RLC & AC circuits

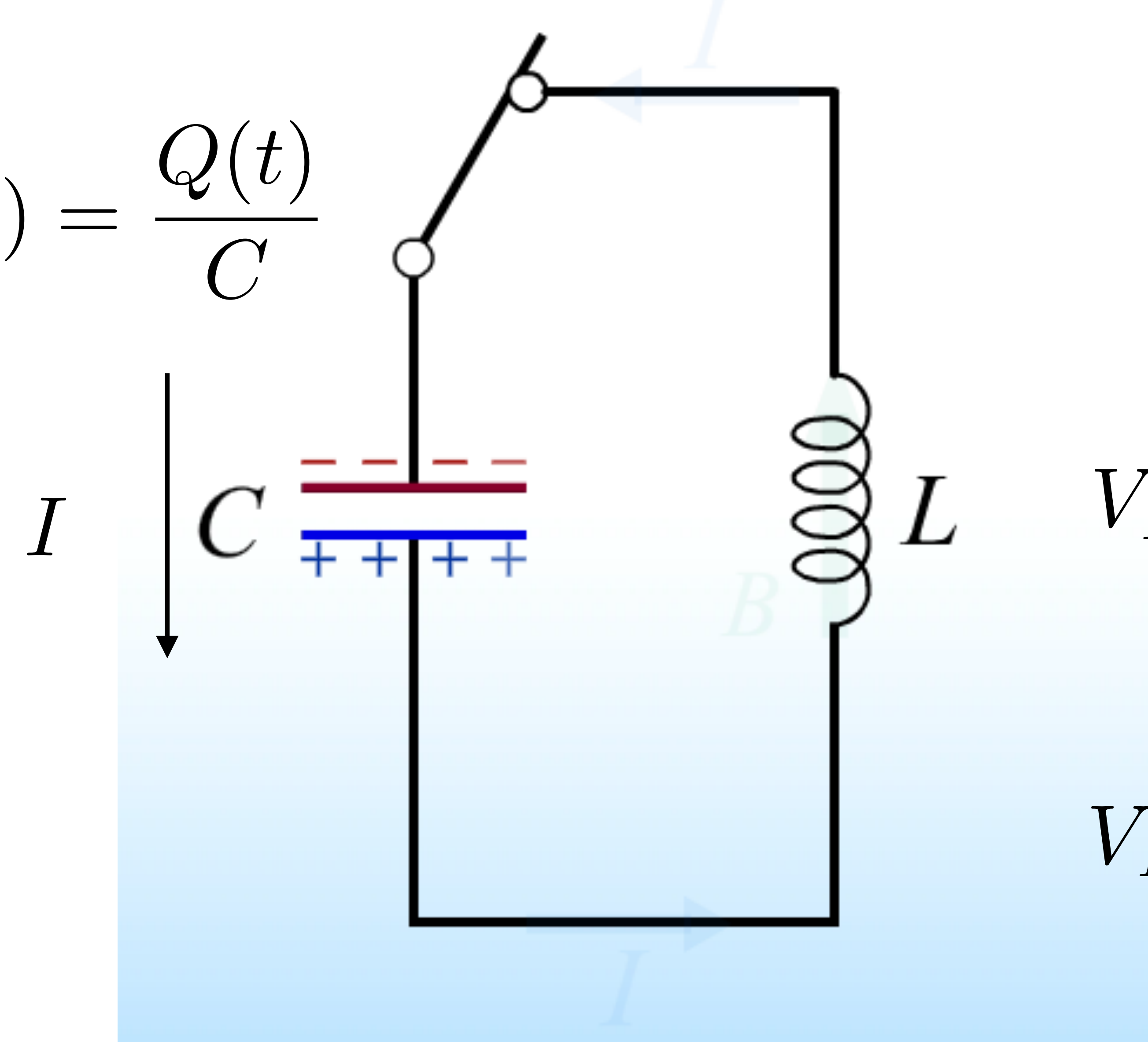
Lecture 26

Announcements

- Reading for Wednesday: 29-5 to 29-6
- Exam pickup: Wednesday

LC Circuits

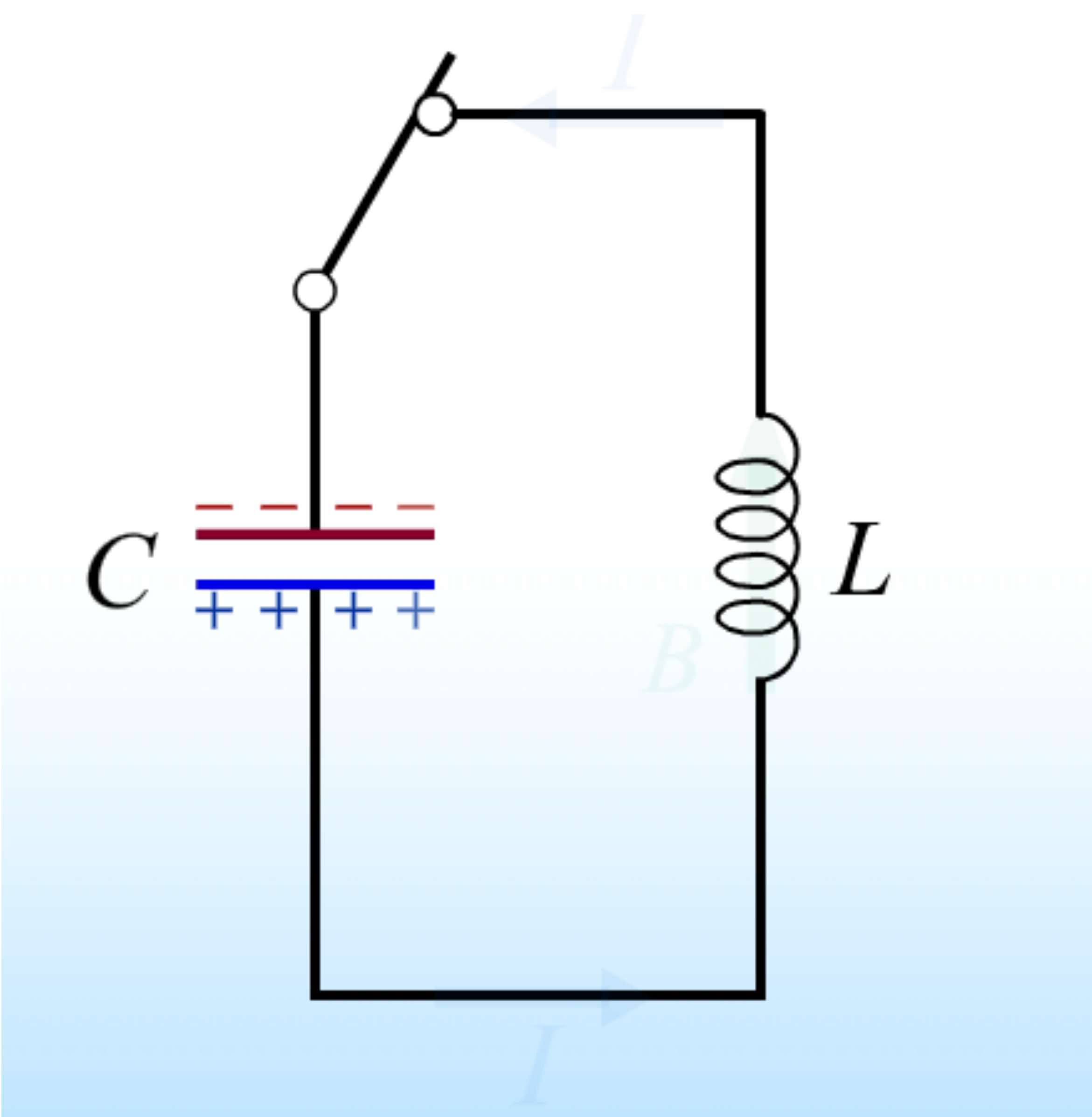
$$V_C(t) = \frac{Q(t)}{C}$$



$$V_L(t) = L\frac{dI(t)}{dt}$$

$$V_L(t) = L\frac{d^2Q(t)}{dt^2}$$

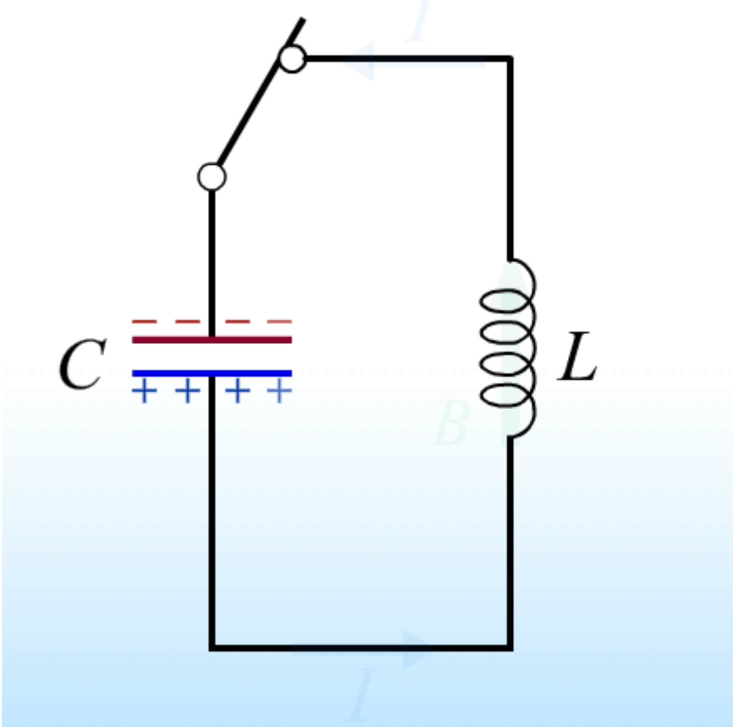
LC Circuits: Detailed Analysis

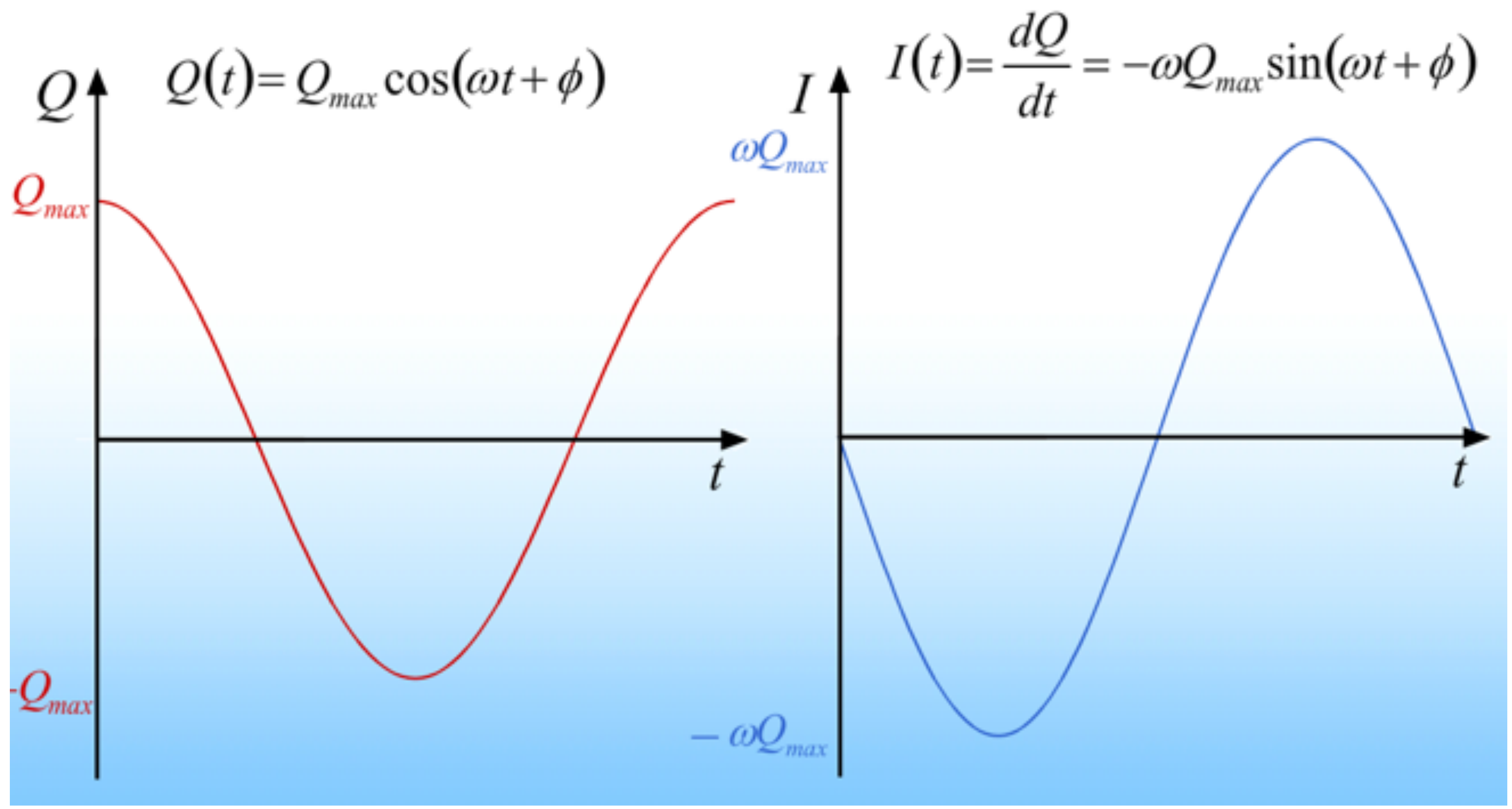


$$\text{KVL}: \quad V_L(t) + V_C(t) = 0$$

