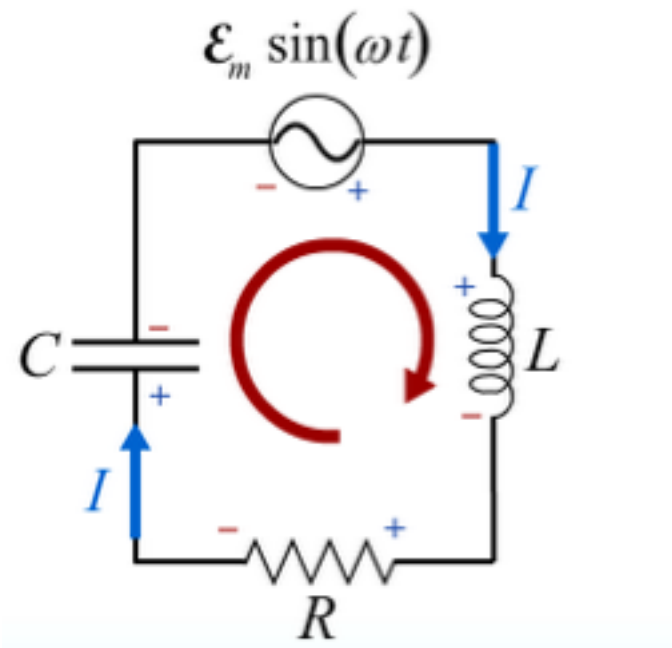
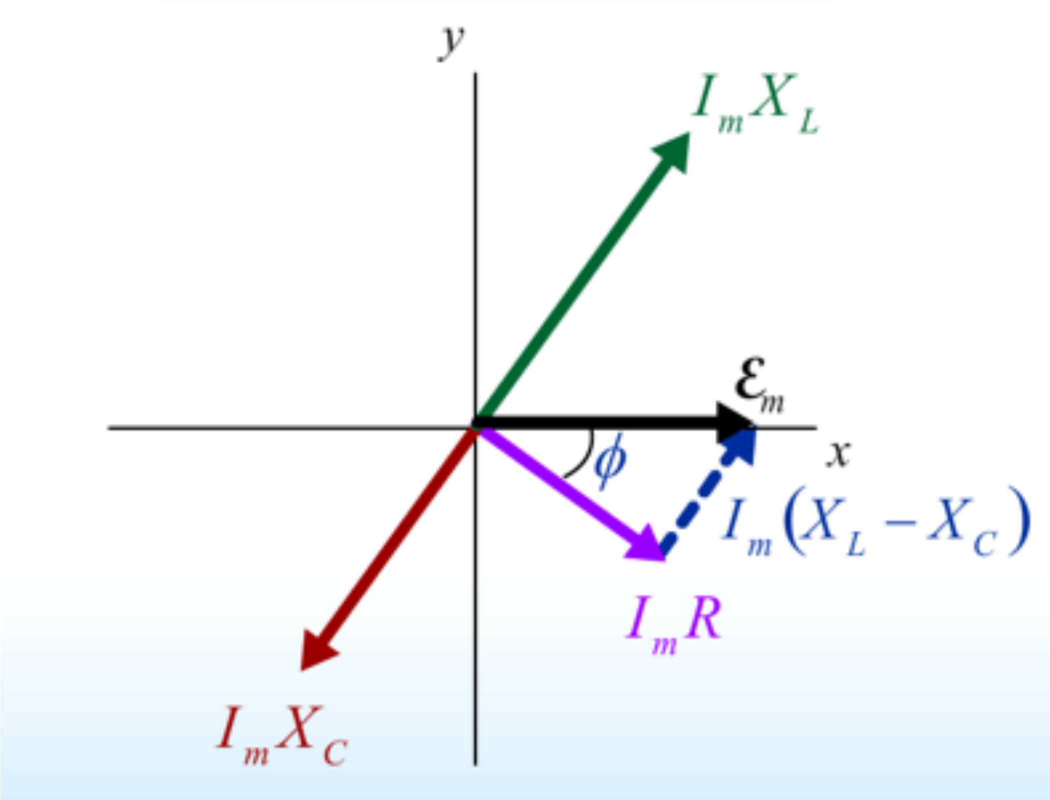
Resonance & Transformers

