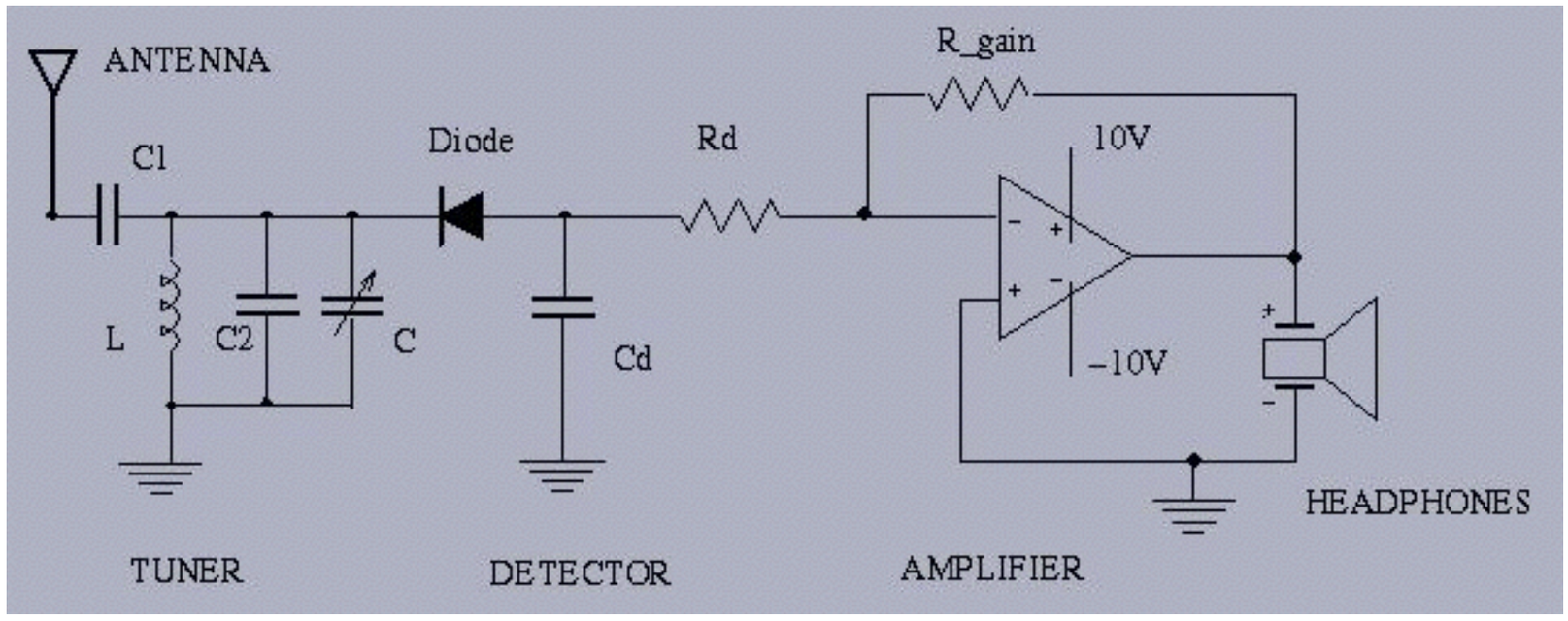
- Root mean square values

$$\mathcal{E}_{\mathrm{rms}} = \frac{1}{\sqrt{2}}\mathcal