$$0 = \frac{Q(t)}{C} + L\frac{d^2Q(t)}{dt^2}$$

LC Circuits: Detailed Analysis

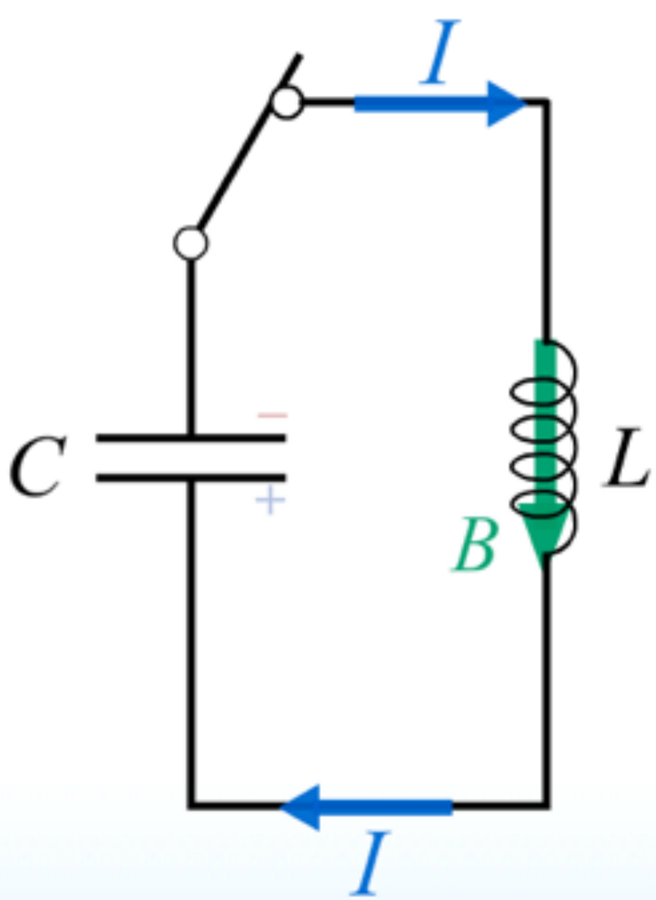




$$\omega = \sqrt{\frac{1}{LC}}$$

LC Circuits: Energy conservation

LR Circuit



Inductor Energy

$$U_L = \frac{1}{2} L I^2 \longrightarrow K = \frac{1}{2} m v^2$$

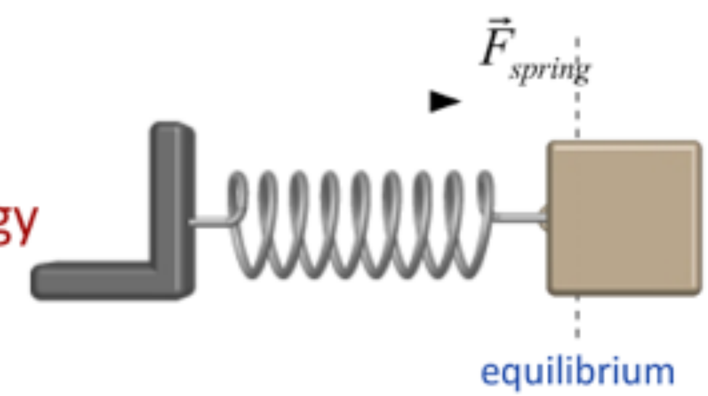
Kinetic Energy

Capacitor Energy

$$U_C = \frac{1}{2} \frac{Q^2}{C} \longrightarrow U_{spring} = \frac{1}{2} k x^2$$