Lecture 26

Review: LRC driven circuits





- Unknowns:

$$I_{\max}, \quad \phi$$

- Choose unknowns such that:

$$\vec{V}_R + \vec{V}_C + \vec{V}_L = \vec{\mathcal{E}}$$

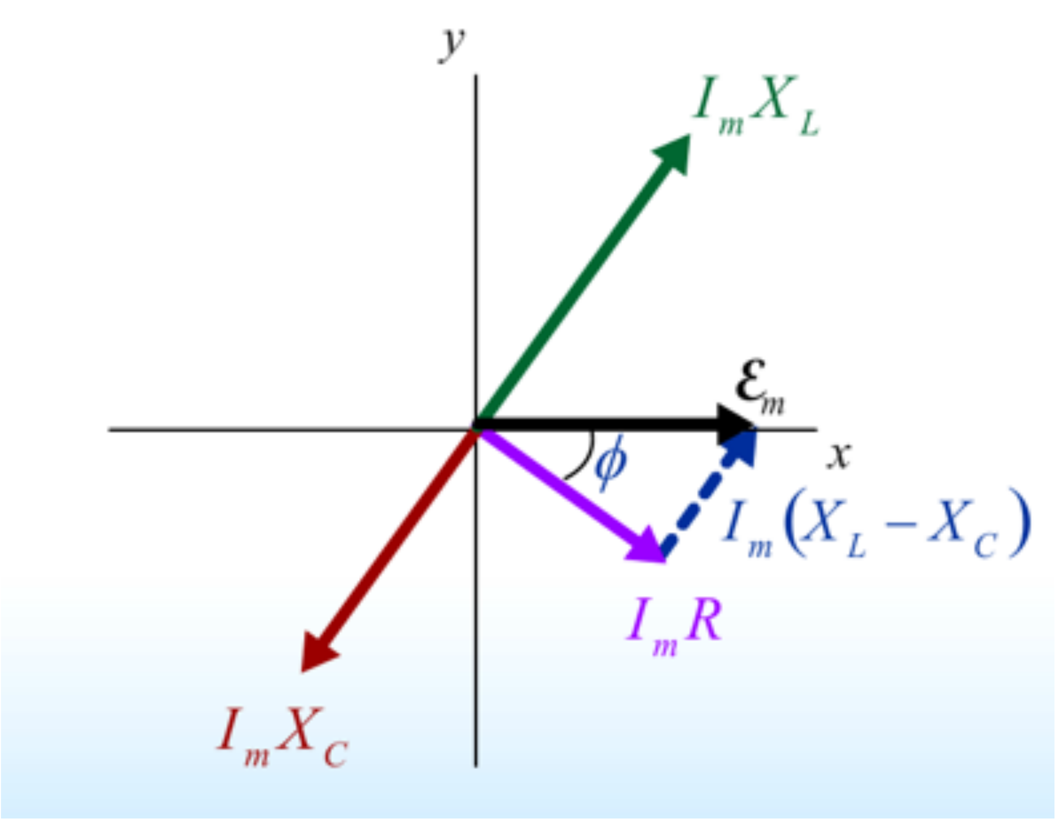
- Result:

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance

Phasor Diagram



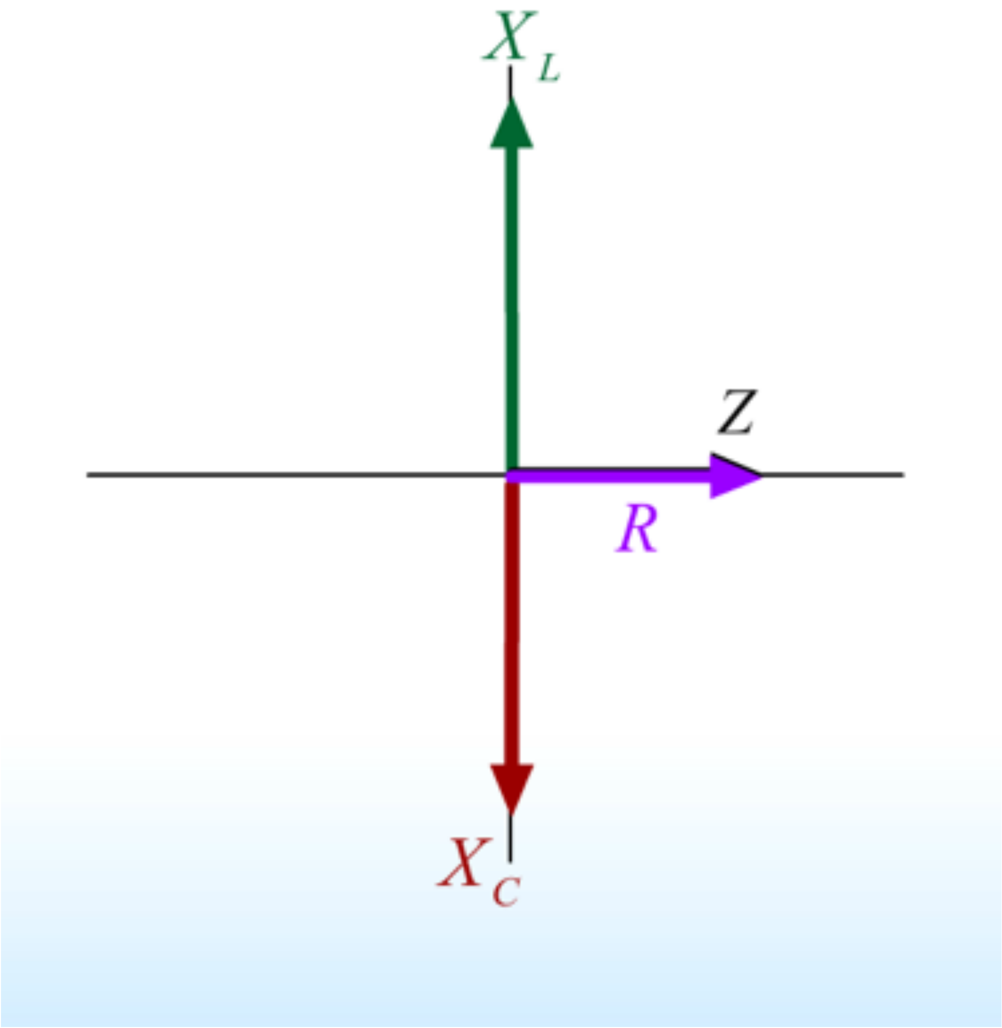
- Define Impedance:

$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{Z}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

Resonance



- Maximize current (power):

- Minimize impedance:

$$I_{\max} = \frac{\mathcal{E}_{\max}}{Z}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

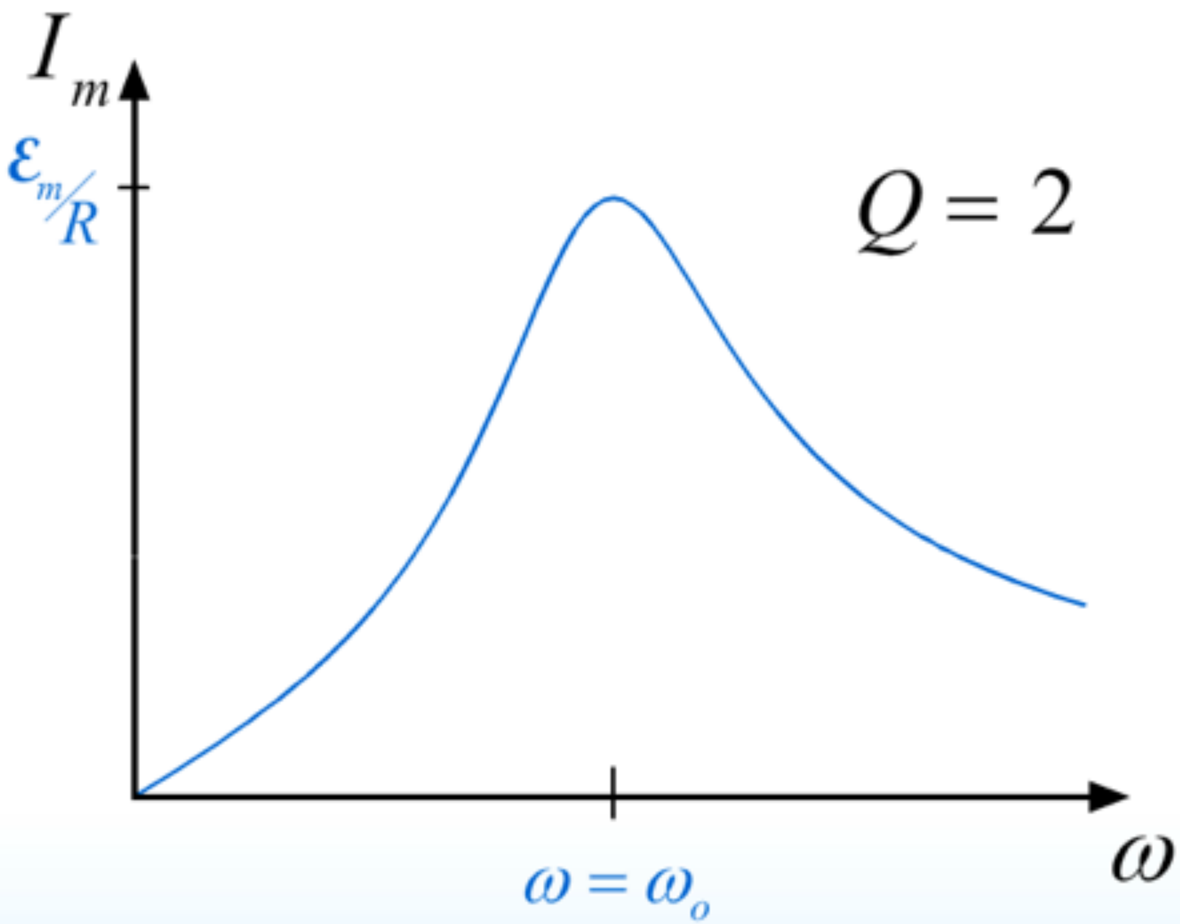
$$X_L = X_C$$

$$\omega = \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