{E}_{\mathrm{max}}$$

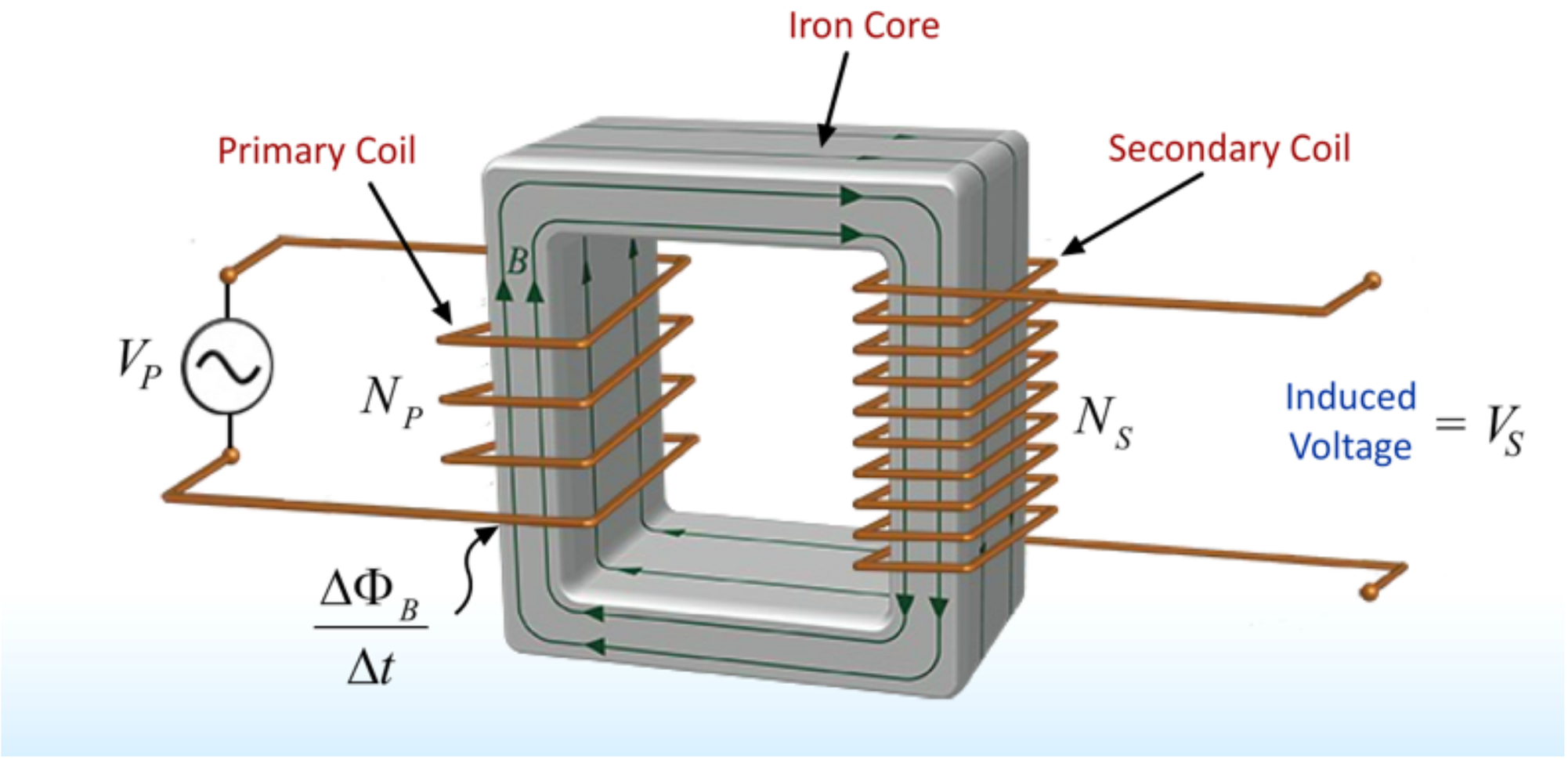
$$I_{\mathrm{rms}} = \frac{1}{\sqrt{2}}I_{\mathrm{max}}$$

