LC Circuits:

$$\frac{Q(t)}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$0 = \left[\left(\frac{d}{dt}\right)^2 + \frac{1}{LC}\right] Q(t)$$

Solutions: $A \sin \omega t + B \cos \omega t$

$$\left[-\omega^2 + \frac{1}{LC}\right] Q(t) = 0$$

$$\omega = \frac{1}{\sqrt{LC}} \quad //.$$

AC Circuits:

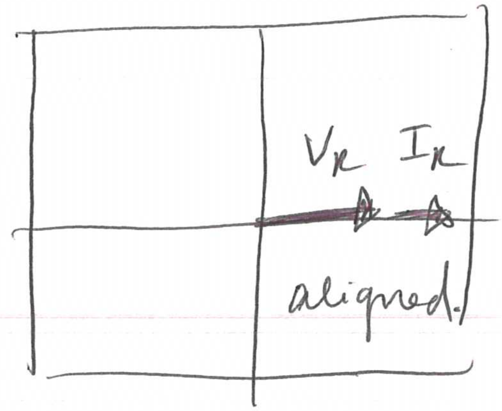
AC voltage source: $\mathcal{E}(t) = \mathcal{E}_{max} \sin \omega t.$

$$\vec{V}(t) = \mathcal{E}_{max}\left[\sin \omega t \; \hat{j} + \cos \omega t \; \hat{i}\right]$$

$$V(t) \equiv \hat{j} \cdot \vec{V}(t).$$

Resistor:

$$I_R = \frac{V_R}{R} =$$



$$\vec{I}_R = \frac{\vec{V}_R}{R} = \frac{\mathcal{E}_{max}}{R}\left[\sin\omega t\,\hat{j} + \cos\omega t\,\hat{i}\right]$$

Define reactance: $X_R \equiv R$.

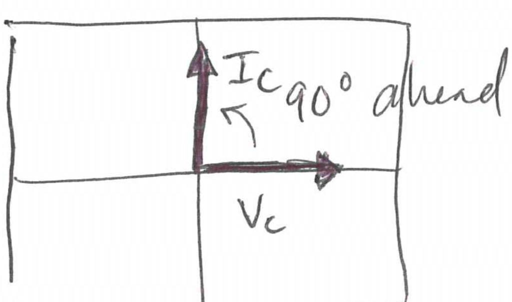
Capacitor:

$$Q_c = CV_c \quad \rightarrow \quad I_c = C\frac{dV_c}{dt}$$

$$\vec{I}_c = \omega C\,\mathcal{E}_{max}\left[\cos\omega t\;\hat{j} - \sin\omega t\;\hat{i}\right]$$

$$= \frac{\mathcal{E}_{max}}{X_c}\left[\sin\left(\omega t + \frac{\pi}{2}\right)\hat{j} + \cos\left(\omega t + \frac{\pi}{2}\right)\hat{i}\right]$$

$$X_c \equiv \frac{1}{\omega C}$$



$$\boxed{\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta + {} - \sin\alpha\sin\beta \end{aligned}}$$

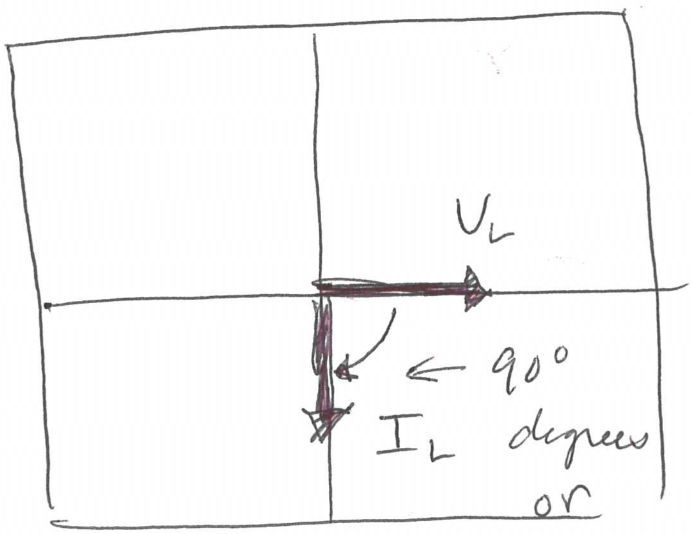
Inductor:

$$V_L = L \frac{dI_L}{dt}$$

$$I_L = \frac{1}{L} \int dt \, V_L$$

$$= + \frac{\mathcal{E}_{max}}{L\omega} \left[-\cos \omega t \, \hat{j} + \sin \omega t \, \hat{i} \right]$$

$$= \frac{\mathcal{E}_{max}}{X_L} \left[\cos\left(\omega t - \frac{\pi}{2}\right) \hat{j} + \sin\left(\omega t - \frac{\pi}{2}\right) \hat{i} \right]$$



I_L $90°$ degrees behind.

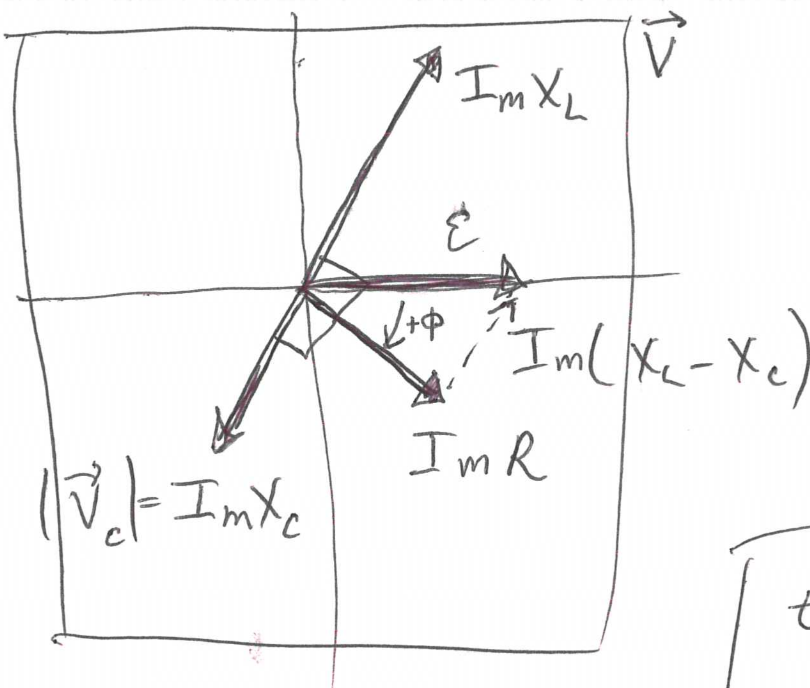
or

V_L is $90°$ ahead of I_L

Putting it all together:

$$\vec{V}_R + \vec{V}_C + \vec{V}_L = \vec{\mathcal{E}}$$

unknown: $\vec{I}$: I_{max} + ϕ ← phase.



$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

$$\mathcal{E}_{max}^2 = (I_m R)^2 + I_m^2 (X_L - X_C)^2$$

$$\boxed{I_m = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}}$$