Resonance

L

$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

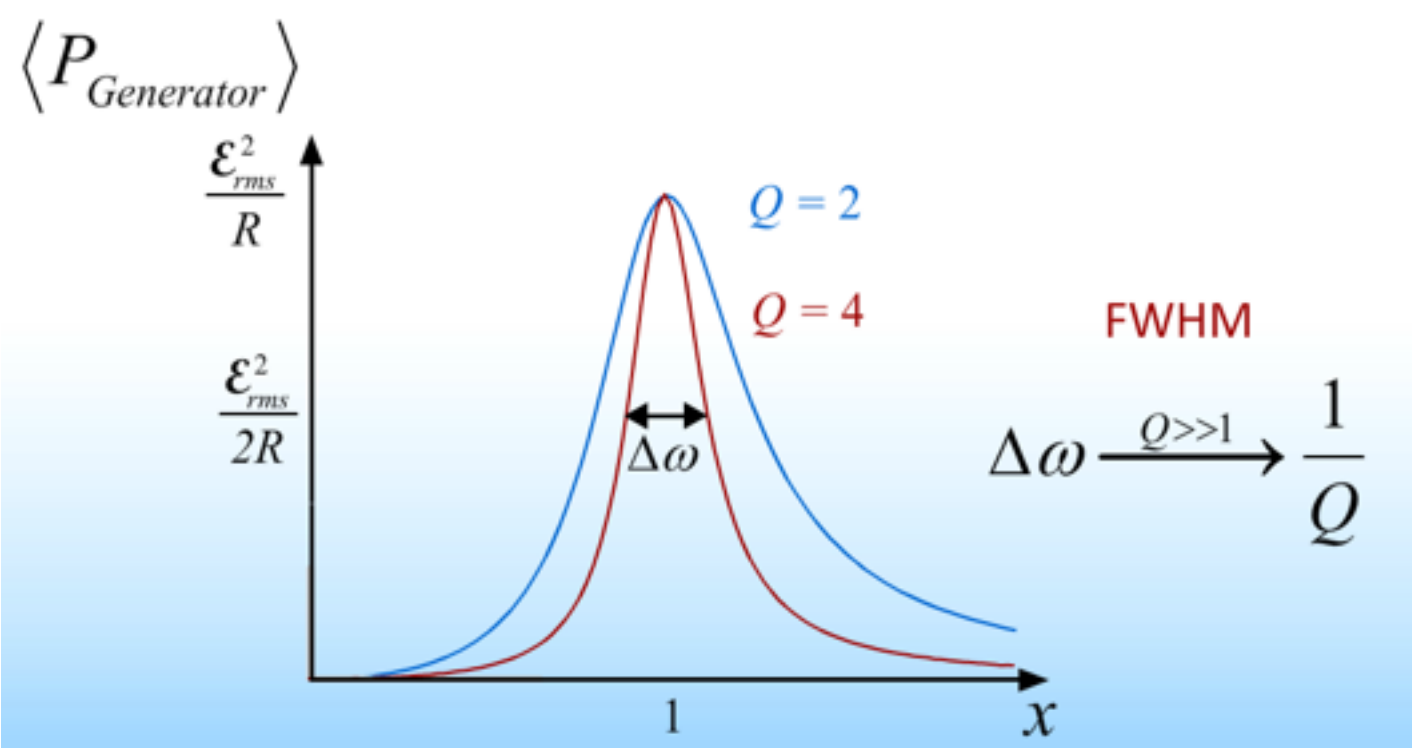


$$I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{\left(x^2 - 1\right)^2}{x^2}}}$$

Resonance

Average Power per Cycle

$$\langle P_{Generator} \rangle = \frac{\mathcal{E}_{rms}^{2}}{R} \frac{x^2}{x^2 + Q^2(x^2-1)^2}$$