- Quality controls the width of the peak…

AM Radio

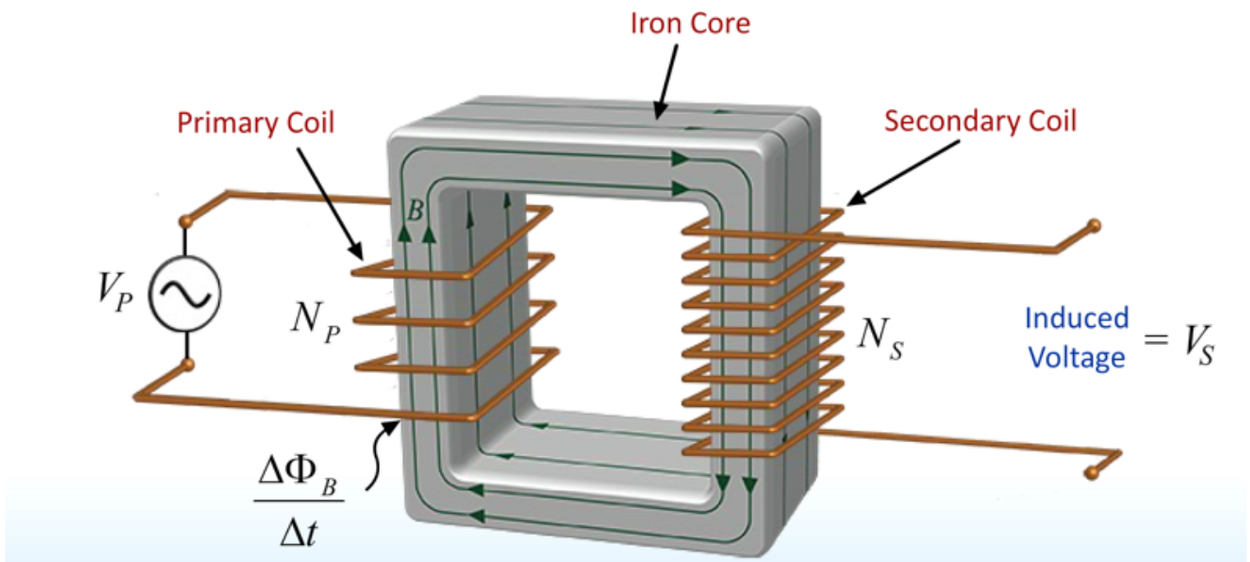

ANTENNA
C1
Diode
Rd
R_gain
10V
L
C2
C
Cd
−10V
HEADPHONES
