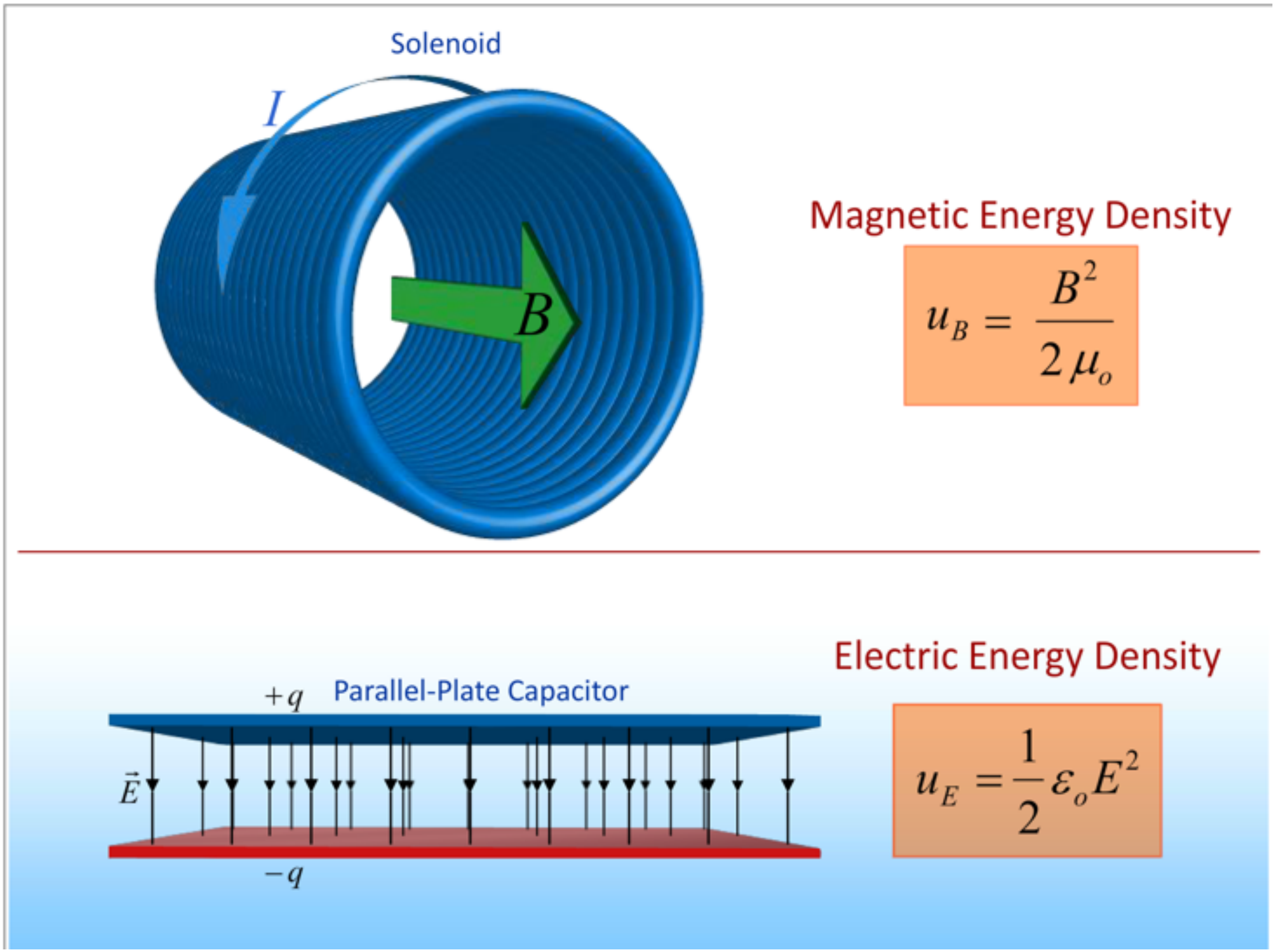
Where is the energy stored?