Spring Potential Energy

Oscillating Block



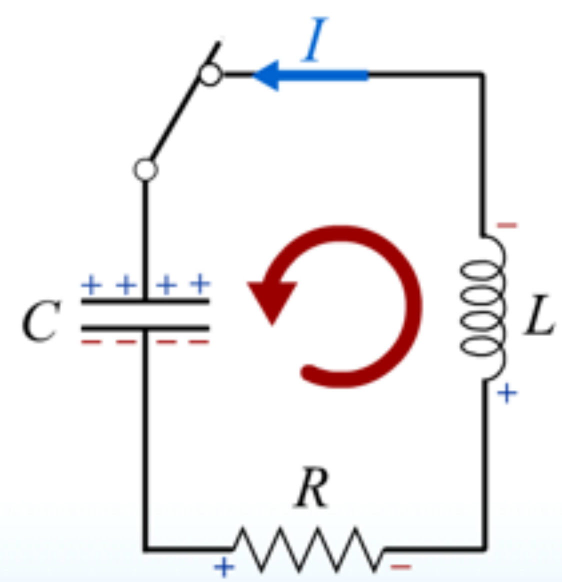
LRC Circuits

KVR

$$\frac{Q}{C} + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} = 0$$

Solution

$$Q(t) = A\,e^{-\beta t}\cos(\omega' t + \phi)$$

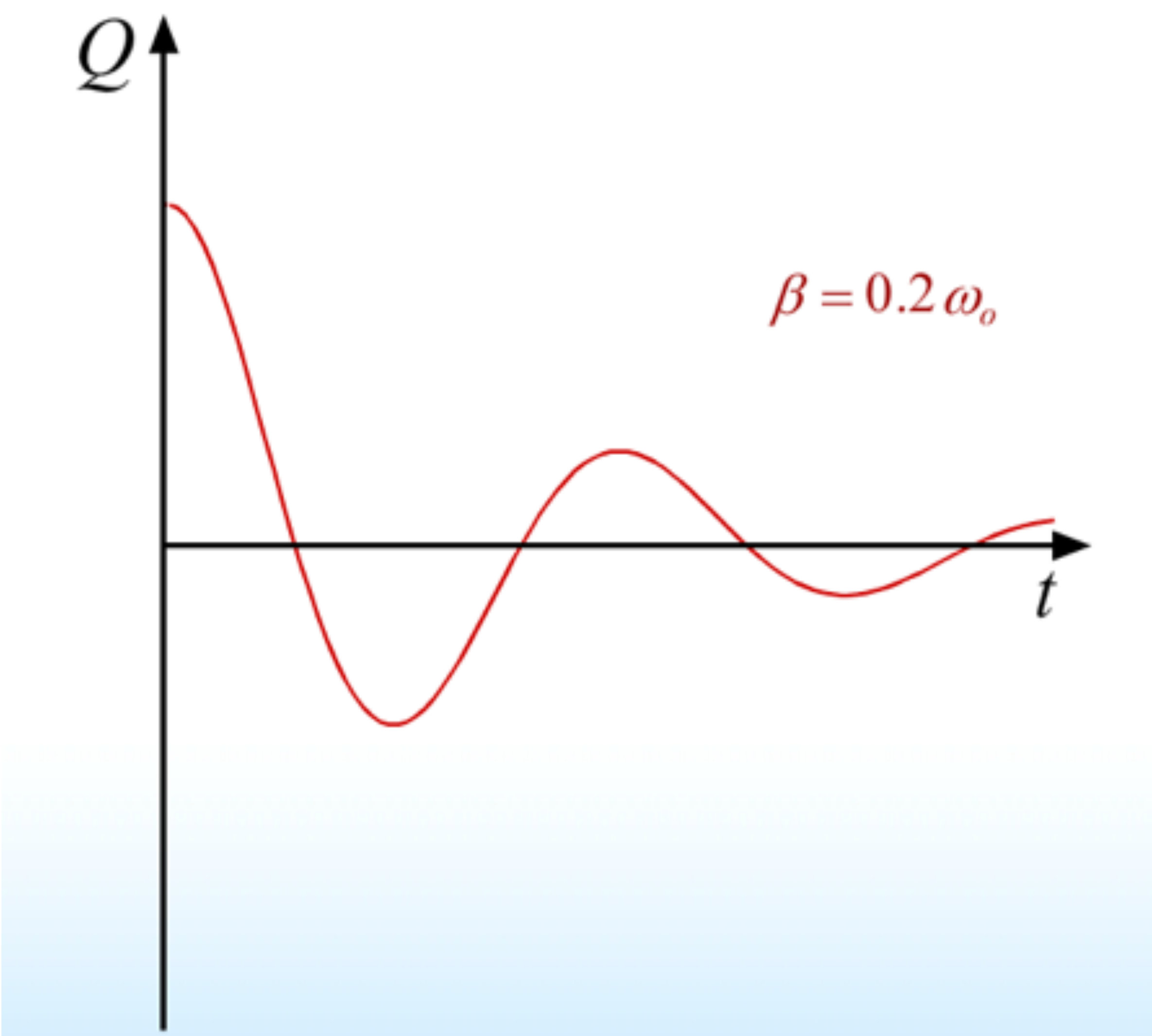