TUNER
DETECTOR
AMPLIFIER

Transformers



- Ideal: Ignore internal resistance…

Transformers



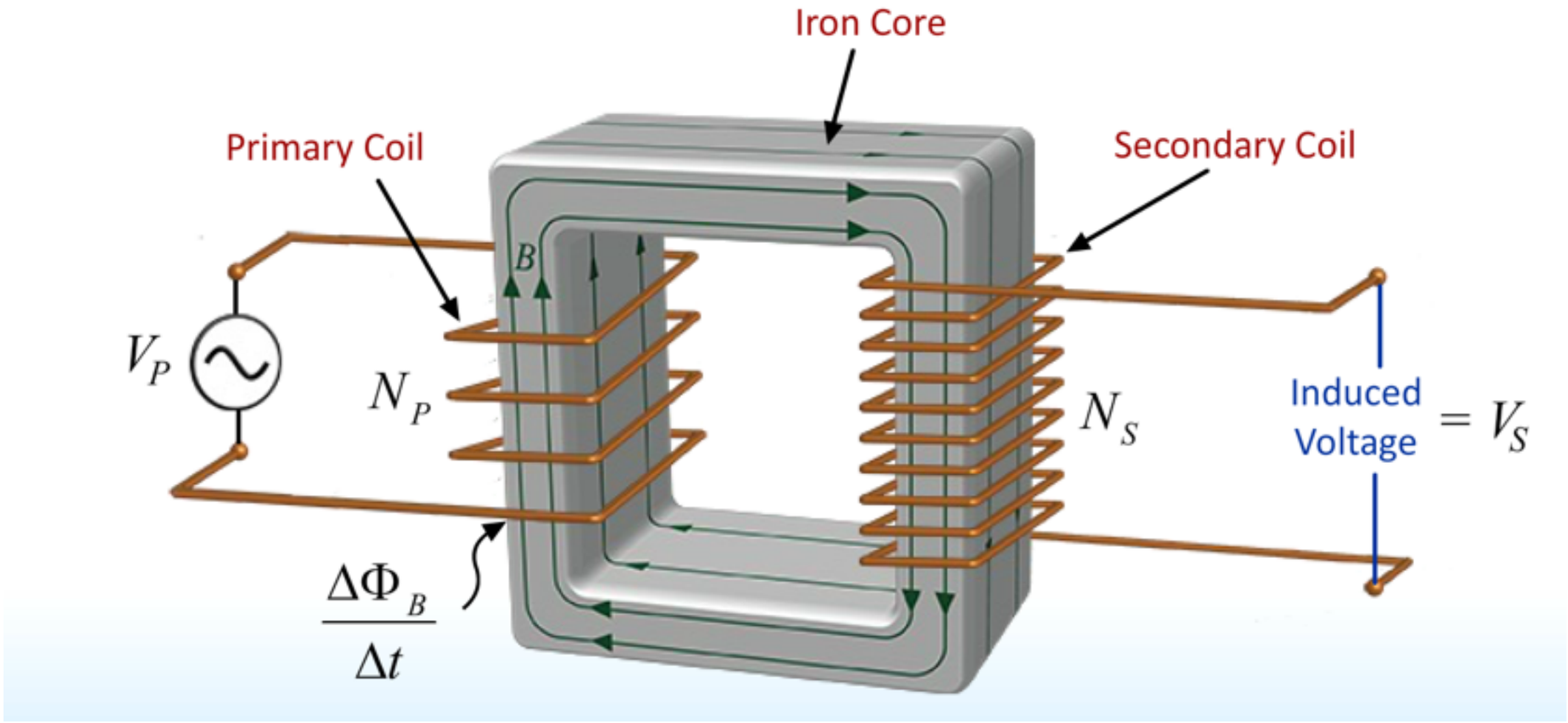
- Complicated… but:

$$\Phi_B^{(P)}(t) = N_P \Phi_B^0(t)$$

$$\Phi_B^{(S)}(t) = N_S \Phi_B^0(t)$$

- Conservation of energy: $P = IV$

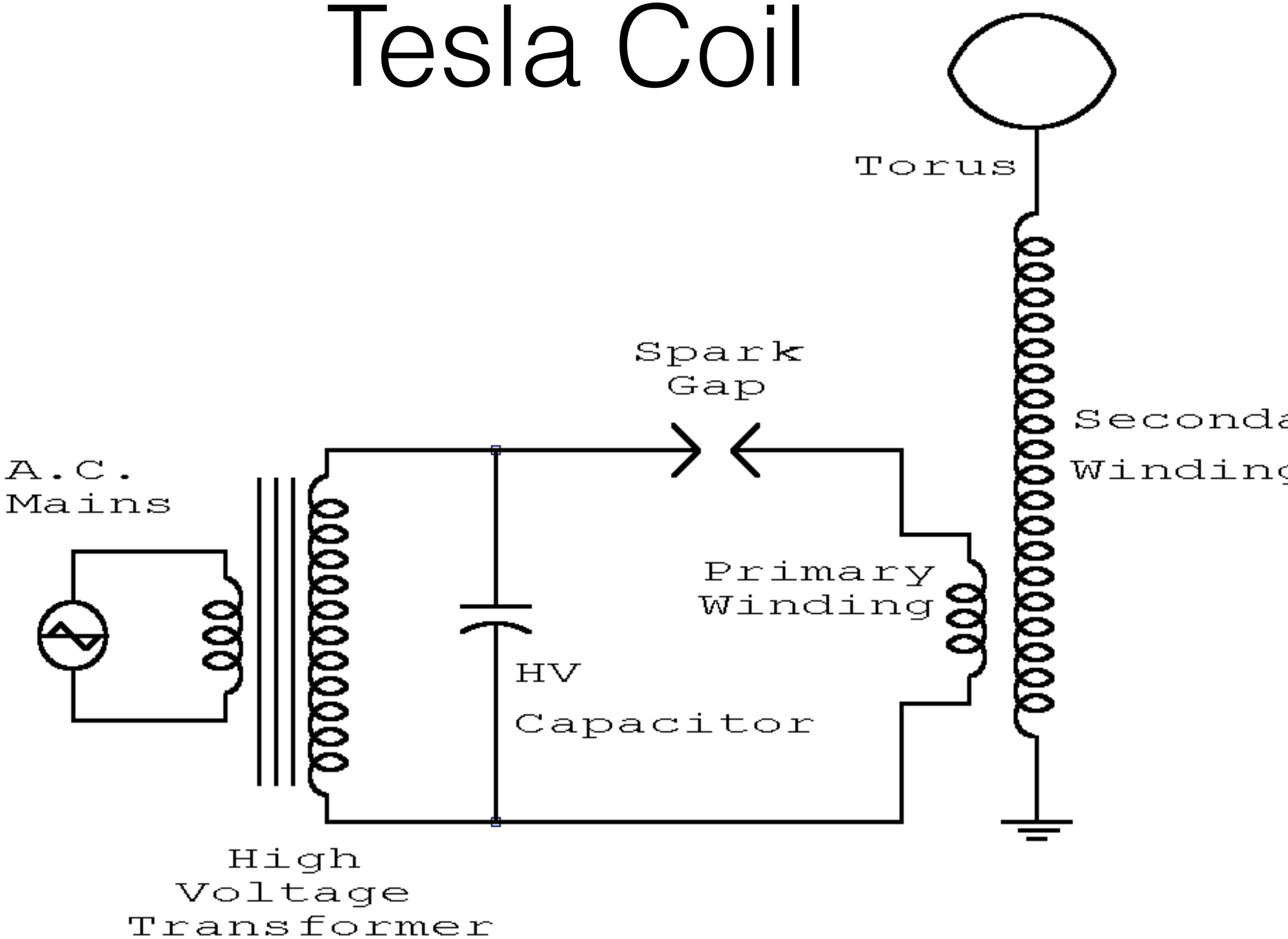
Key Transformer Eqns



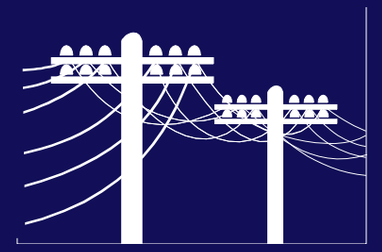
$$V_s = \frac{N_s}{N_p} V_p$$

$$I_s = \frac{N_p}{N_s} I_p$$

Tesla Coil


Torus
Spark
Gap
Seconda
Winding
A.C.
Mains
Primary
Winding
HV
Capacitor
High
Voltage
Transformer

Power Transmission



- **Why use high-voltage to transport electricity ?**
  - **At home: 120V AC at 60Hz.**
  - **Transmission typically ~500 kV)**
  - **Transformers raise voltage for transmission; lower for use**
- **Why do we do that?**
  - **Calculate ohmic losses in the transmission lines:**
  - **Define *efficiency* of transmission:**

$$\varepsilon ff \equiv \frac{P_{out}}{P_{in}} = \frac{iV_{in} - i^2R}{iV_{in}} = 1 - \frac{iR}{V_{in}}\left(\frac{V_{in}}{V_{in}}\right) = 1 - \frac{P_{in}R}{V^2{}_{in}}$$

Keep R small

Make V_{in} big