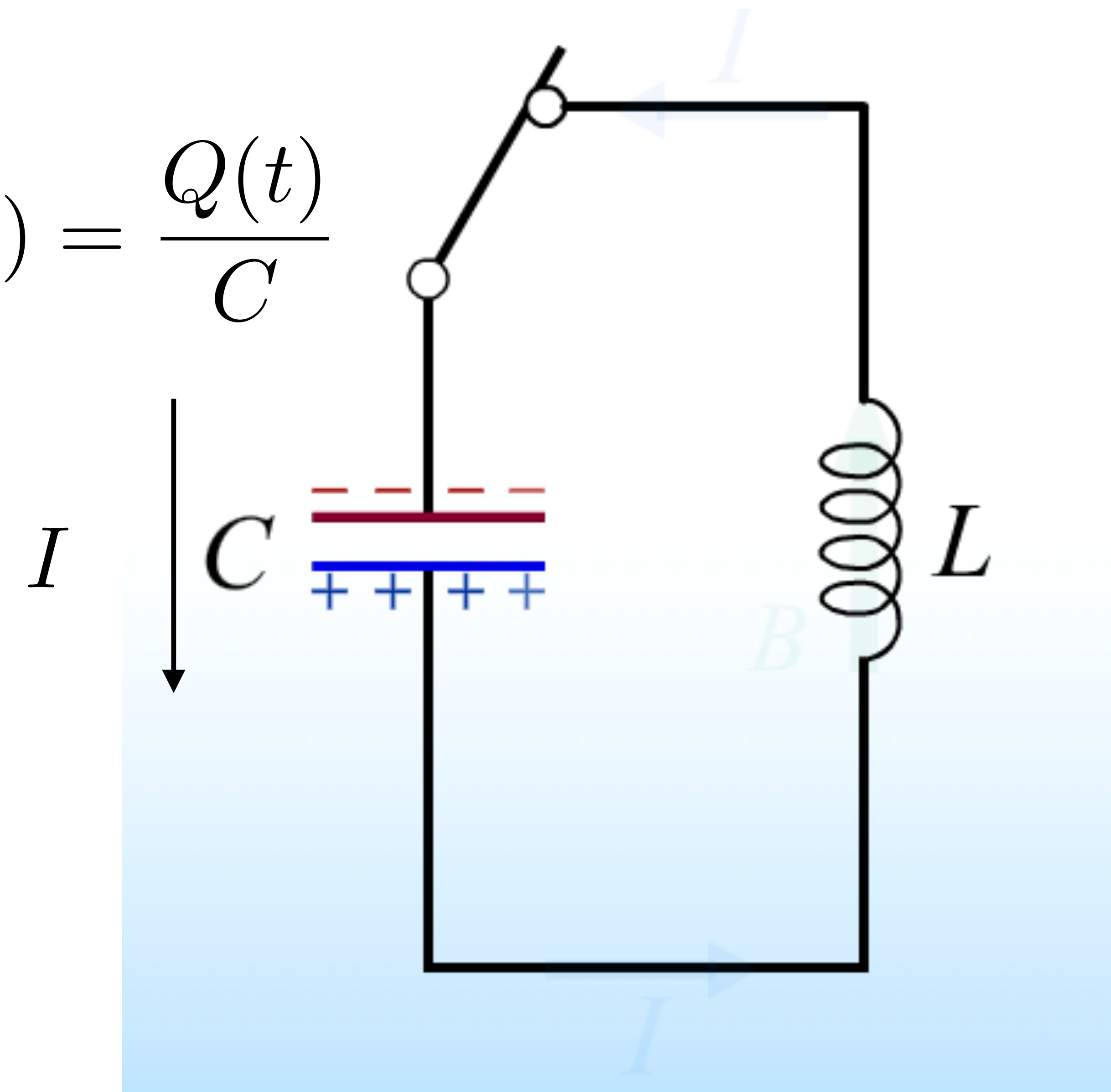


LC & RLC circuits

Lecture 23

LC Circuits

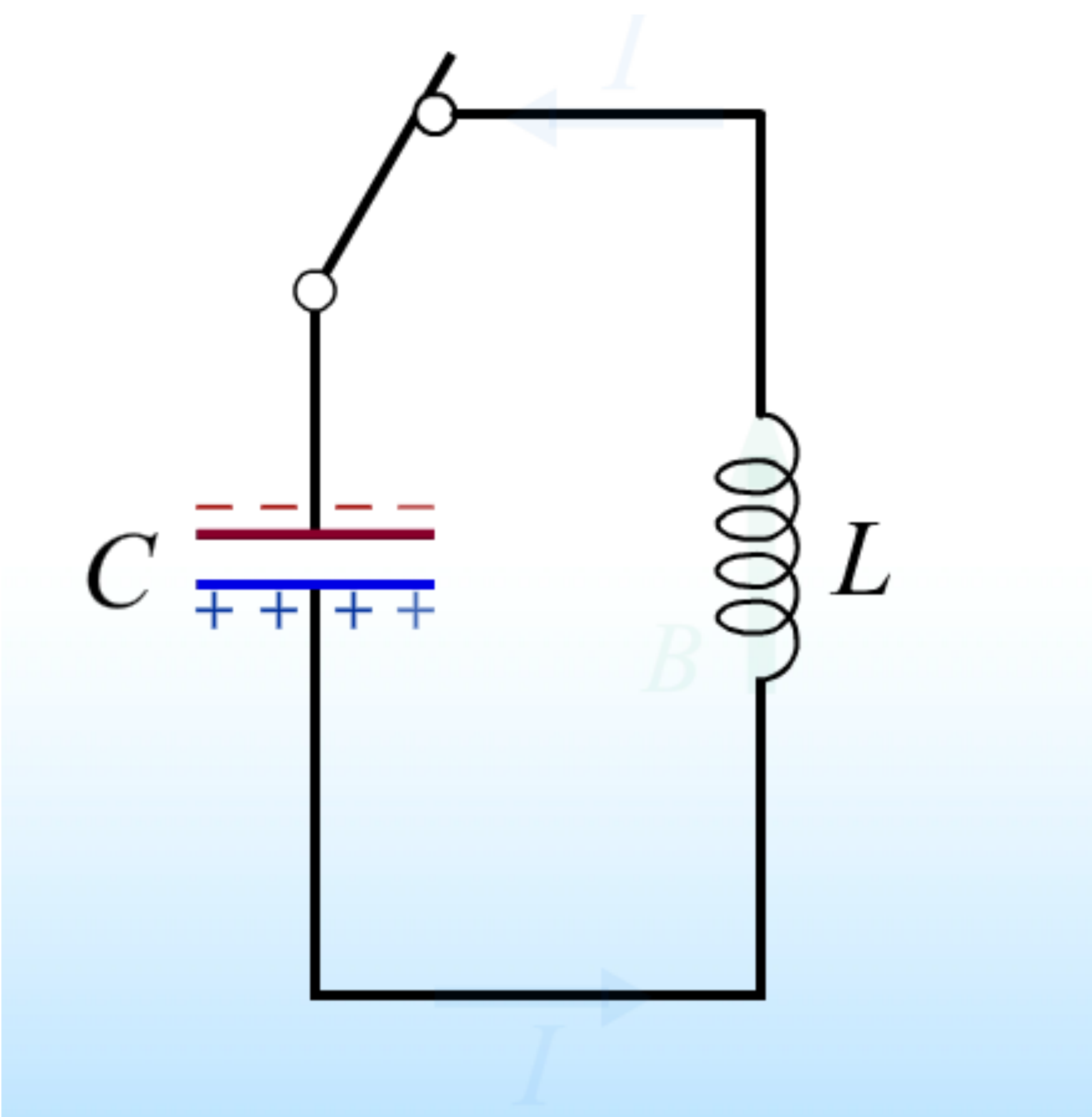
$$V_C(t) = \frac{Q(t)}{C}$$



$$V_L(t) = L\frac{dI(t)}{dt}$$

$$V_L(t) = L\frac{d^2Q(t)}{dt^2}$$

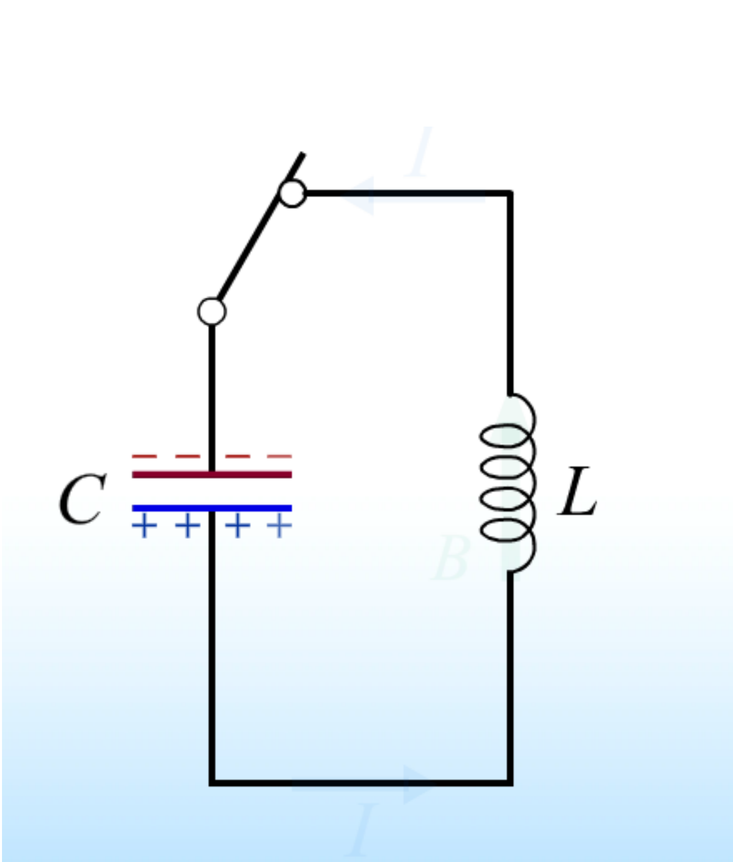
LC Circuits: Detailed Analysis

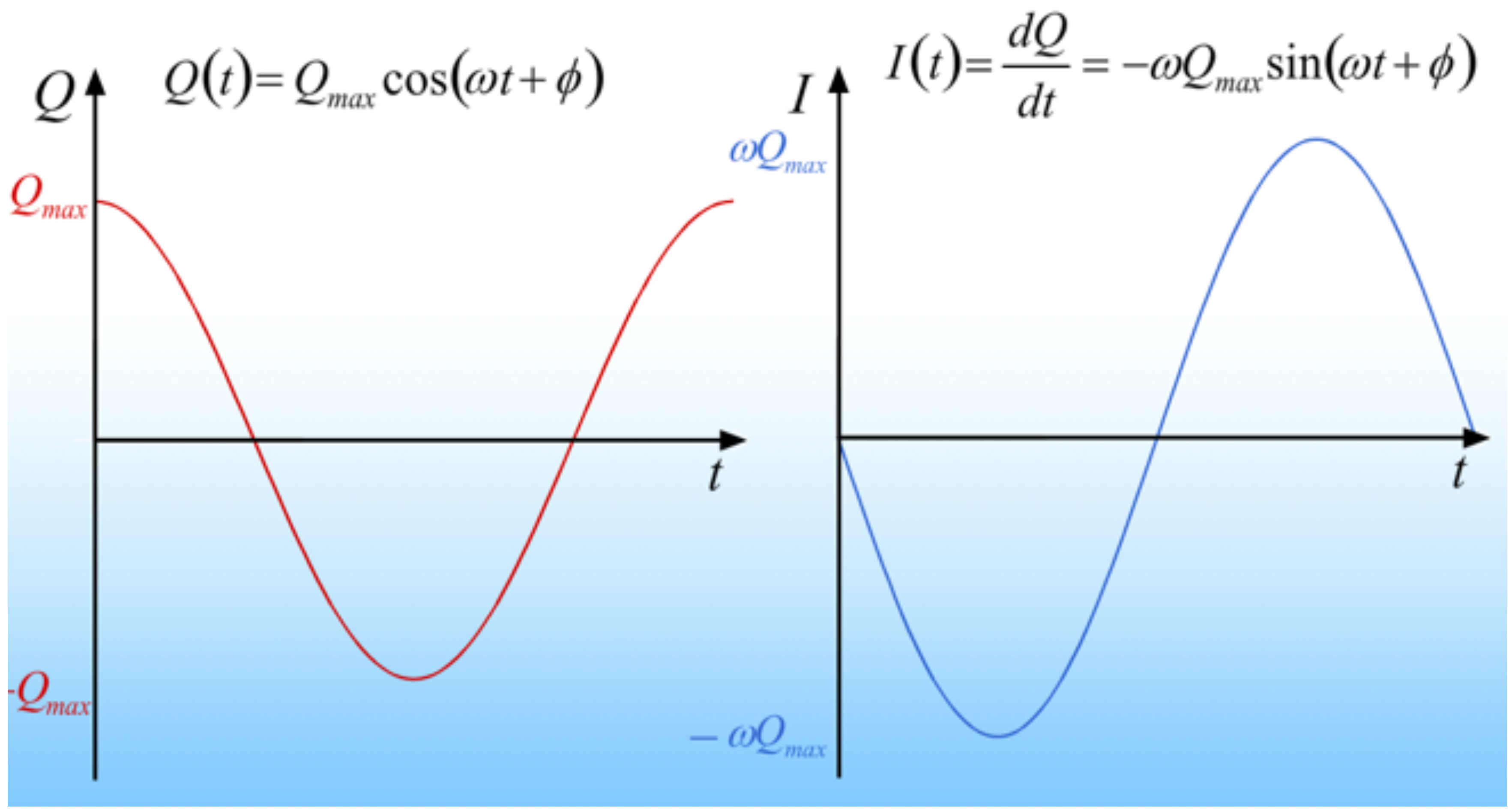


$$\text{KVL}: \quad V_L(t) + V_C(t) = 0$$

$$0 = \frac{Q(t)}{C} + L\frac{d^2Q(t)}{dt^2}$$

LC Circuits: Detailed Analysis

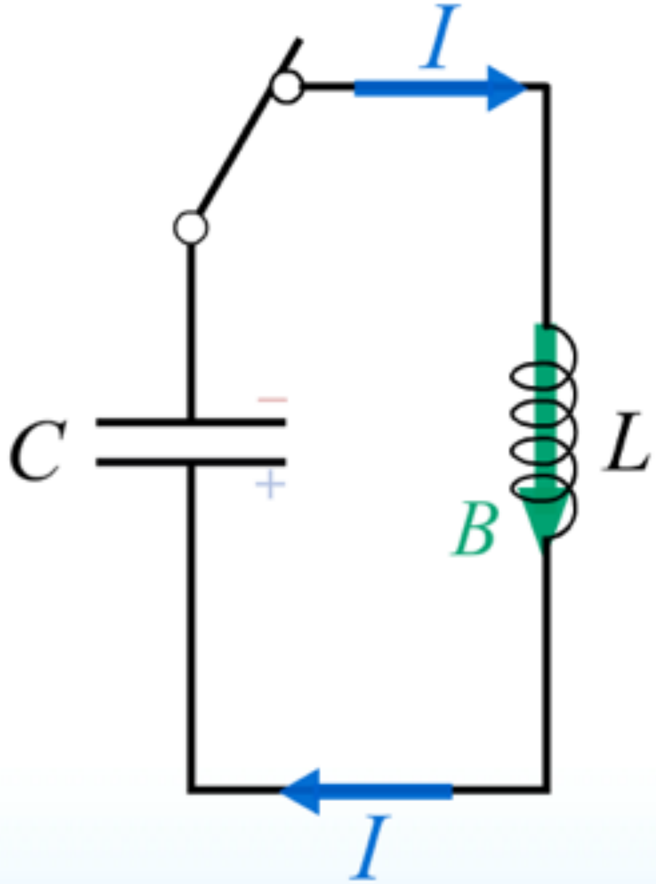




$$\omega = \sqrt{\frac{1}{LC}}$$

LC Circuits: Energy conservation

LR Circuit



Inductor Energy

$$U_L = \frac{1}{2} L I^2 \longrightarrow K = \frac{1}{2} m v^2$$

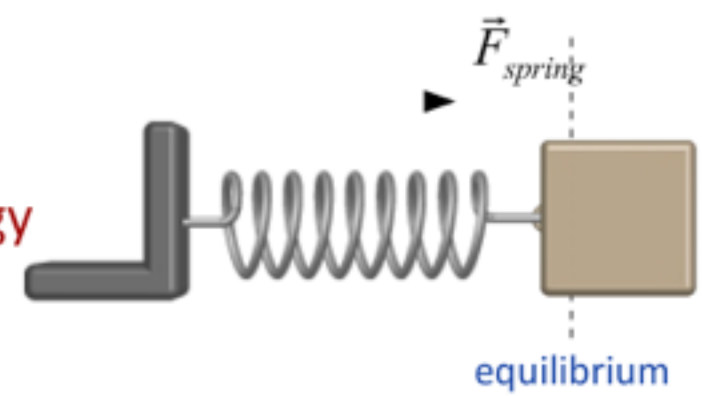
Kinetic Energy

Capacitor Energy

$$U_C = \frac{1}{2} \frac{Q^2}{C} \longrightarrow U_{spring} = \frac{1}{2} k x^2$$