Damping Factor $\beta = \frac{R}{2L}$

Oscillation Frequency $\omega'^2 = \omega_o^2 - \beta^2$

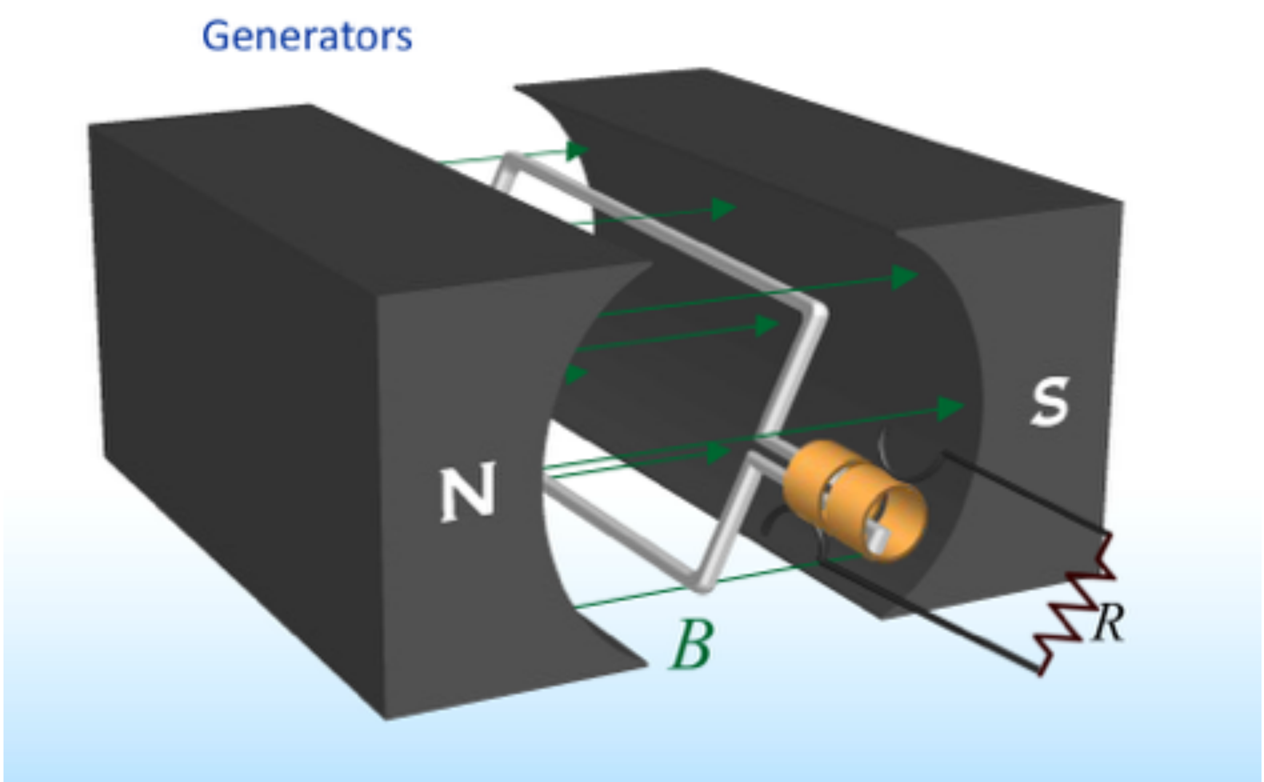
Natural Frequency $\omega_o = \frac{1}{\sqrt{LC}}$

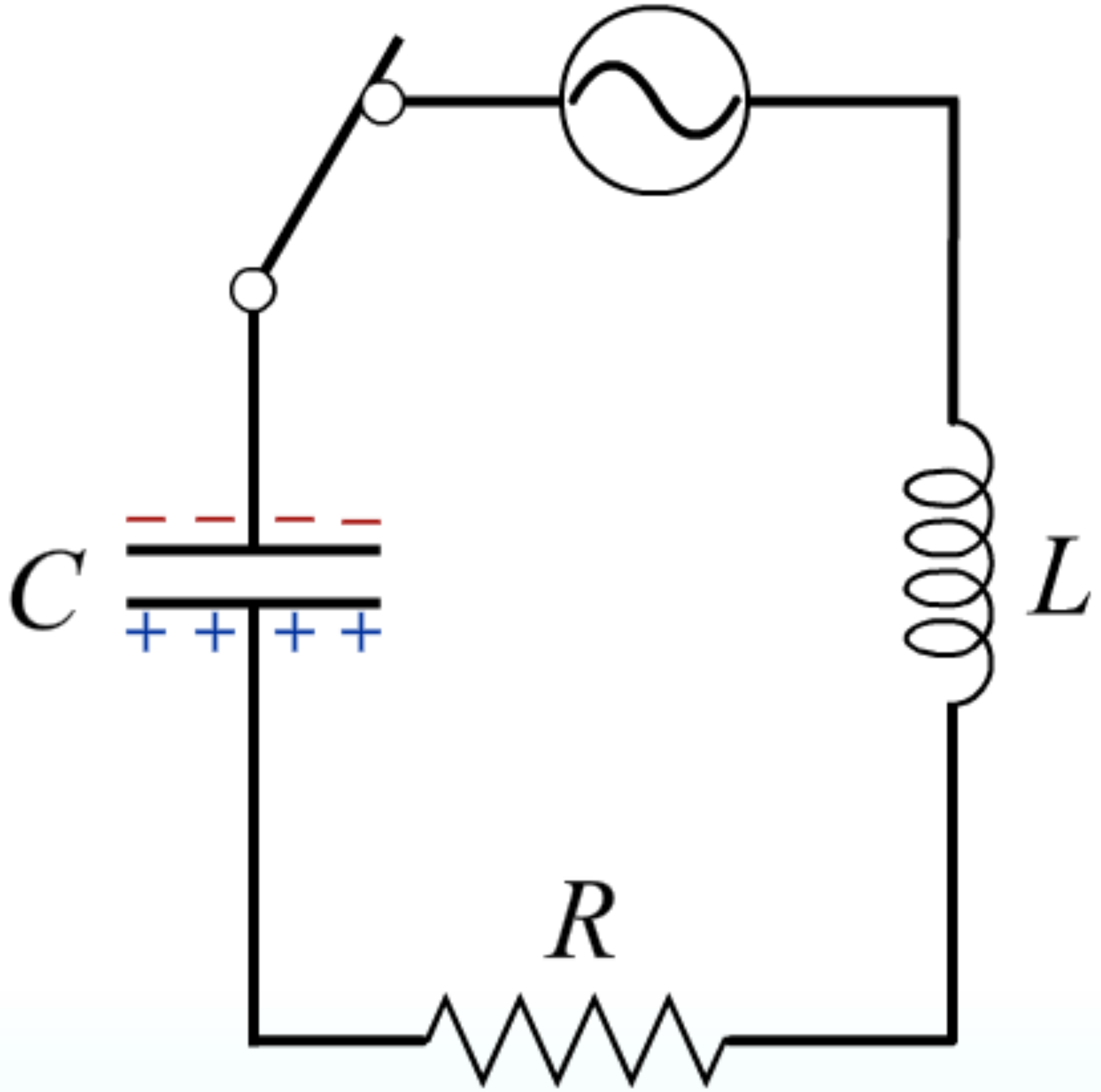
LRC Circuits



- Oscillations decay due to energy loss in the resistor!