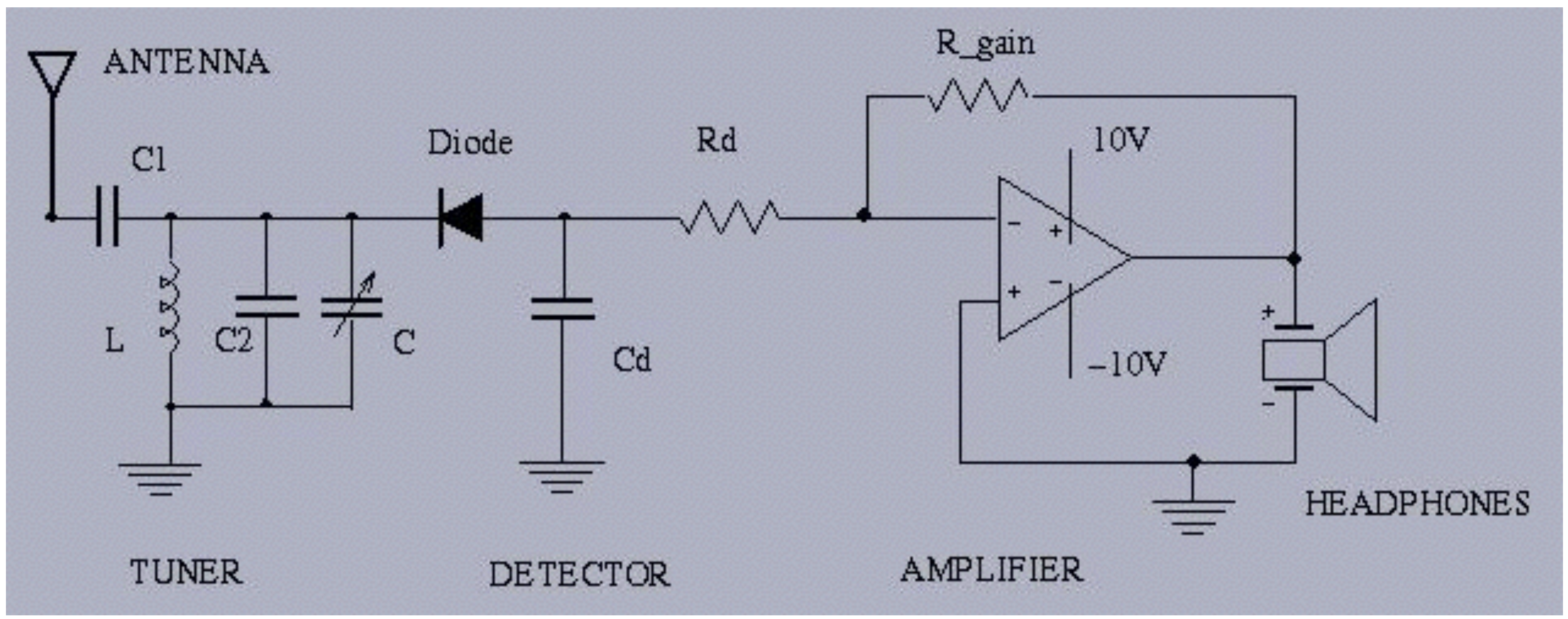


- Root mean square values

$$\mathcal{E}_{\mathrm{rms}} = \frac{1}{\sqrt{2}}\mathcal{E}_{\mathrm{max}}$$
$$I_{\mathrm{rms}} = \frac{1}{\sqrt{2}}I_{\mathrm{max}}$$