Spring Potential Energy

Oscillating Block



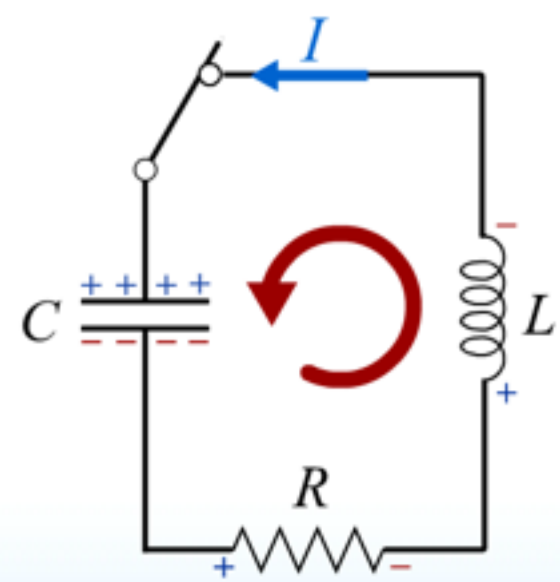
LRC Circuits

KVR

$$\frac{Q}{C} + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} = 0$$

Solution

$$Q(t) = A\,e^{-\beta t}\cos(\omega' t + \phi)$$

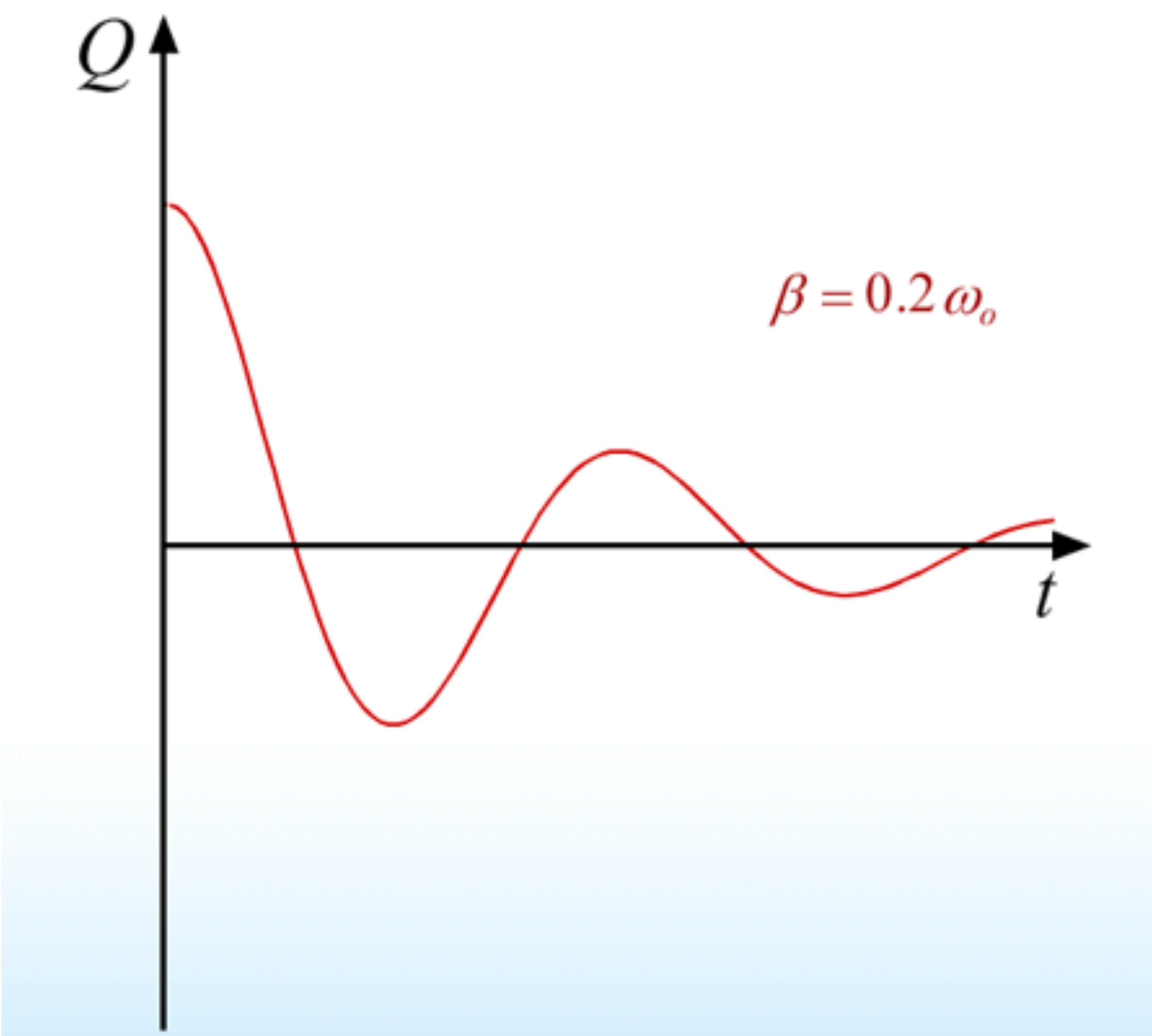


Damping Factor $\beta = \frac{R}{2L}$

Oscillation Frequency $\omega'^2 = \omega_o^2 - \beta^2$

Natural Frequency $\omega_o = \frac{1}{\sqrt{LC}}$

LRC Circuits



- Oscillations decay due to energy loss in the resistor!

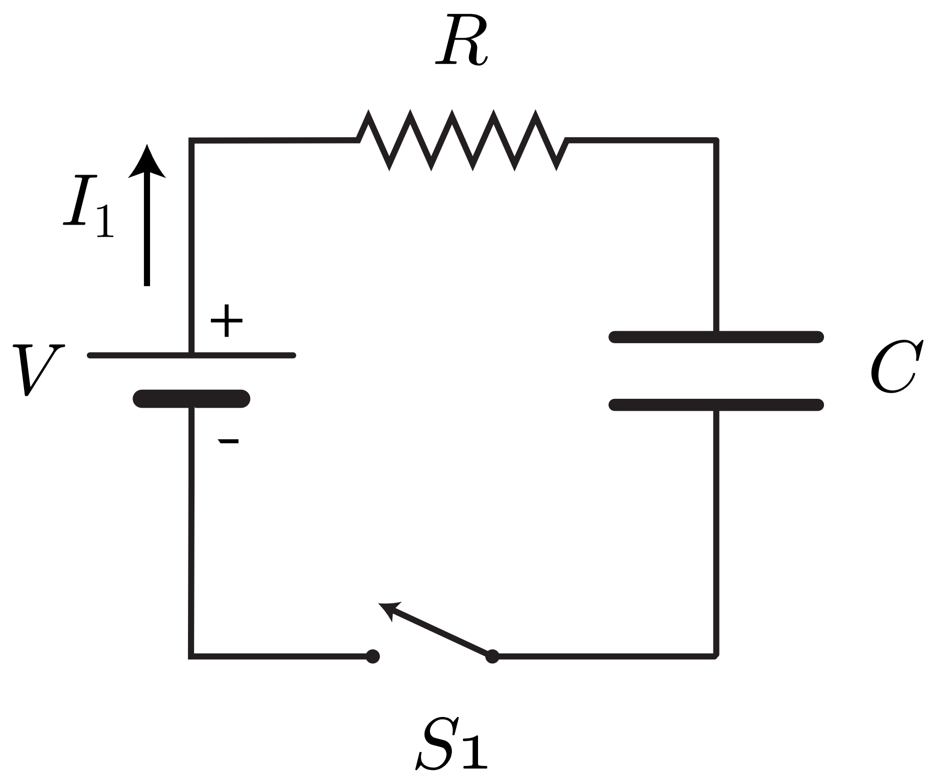
MT2 Review

Lecture 24

Units Units Units

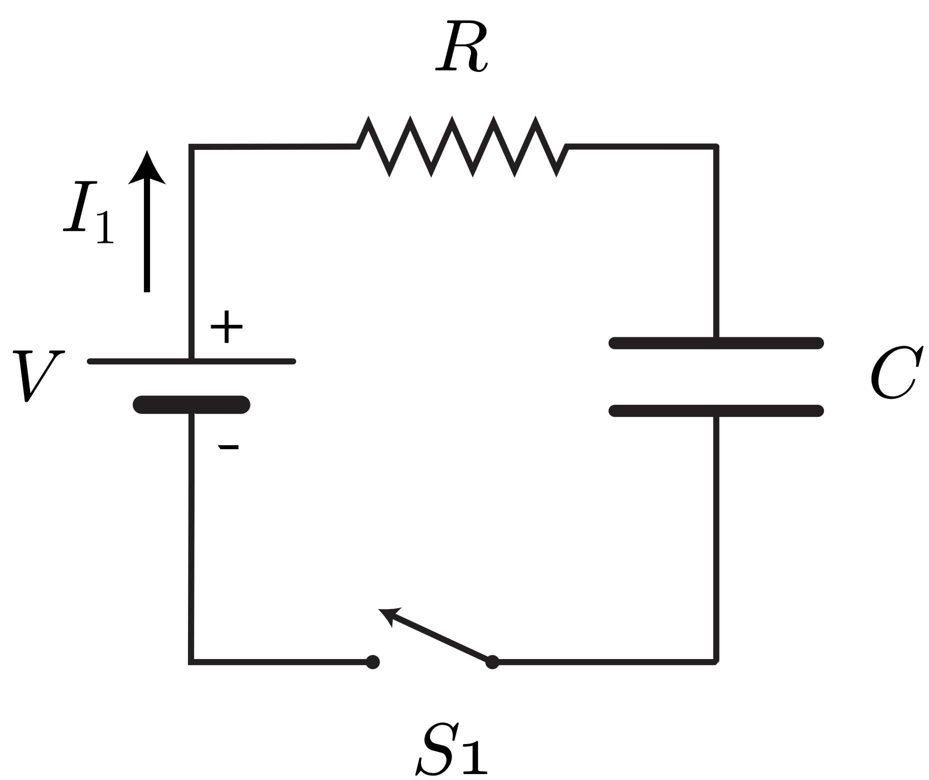
- Always Always Always convert to MKS
  - cm -> m
  - ms -> s
  - mA -> A
  - etc…

RC Circuits



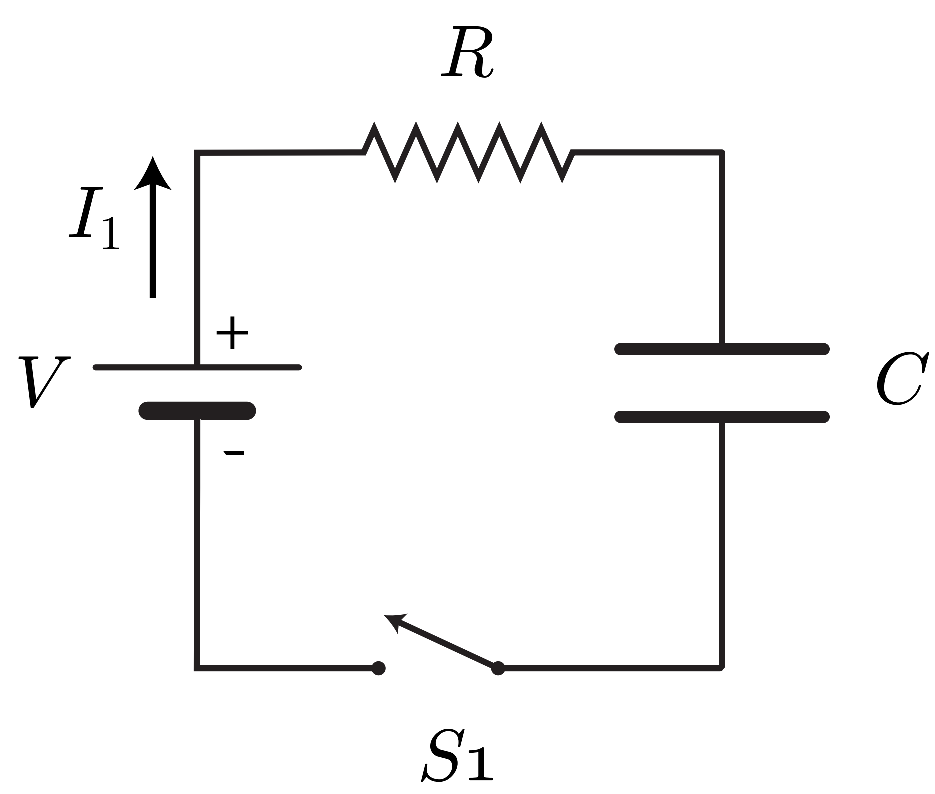
- C is uncharge at t = 0.
- What is the current the instant after S1 is closed?
- Why?