AC circuits





$$\mathcal{E}(t) = \mathcal{E}_{\max} \sin \omega t$$

Kircchoff Voltage Law (KVL):

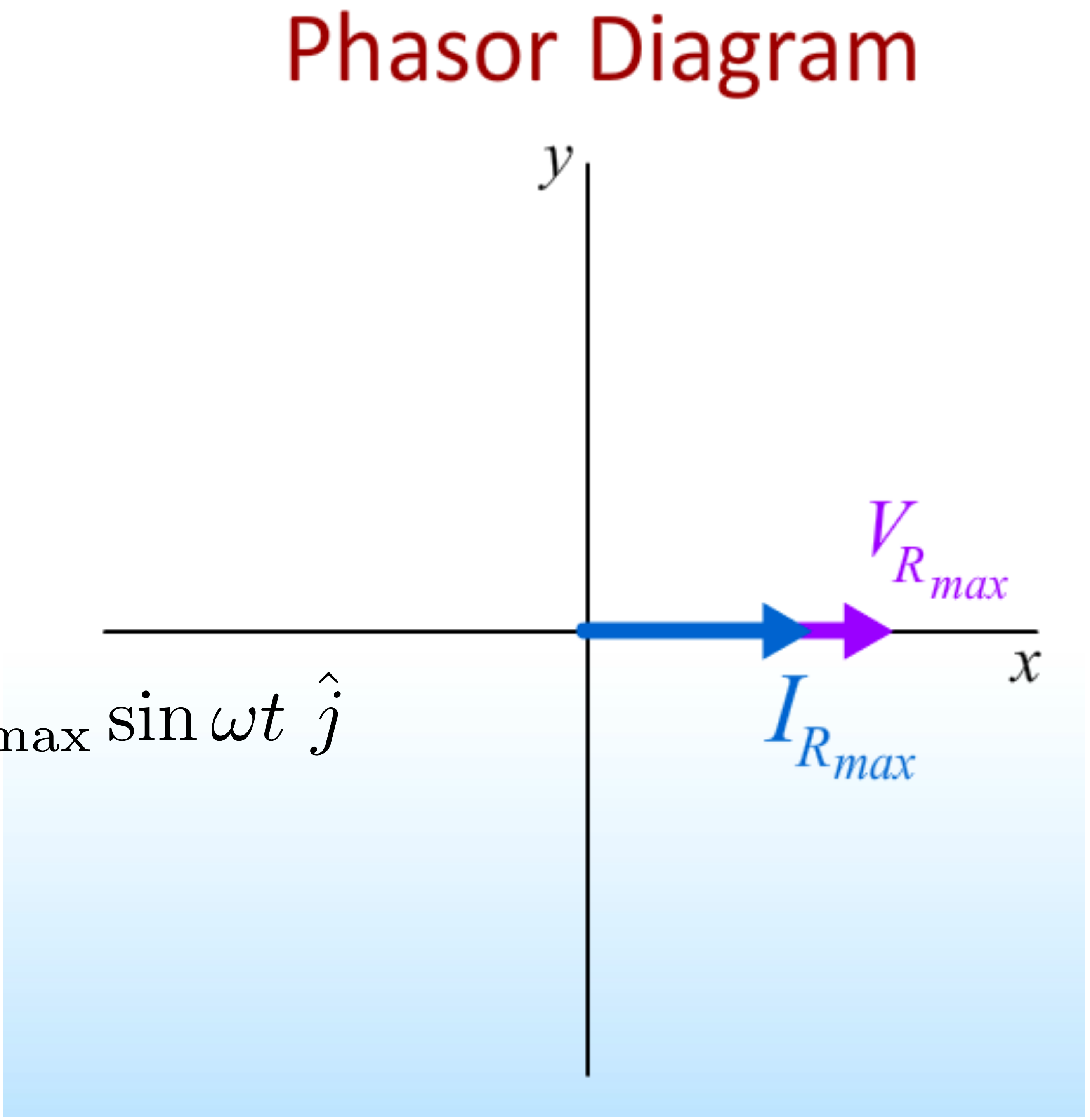
Phasors

- No one uses them…

$$V(t) = \mathcal{E}_{\max} \sin \omega t$$

- y-axis projection of the vector

$$\vec{V}(t) = \mathcal{E}_{\max} \cos \omega t \ \hat{i} + \mathcal{E}_{\max} \sin \omega t \ \hat{j}$$