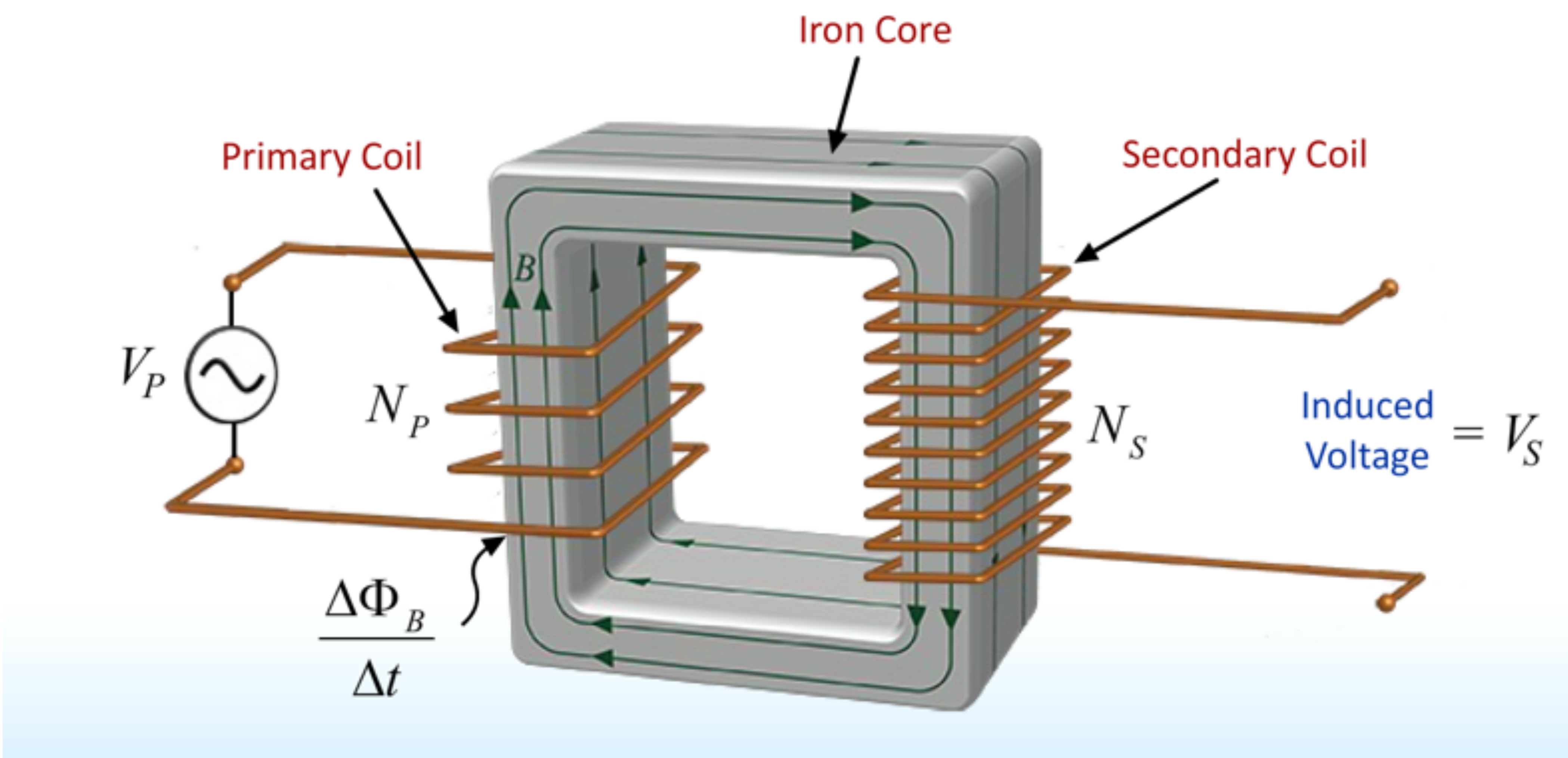
- Quality controls the width of the peak…

AM Radio



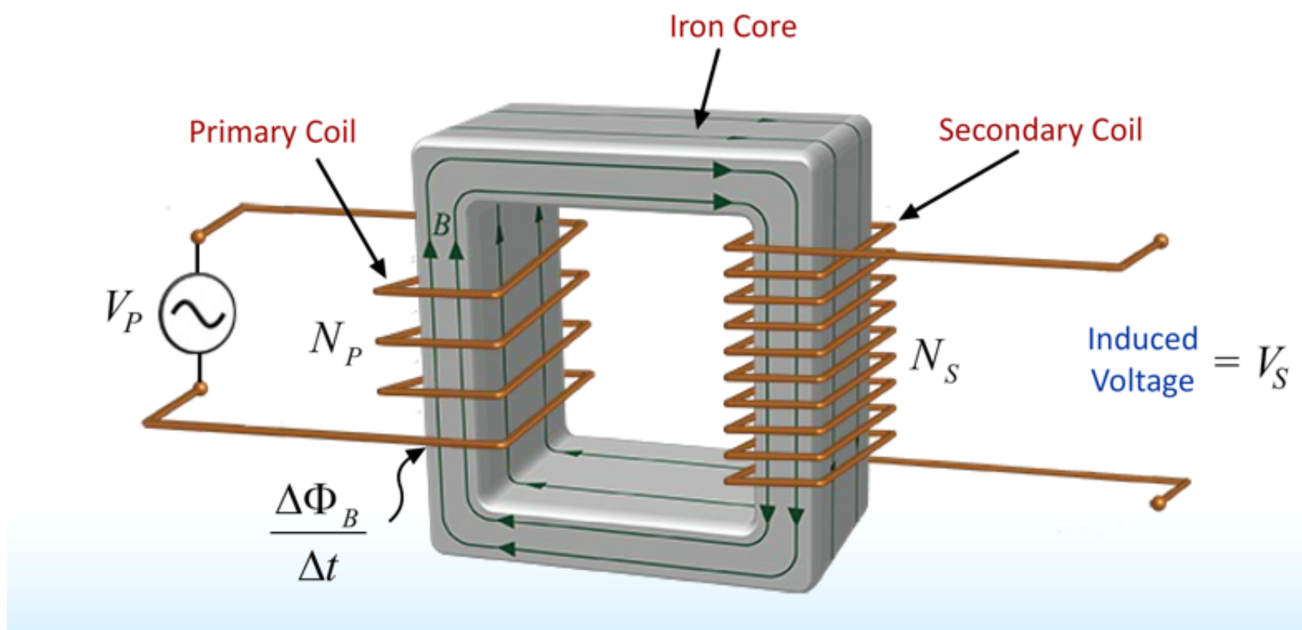
ANTENNA
C1
Diode
Rd
R_gain
10V
−10V
L
C2
C
Cd
HEADPHONES
TUNER
DETECTOR
AMPLIFIER