RC Circuits



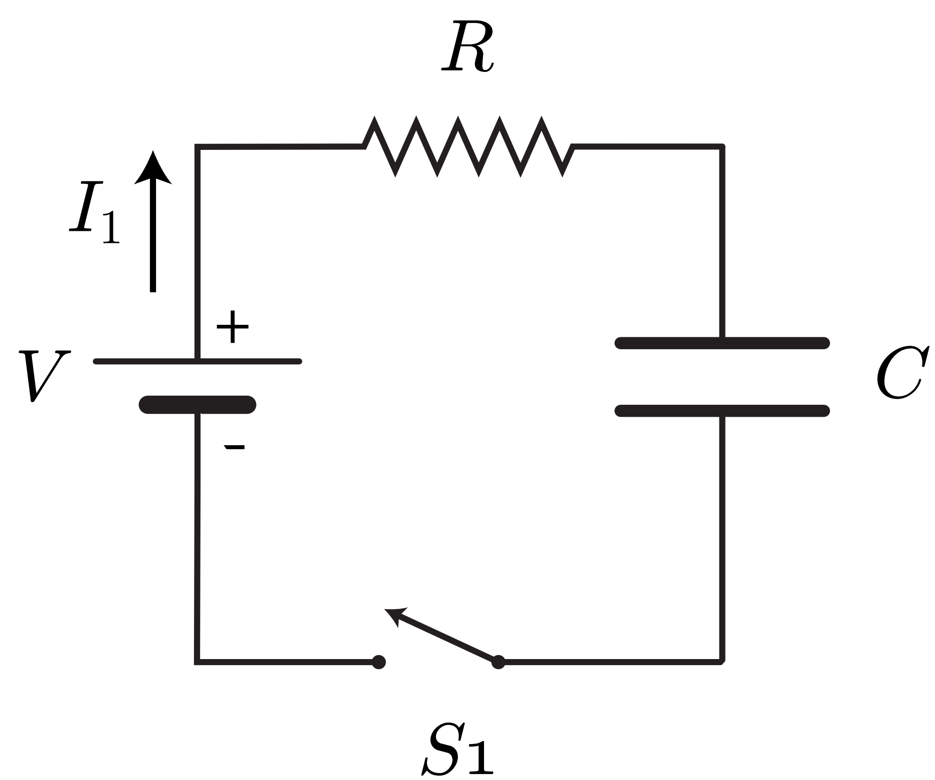
- C is uncharge at t = 0.
- What is the current at long times after S1 is closed?
- Why?

RC Circuits



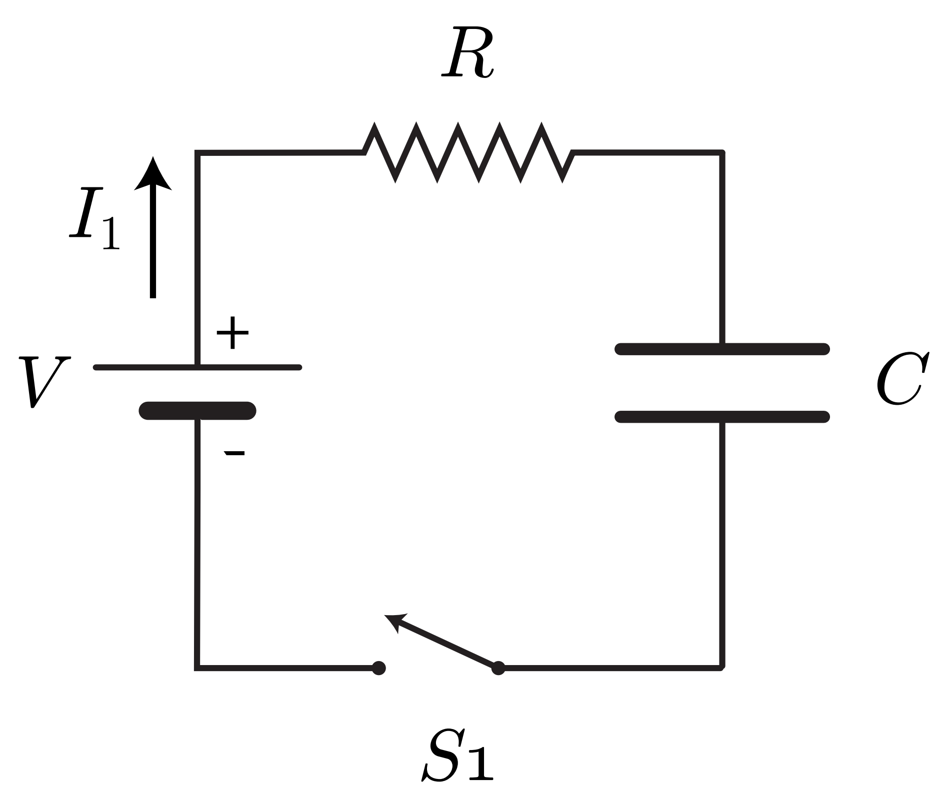
- C is uncharge at t = 0.
- What is the time constant?

RC Circuits



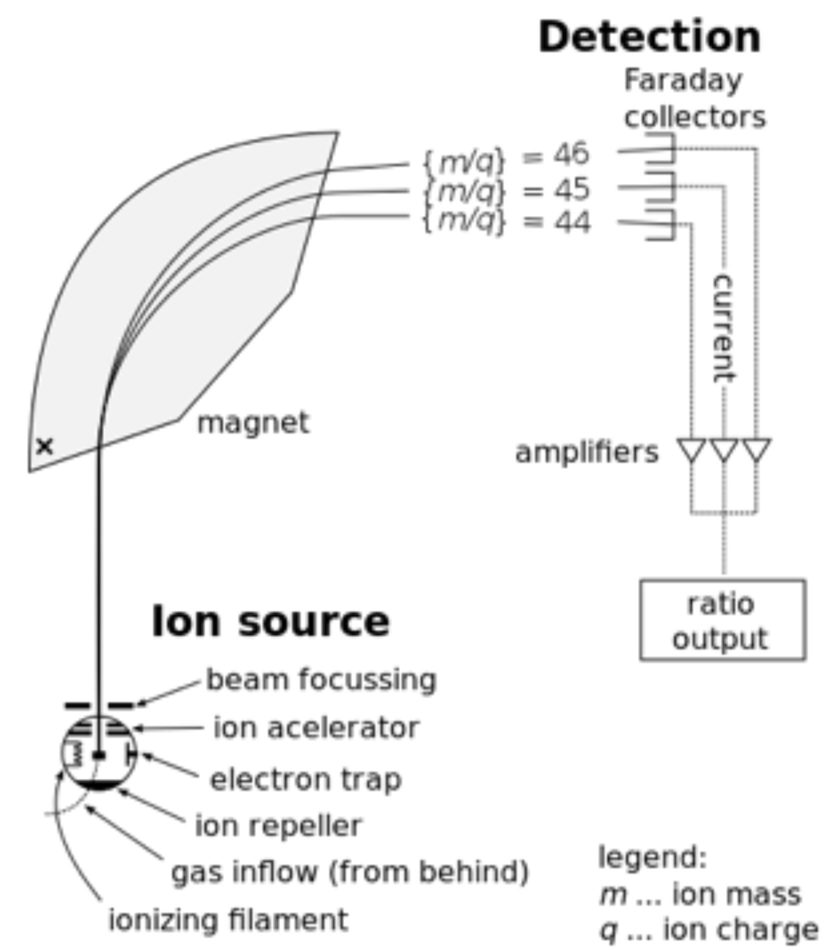
- C is uncharge at t = 0.
- Sketch the voltage across the capacitor as a function of time

RC Circuits



- If R is doubled, how should the components be changed to get the same V_c(t) curve?
- Do I(t) and Q(t) change?

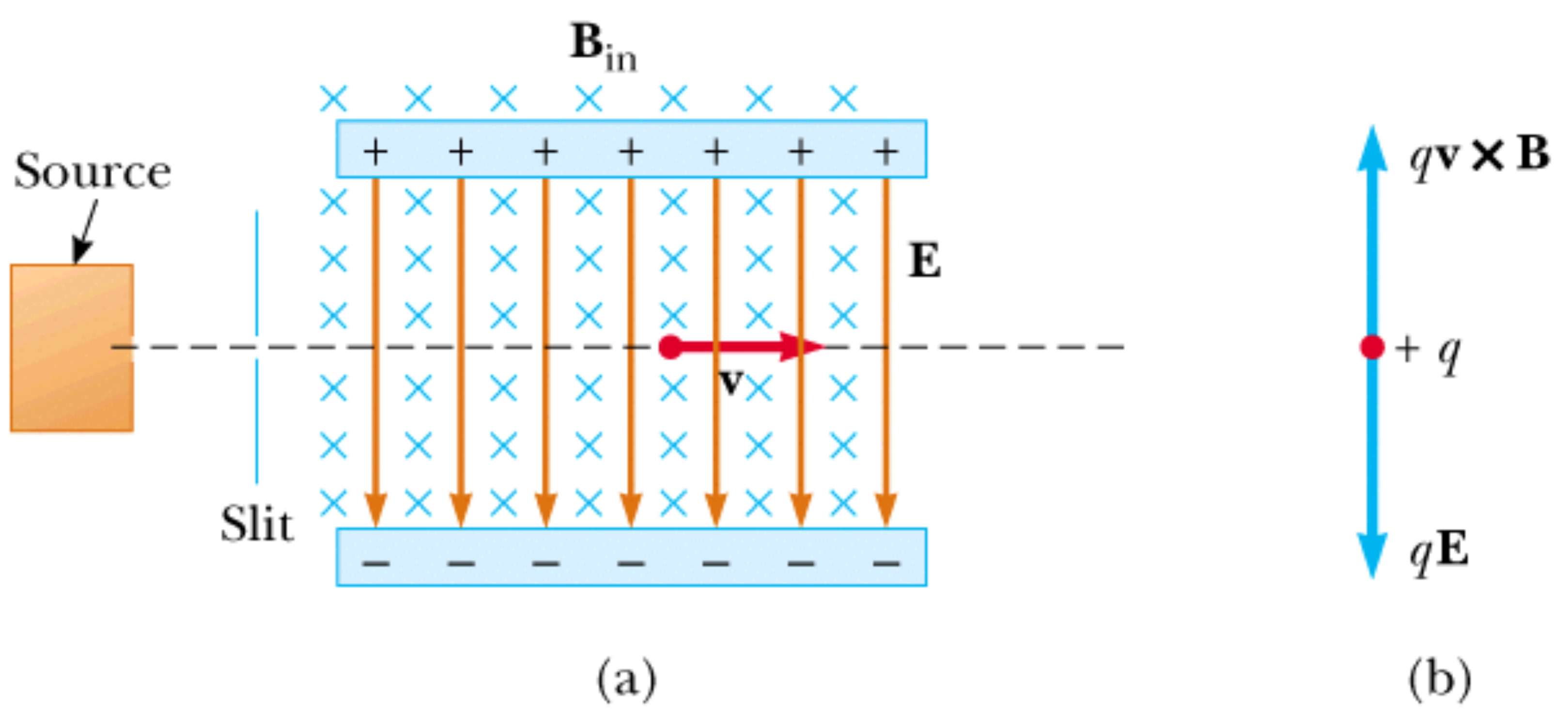
Charges Moving in a B Field: Mass Spectrometer



If the mass of the particle is doubled, how must the magnitude of the charge change such that the deflection of the particle is unchanged?