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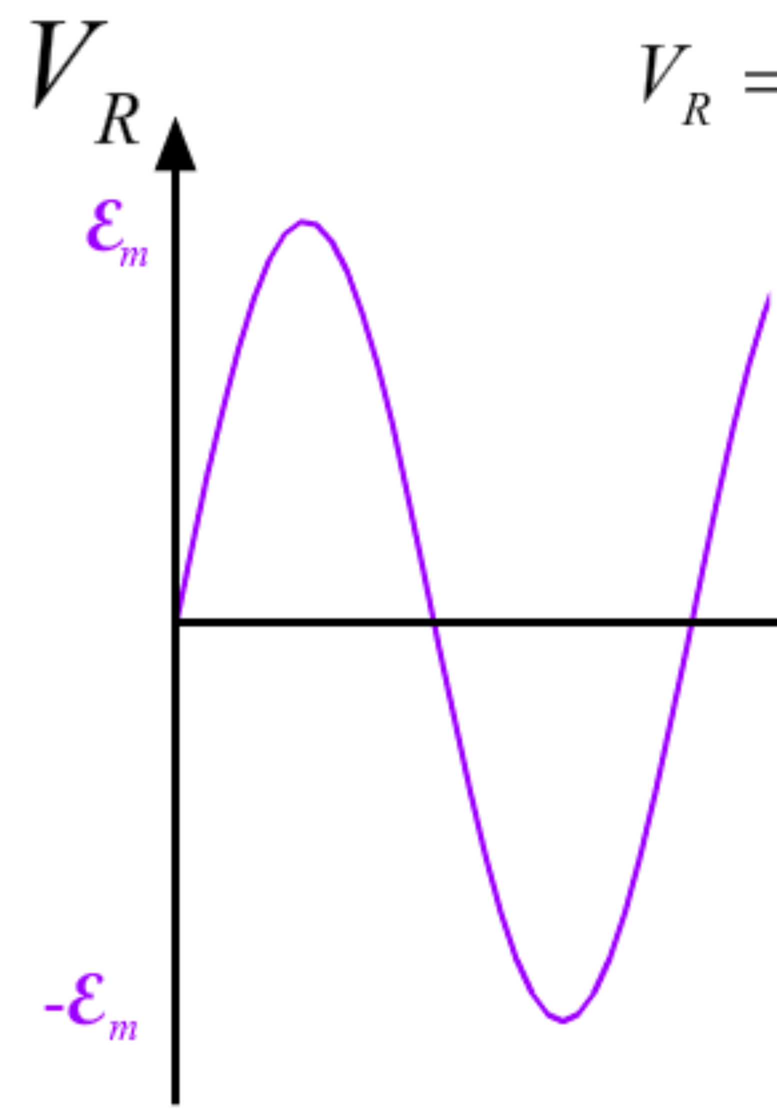
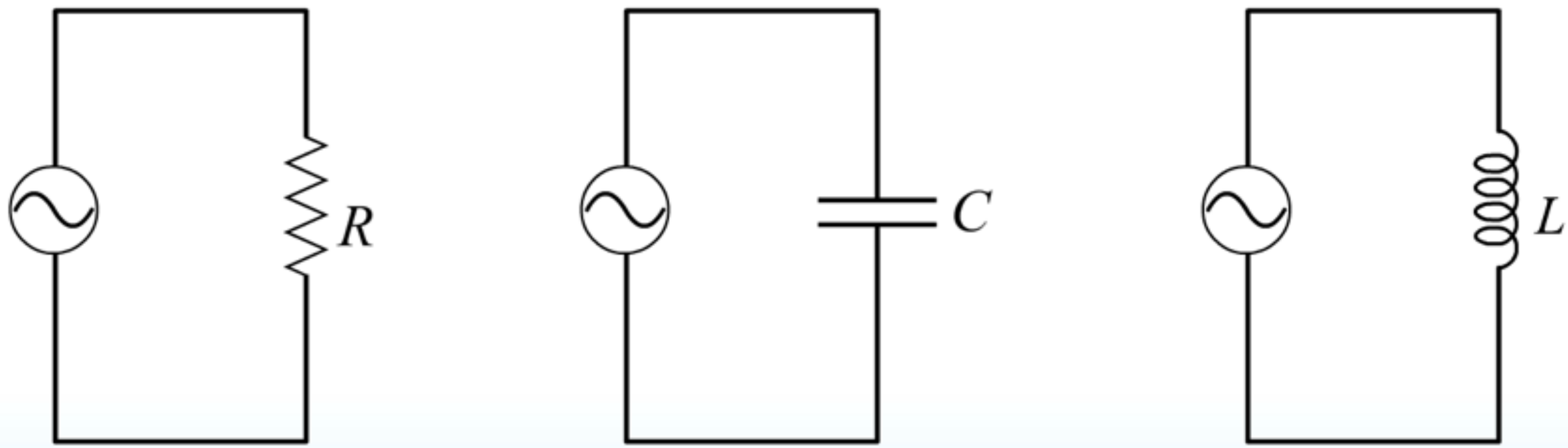


Overview of circuit components: Reactance

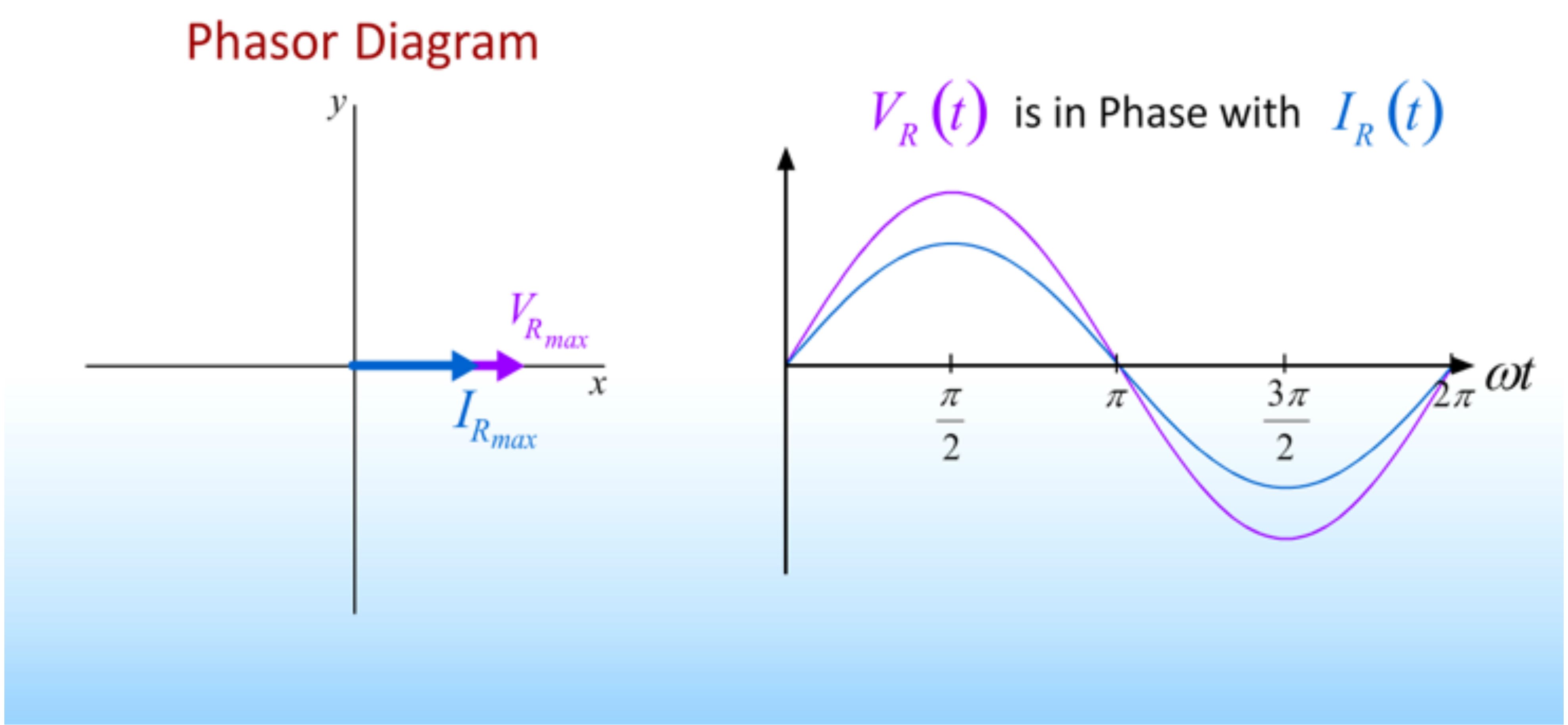


$$V = IR \qquad V = Q/C \qquad V = L\frac{dI}{dt}$$

$$I_R = \frac{\mathcal{E}_{\max}}{R}\sin\omega t \qquad I_C = \frac{\mathcal{E}_{\max}}{X_C}\cos\omega t \qquad I_L = -\frac{\mathcal{E}_{\max}}{X_L}\cos\omega t$$