Transformers



- Ideal: Ignore internal resistance…

Transformers



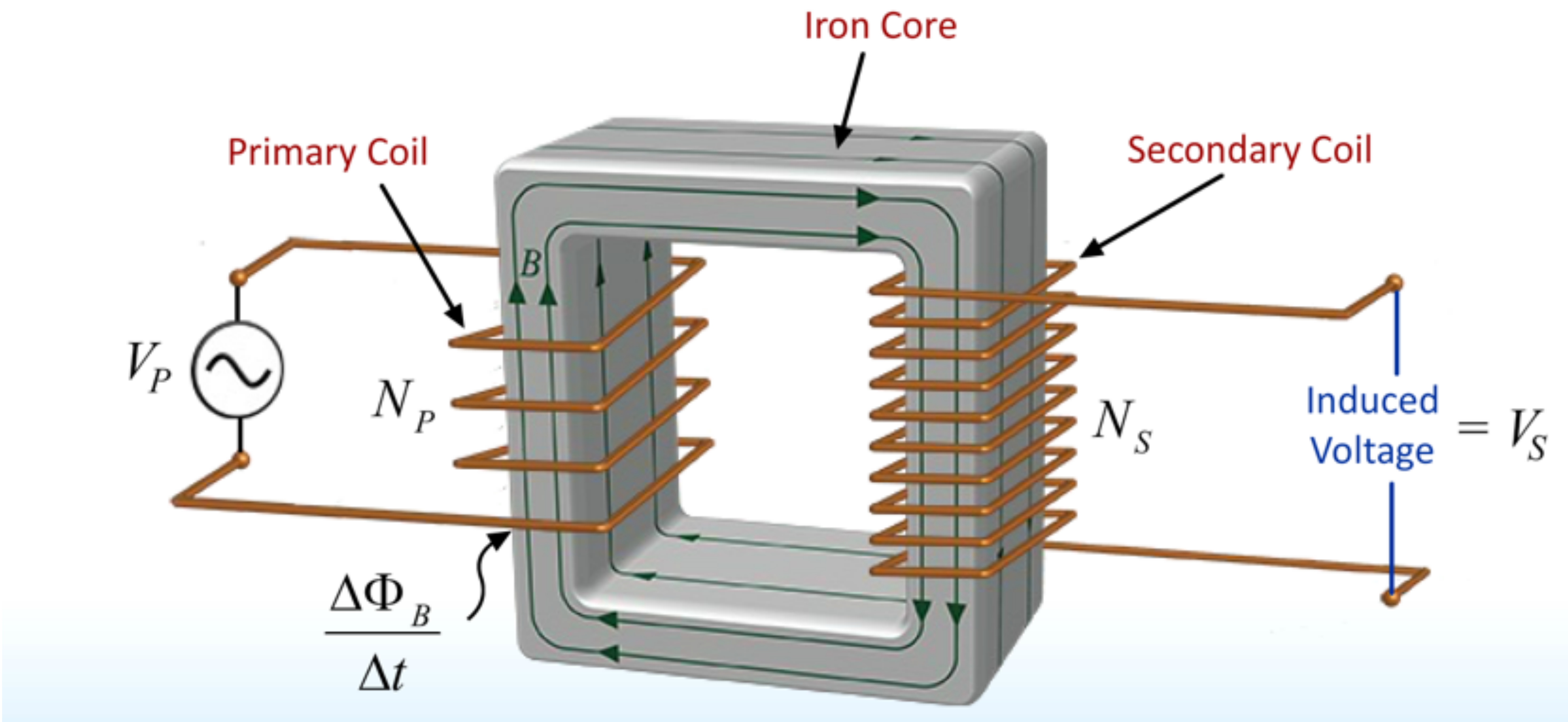
- Complicated… but:

$$\Phi_B^{(P)}(t) = N_P \Phi_B^0(t)$$

$$\Phi_B^{(S)}(t) = N_S \Phi_B^0(t)$$

- Conservation of energy: $P = IV$

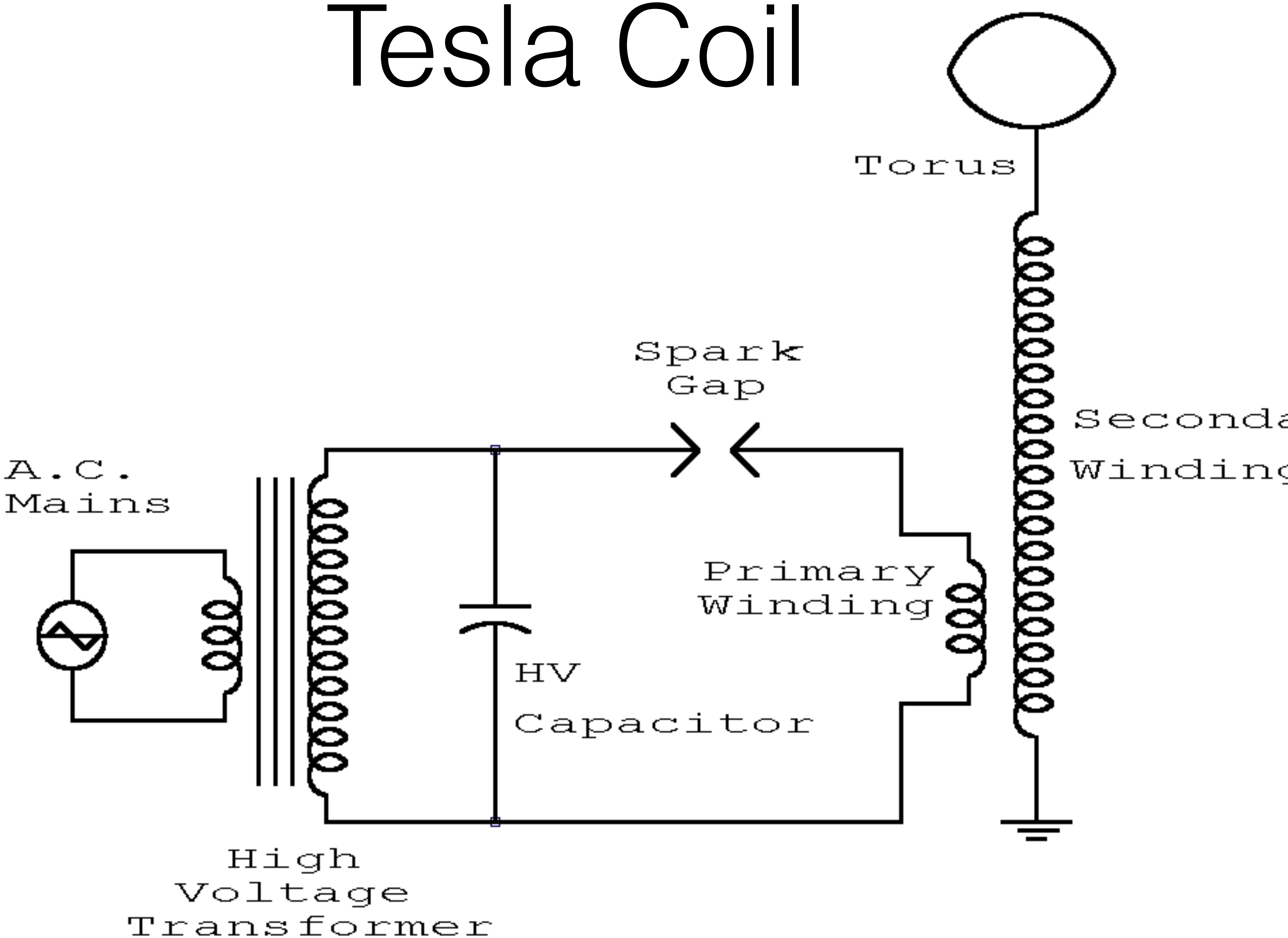
Key Transformer Eqns



$$V_s = \frac{N_s}{N_p} V_p$$

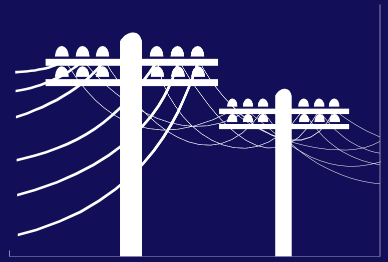
$$I_s = \frac{N_p}{N_s} I_p$$

Tesla Coil



Torus
Spark
Gap
Seconda
Winding
A.C.
Mains
Primary
Winding
HV
Capacitor
High
Voltage
Transformer

Power Transmission



- Why use high-voltage to transport electricity ?
  - At home: 120V AC at 60Hz.
  - Transmission typically ~500 kV)
  - Transformers raise voltage for transmission; lower for use
- Why do we do that?
  - Calculate ohmic losses in the transmission lines:
  - Define *efficiency* of transmission:

$$\varepsilon ff \equiv \frac{P_{out}}{P_{in}} = \frac{iV_{in} - i^2 R}{iV_{in}} = 1 - \frac{iR}{V_{in}}\left(\frac{V_{in}}{V_{in}}\right) = 1 - \frac{P_{in} R}{V^2_{in}}$$

Keep R small

Make V_{in} big