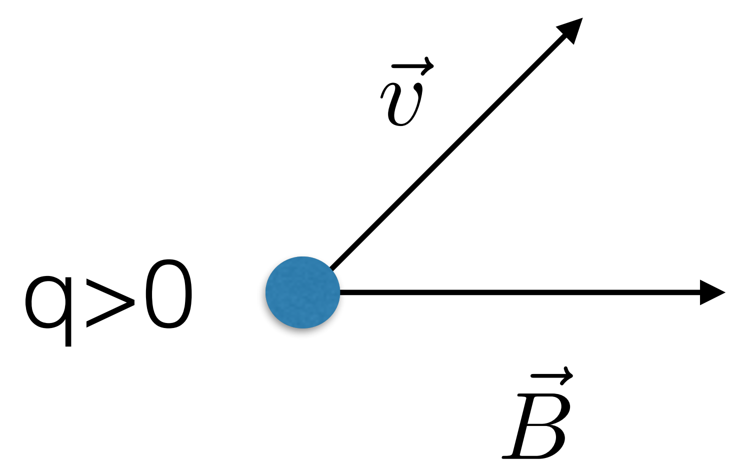
Charges Moving in a B Field: Velocity Selector

How does a velocity selector work?



Charges Moving in a B Field

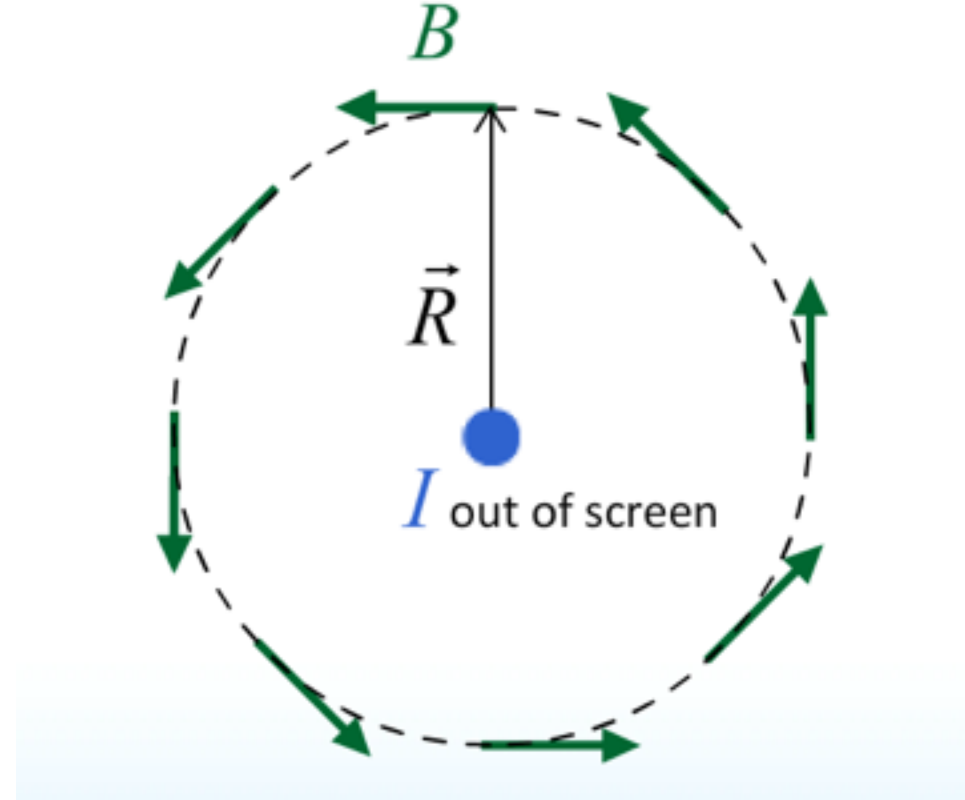
- What is the trajectory of this particle?



Ampère Law: Infinite Wire (1)

(Overhead)

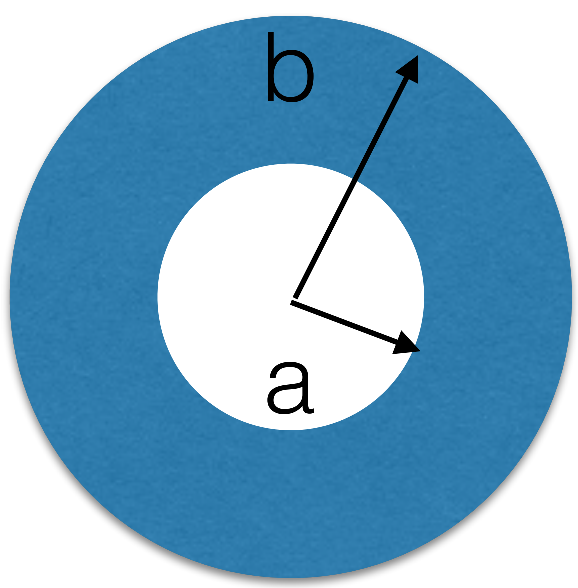
1. Identify **symmetry**
2. Draw B field/field lines
3. Choose a Ampère Loop
4. Compute B



$$B = \frac{\mu_0 I}{2\pi R}$$

Ampère Law: Weird Wires

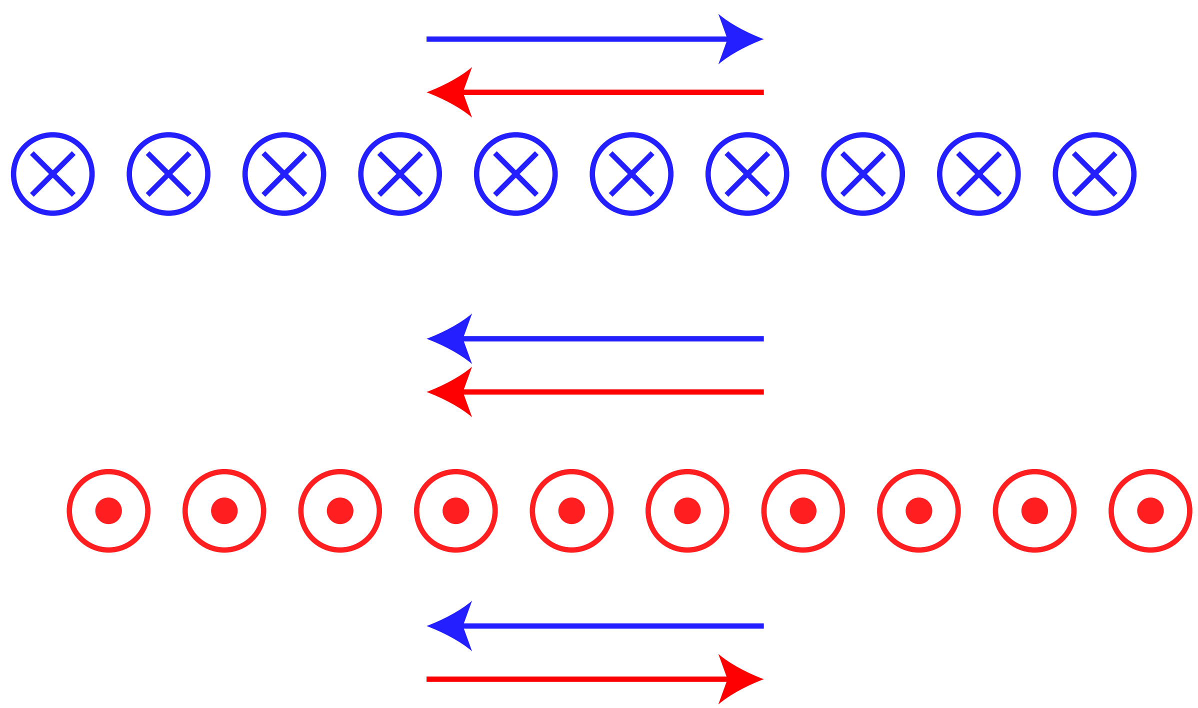
I (out of the board)

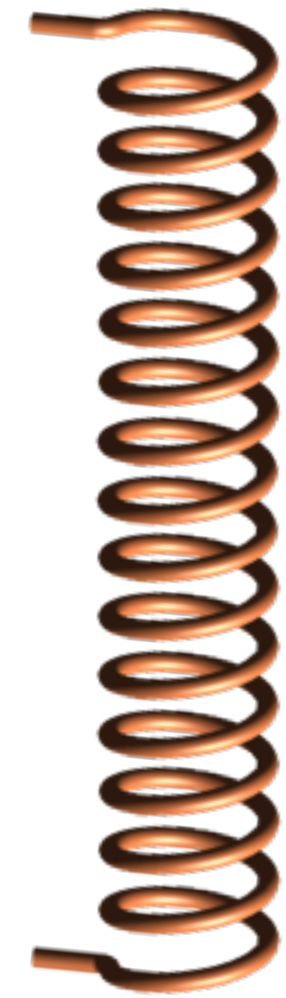


- A wire cross section is shown to the left. What is the B field as a function or the displacement r?

Ampère Law: B field for an ∞ solenoid

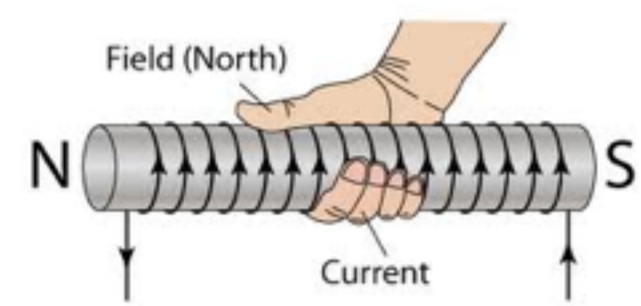
- Intuitive picture: ~ 2 infinite sheets