$$X_R = R \qquad X_C = \frac{1}{\omega C} \qquad X_L = \omega L$$

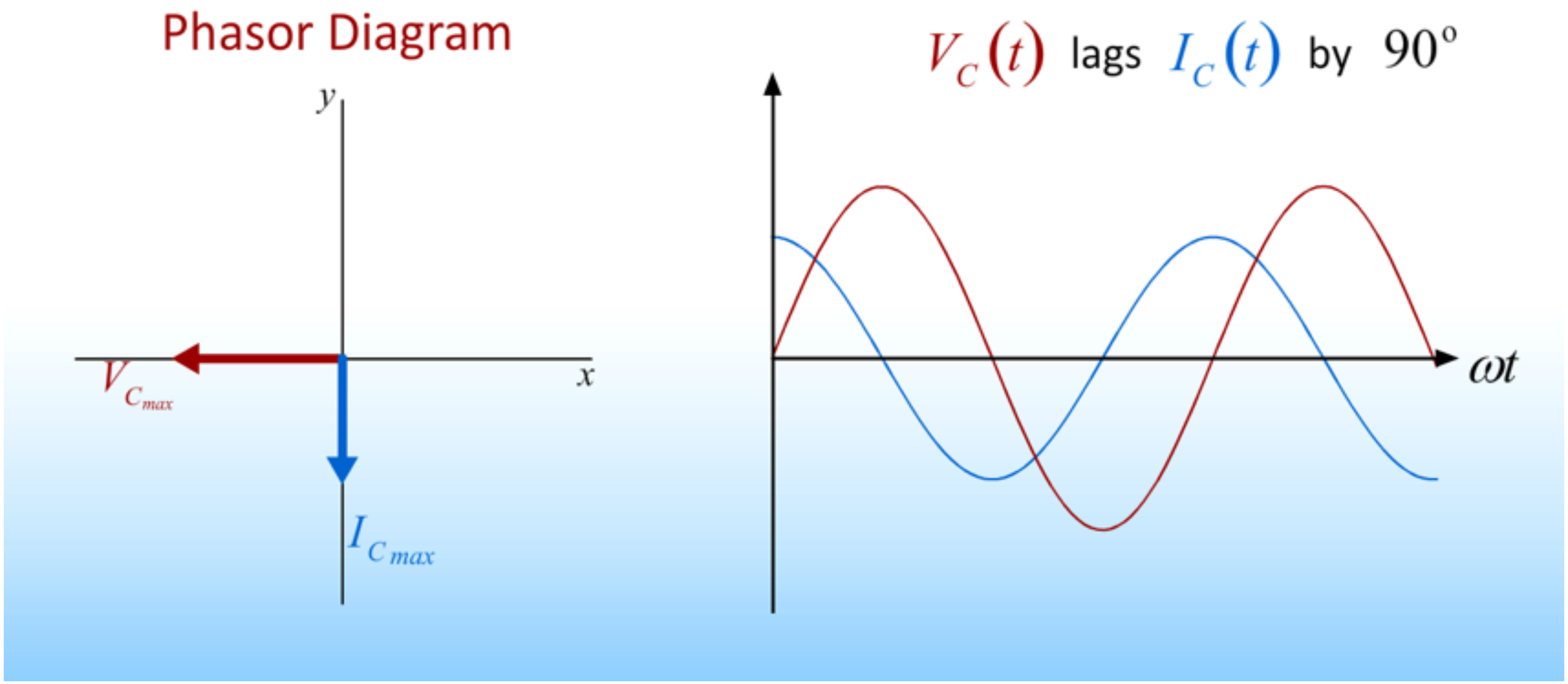
Resistor



$$I_R = \frac{\mathcal{E}_{\max}}{R} \sin \omega t$$

$$\vec{I}_R(t) = \frac{\mathcal{E}_{\max}}{X_R} \left[\cos(\omega t)\ \hat{i} + \sin(\omega t)\ \hat{j}\right]$$

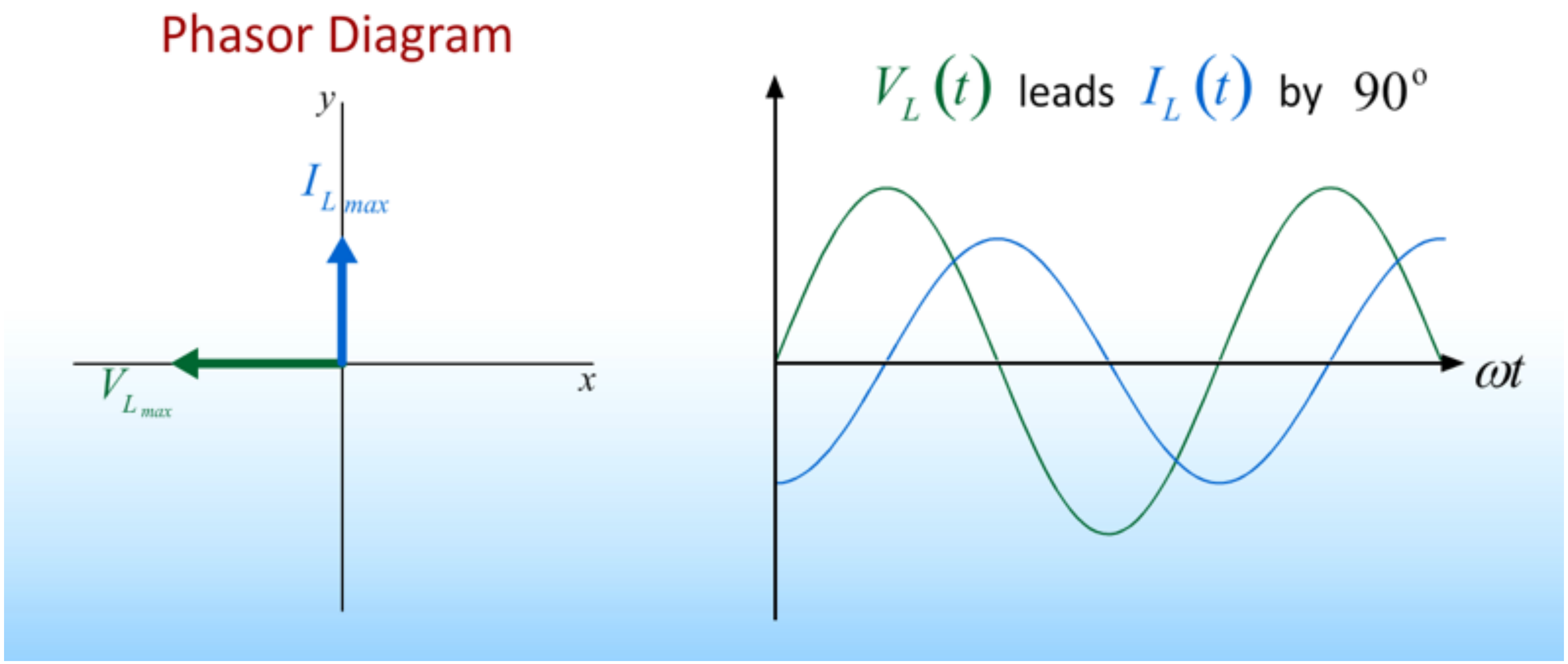
Capacitor



$$I_C = \frac{\mathcal{E}_{\max}}{X_C} \cos \omega t$$

$$\vec{I}_C(t) = \frac{\mathcal{E}_{\max}}{X_C} \left[\cos(\omega t + \tfrac{\pi}{2})\ \hat{i} + \sin(\omega t + \tfrac{\pi}{2})\ \hat{j} \right]$$