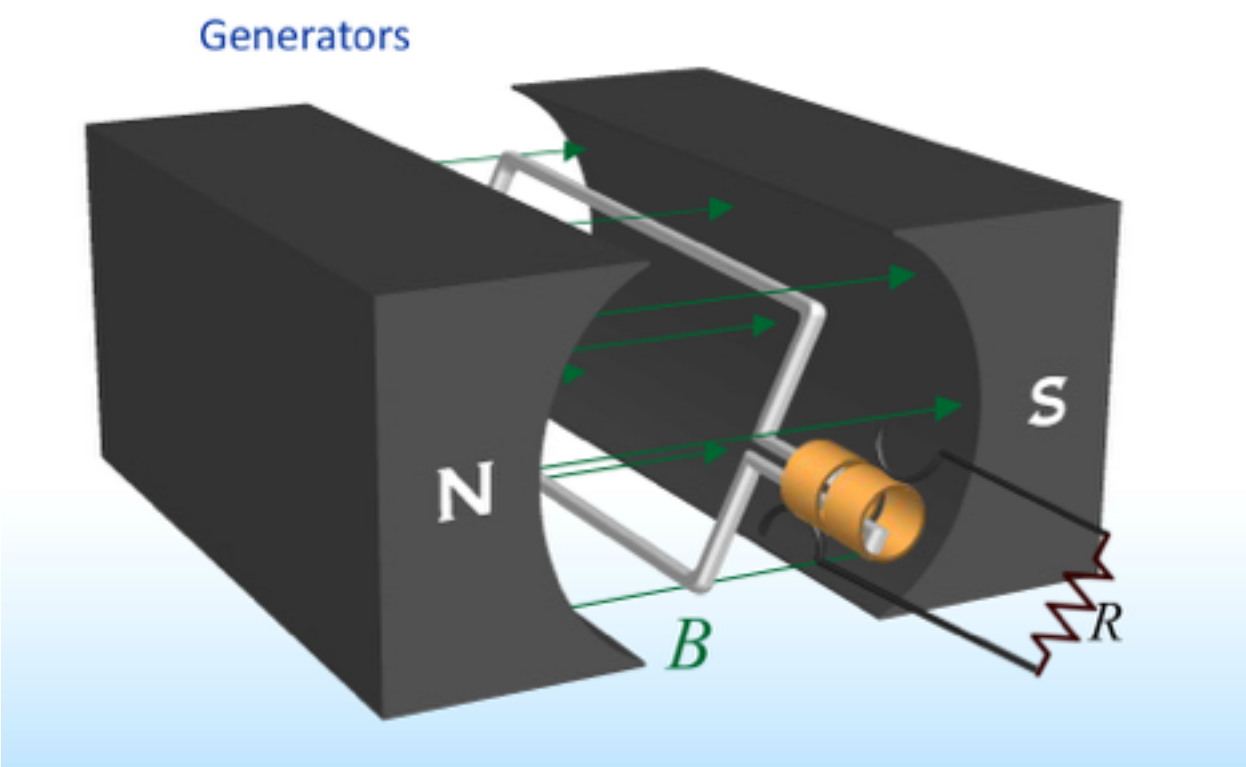


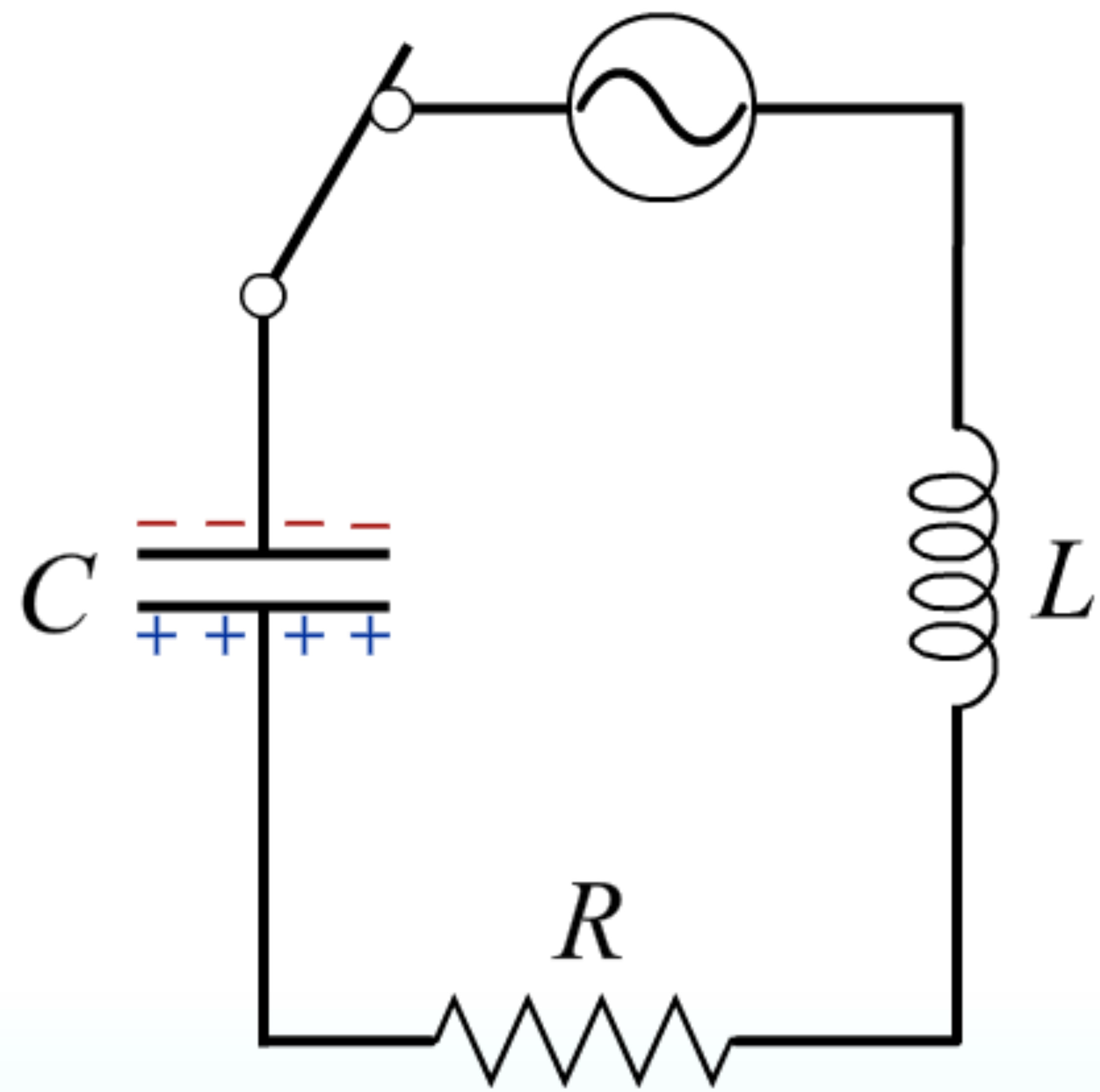
- Field is ~ zero outside

$$B = \mu_0 n I$$



AC circuits





$$\mathcal{E}(t) = \mathcal{E}_{\max} \sin \omega t$$

Kircchoff Voltage Law (KVL):

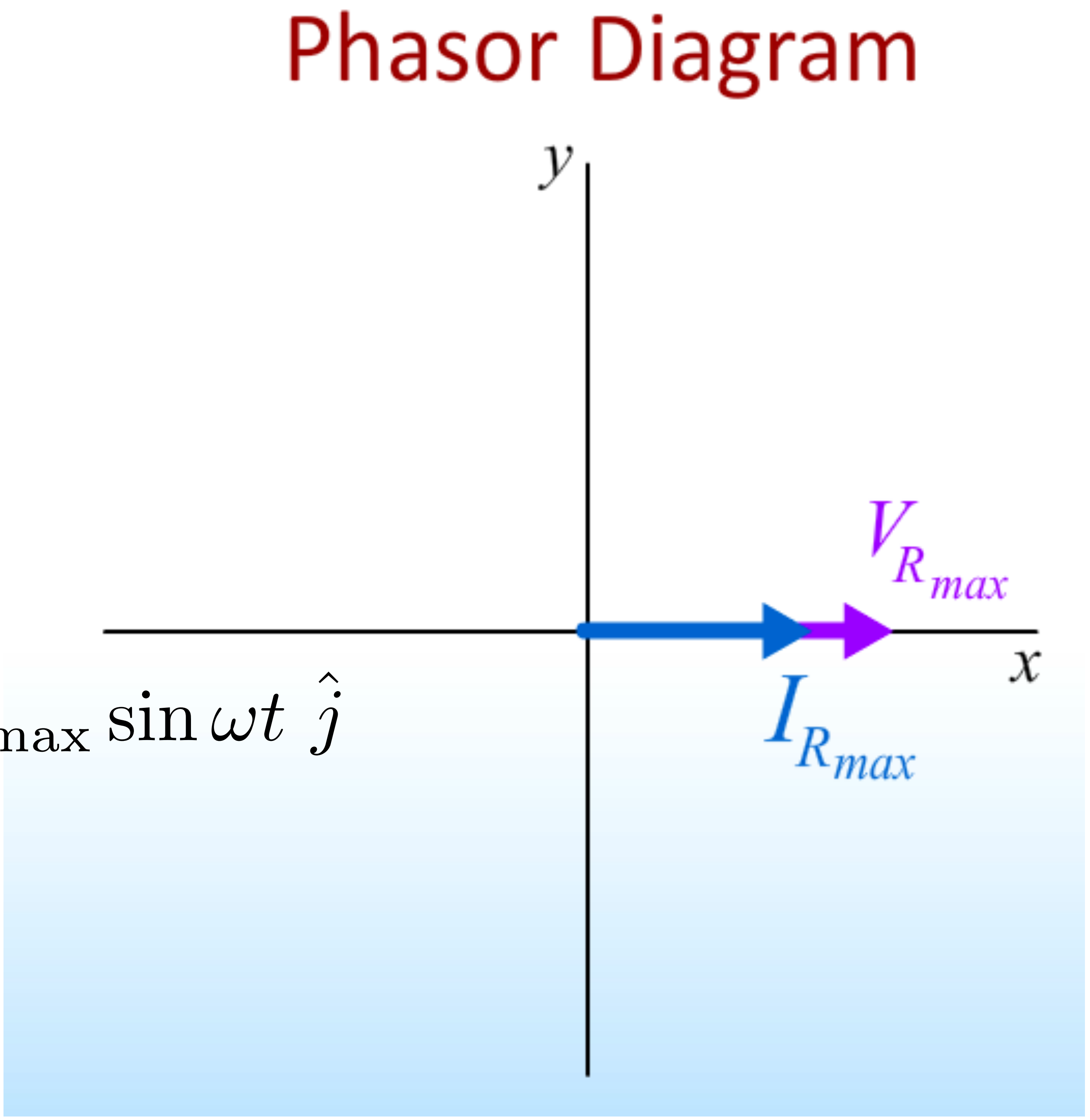
Phasors

- No one uses them…

$$V(t) = \mathcal{E}_{\max} \sin \omega t$$

- y-axis projection of the vector

$$\vec{V}(t) = \mathcal{E}_{\max} \cos \omega t \; \hat{i} + \mathcal{E}_{\max} \sin \omega t \; \hat{j}$$



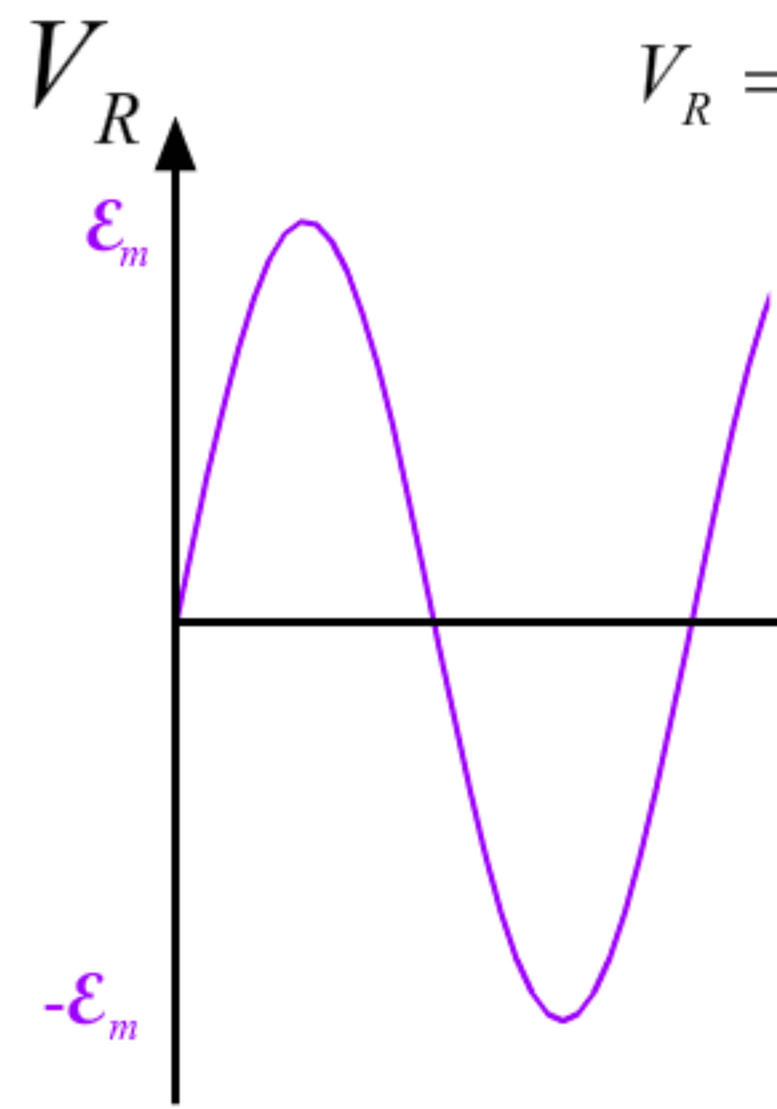
Phasors

- No one uses them…

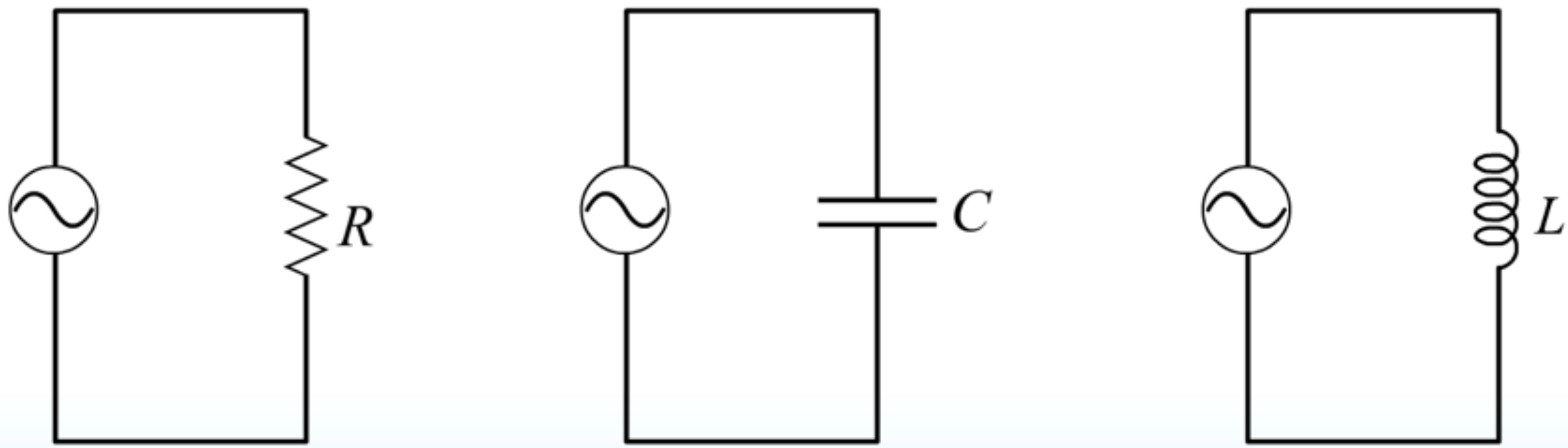
$$V(t) = \mathcal{E}_{\max} \sin \omega t$$

- y-axis projection of the vector

$$\vec{V}(t) = \mathcal{E}_{\max} \cos \omega t \ \hat{i} + \mathcal{E}_{\max} \sin \omega t \ \hat{j}$$



Overview of circuit components: Reactance



$$V = IR$$

$$V = Q/C$$

$$V = L\frac{dI}{dt}$$

$$I_R = \frac{\mathcal{E}_{\max}}{R}\sin\omega t$$

$$I_C = \frac{\mathcal{E}_{\max}}{X_C}\cos\omega t$$

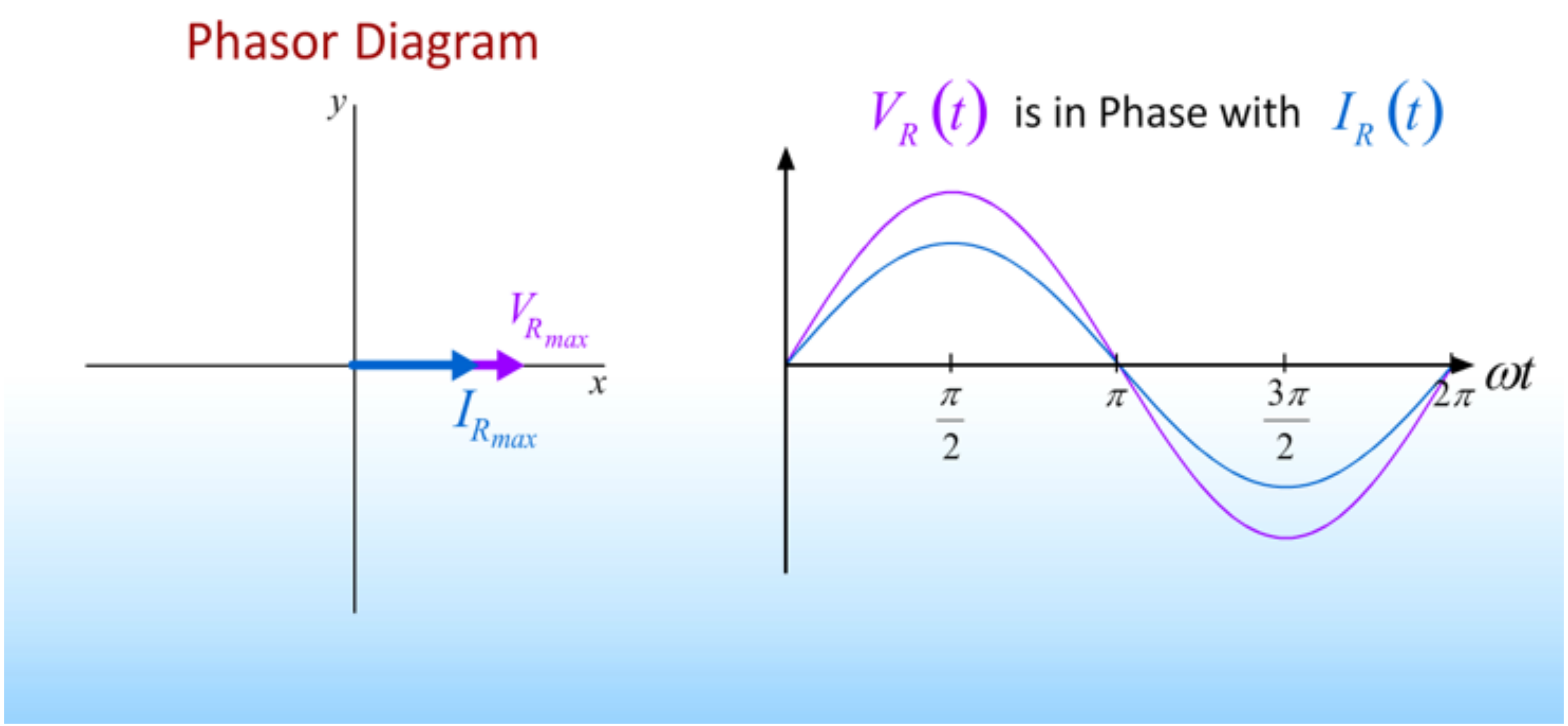
$$I_L = -\frac{\mathcal{E}_{\max}}{X_L}\cos\omega t$$

$$X_R = R$$

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

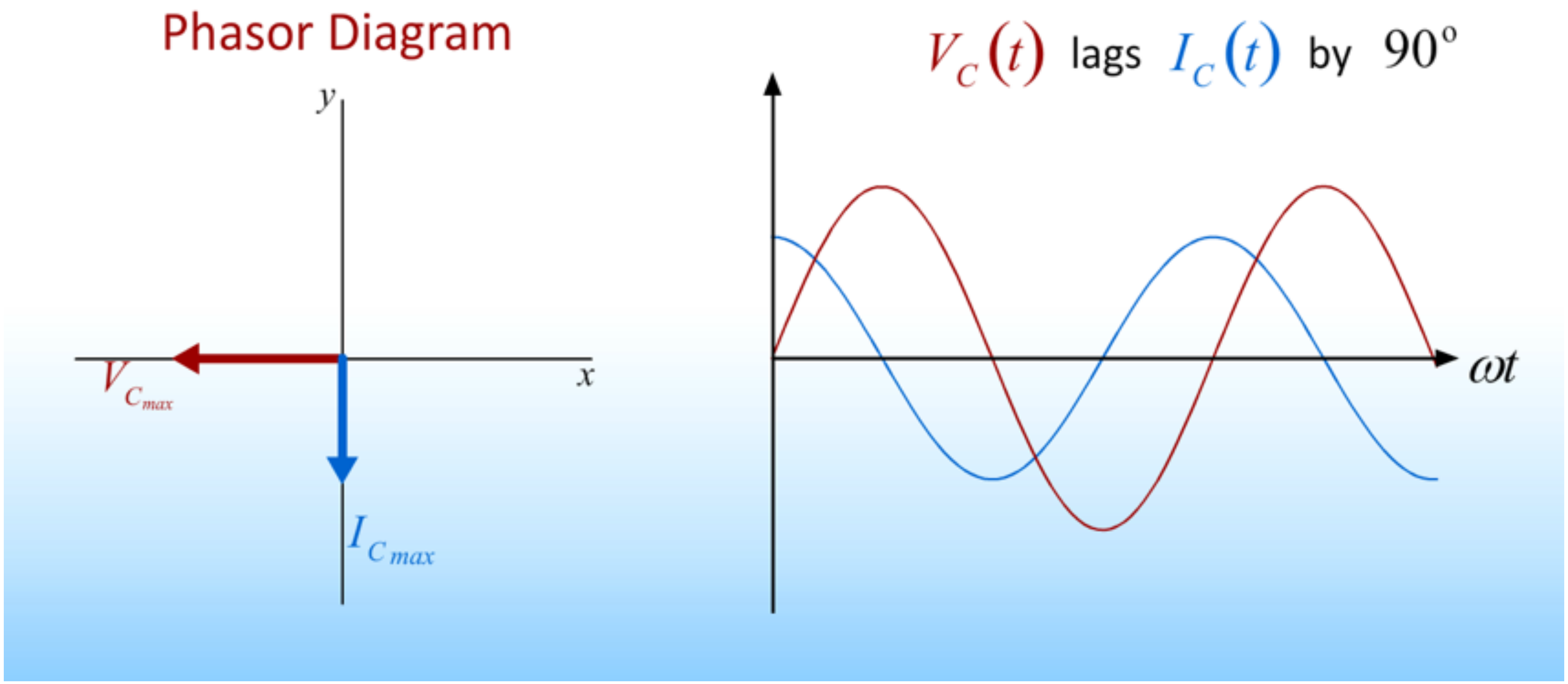
Resistor



$$I_R = \frac{\mathcal{E}_{\max}}{R} \sin \omega t$$

$$\vec{I}_R(t) = \frac{\mathcal{E}_{\max}}{X_R} \left[\cos(\omega t)\ \hat{i} + \sin(\omega t)\ \hat{j}\right]$$