Inductor



$$I_L = \frac{\mathcal{E}_{\max}}{X_L} \cos \omega t$$

$$\vec{I}_L(t) = \frac{\mathcal{E}_{\max}}{X_L} \left[\cos(\omega t - \tfrac{\pi}{2})\ \hat{i} + \sin(\omega t - \tfrac{\pi}{2})\ \hat{j} \right]$$

Putting it all together…

Current

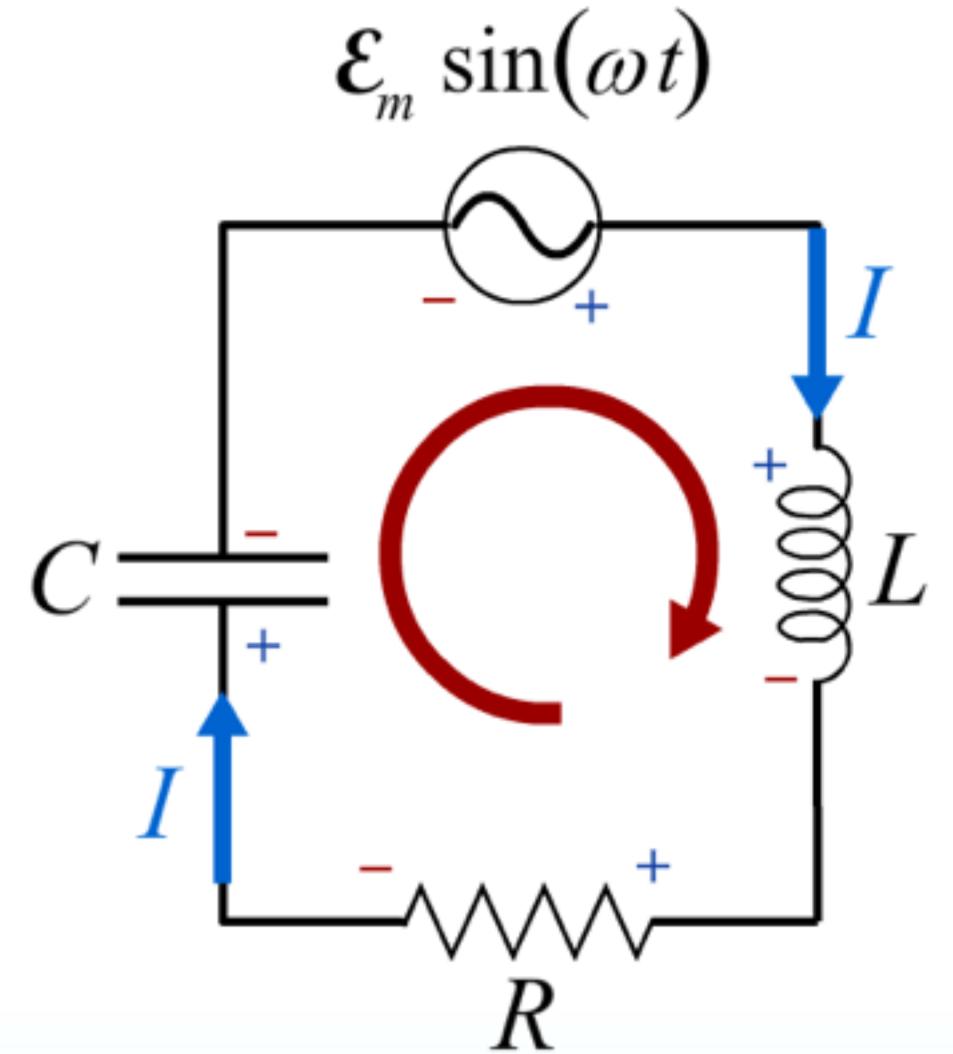
$$I = I_m \sin(\omega t - \phi)$$

KVR

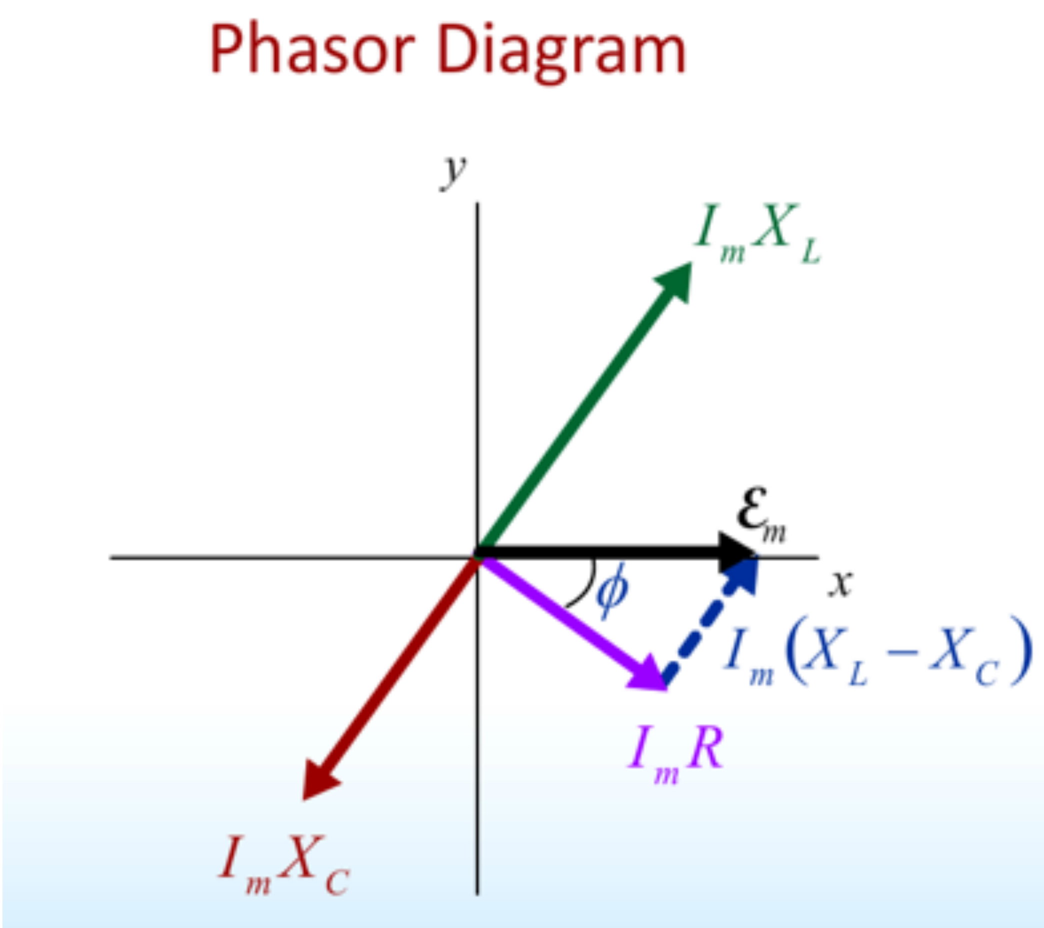
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} - \mathcal{E}_m \sin(\omega t) = 0$$

- Unknowns:

$$I_{\max}, \quad \phi$$



Putting it all together…



- Unknowns:

$$I_{\max}, \quad \phi$$

- Choose unknowns such that:

$$\vec{V}_R + \vec{V}_C + \vec{V}_L = \vec{\mathcal{E}}$$

- Result:

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$