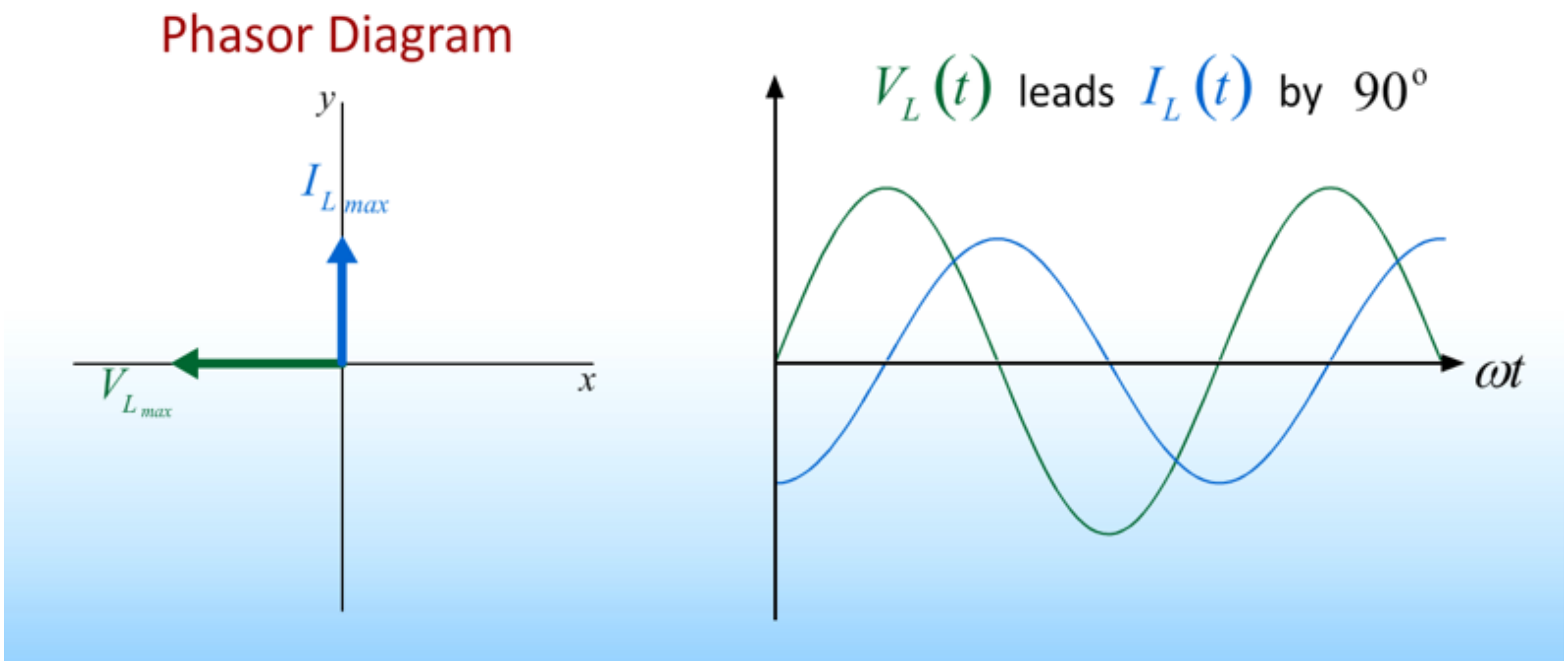
Capacitor



$$I_C = \frac{\mathcal{E}_{\max}}{X_C} \cos \omega t$$

$$\vec{I}_C(t) = \frac{\mathcal{E}_{\max}}{X_C} \left[\cos(\omega t + \tfrac{\pi}{2})\ \hat{i} + \sin(\omega t + \tfrac{\pi}{2})\ \hat{j} \right]$$

Inductor



$$I_L = \frac{\mathcal{E}_{\max}}{X_L}\cos\omega t$$

$$\vec{I}_L(t) = \frac{\mathcal{E}_{\max}}{X_L}\left[\cos(\omega t - \tfrac{\pi}{2})\ \hat{i} + \sin(\omega t - \tfrac{\pi}{2})\ \hat{j}\right]$$

Putting it all together…

Current

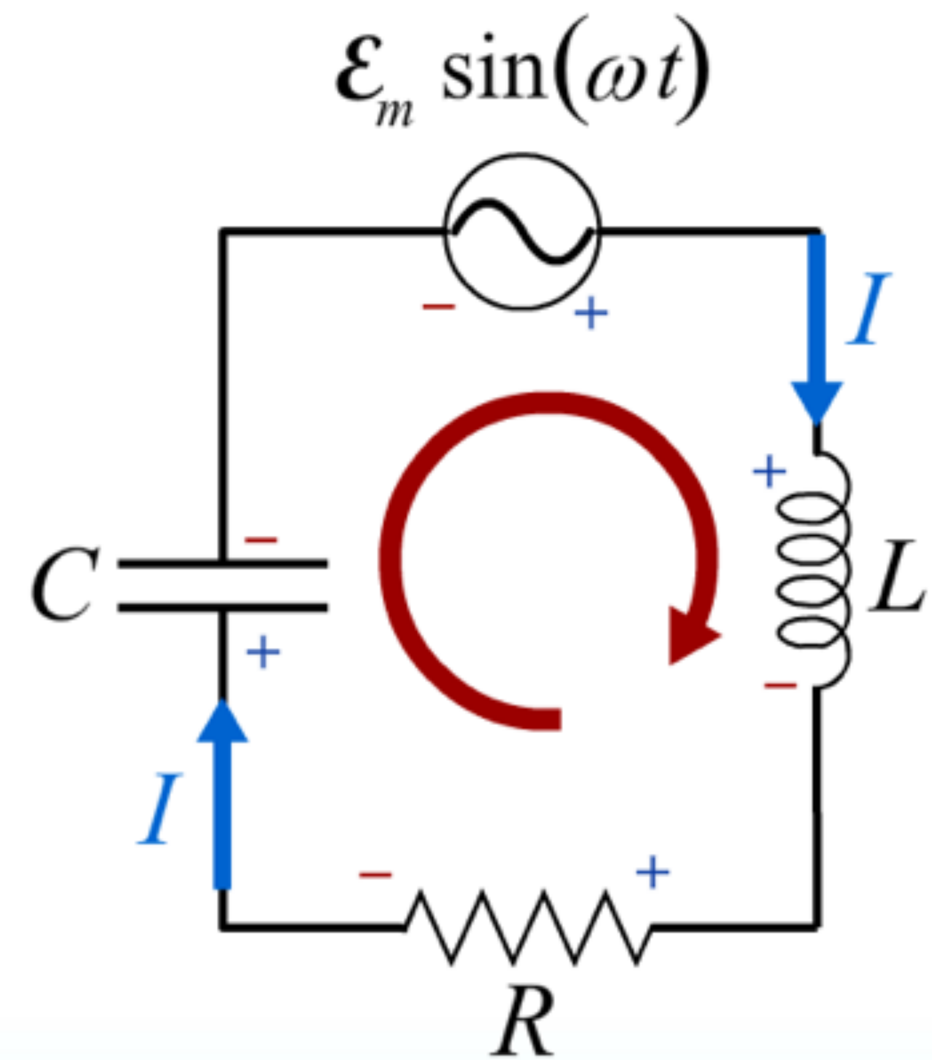
$$I = I_m \sin(\omega t - \phi)$$

KVR

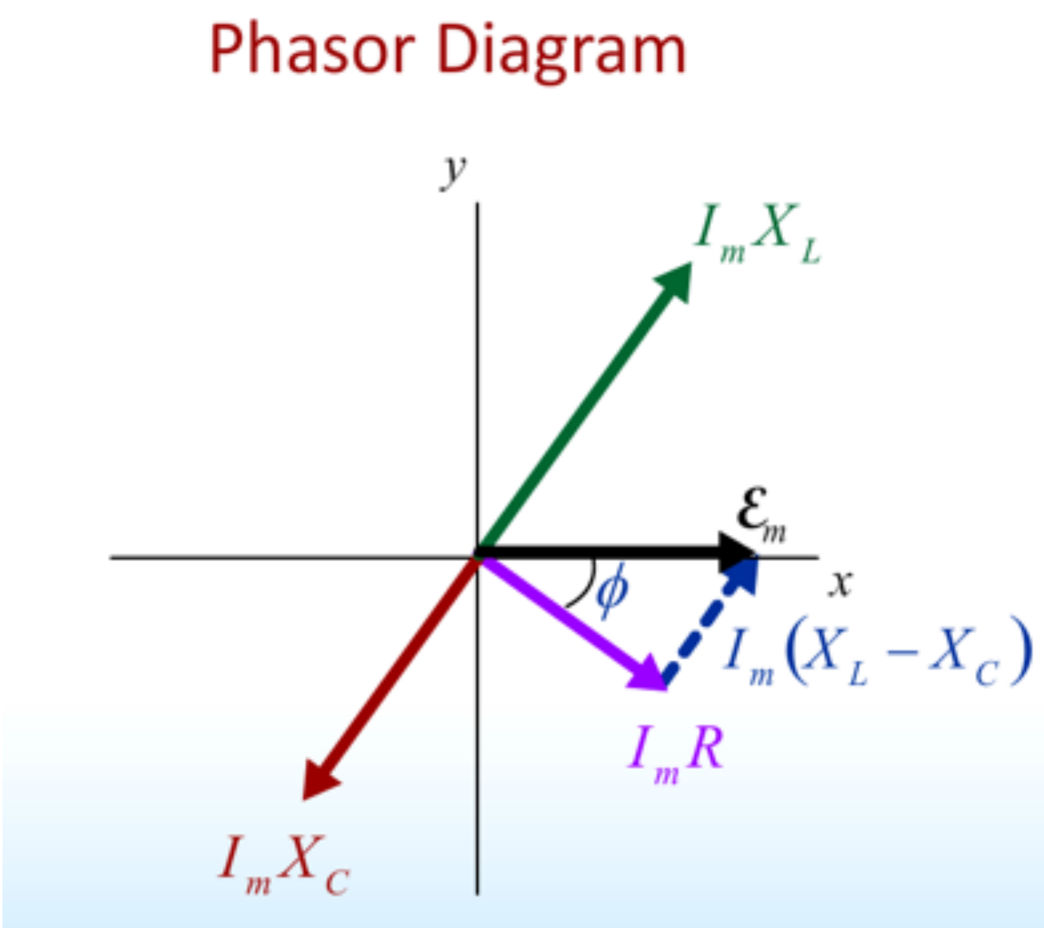
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} - \mathcal{E}_m \sin(\omega t) = 0$$

- Unknowns:

$$I_{\max}, \quad \phi$$



Putting it all together…



- Unknowns:

$$I_{\max}, \quad \phi$$

- Choose unknowns such that:

$$\vec{V}_R + \vec{V}_C + \vec{V}_L = \vec{\mathcal{E}}$$

- Result:

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$