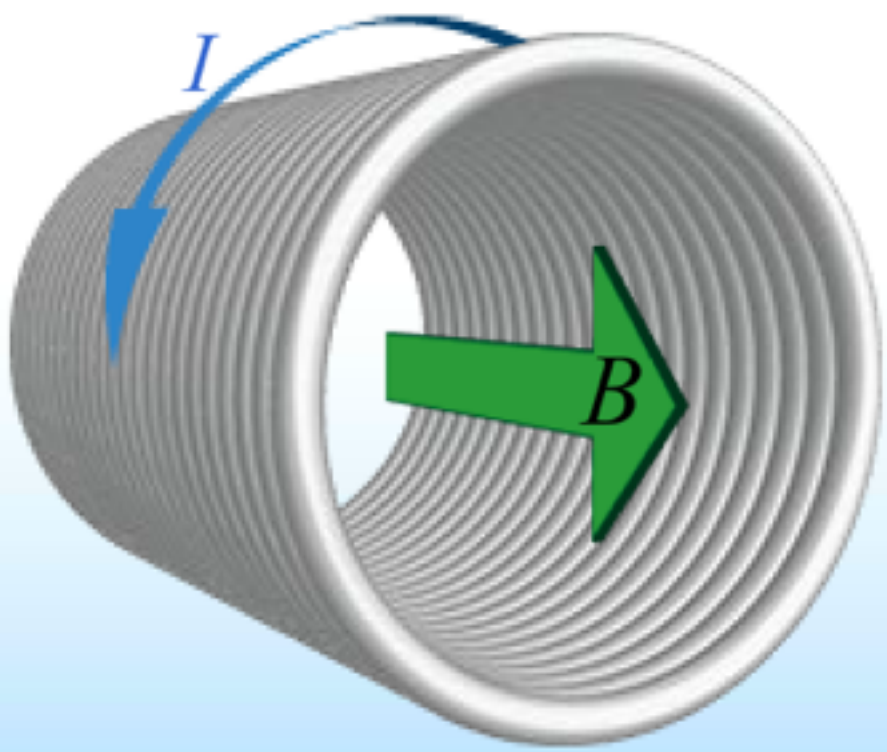
Inductance, inductors, RL circuits

Lecture 23

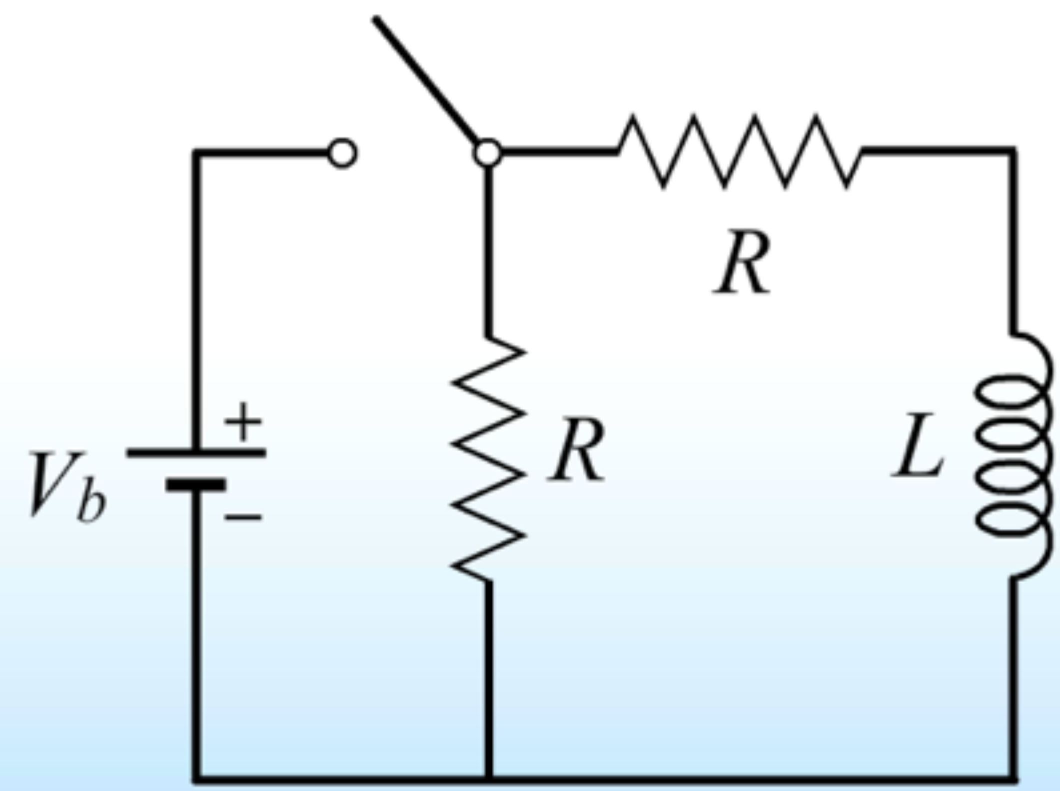
Induction/Inductors

Solenoids



$$L = \Phi_B / I$$

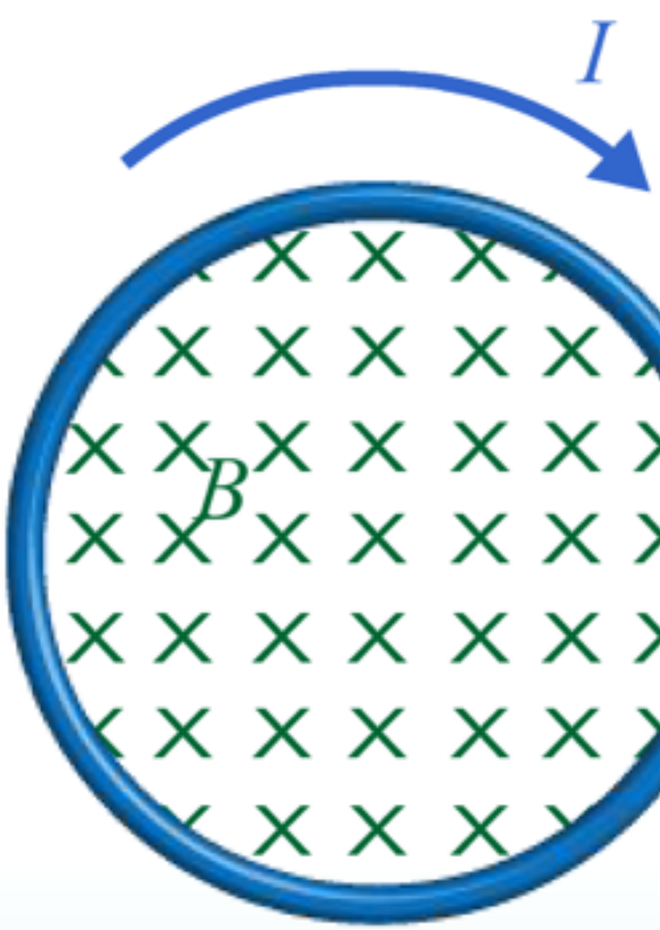
RL Circuit



$$V_L = L\frac{dI}{dt}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} = -V_L$$

Self-Inductance



Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

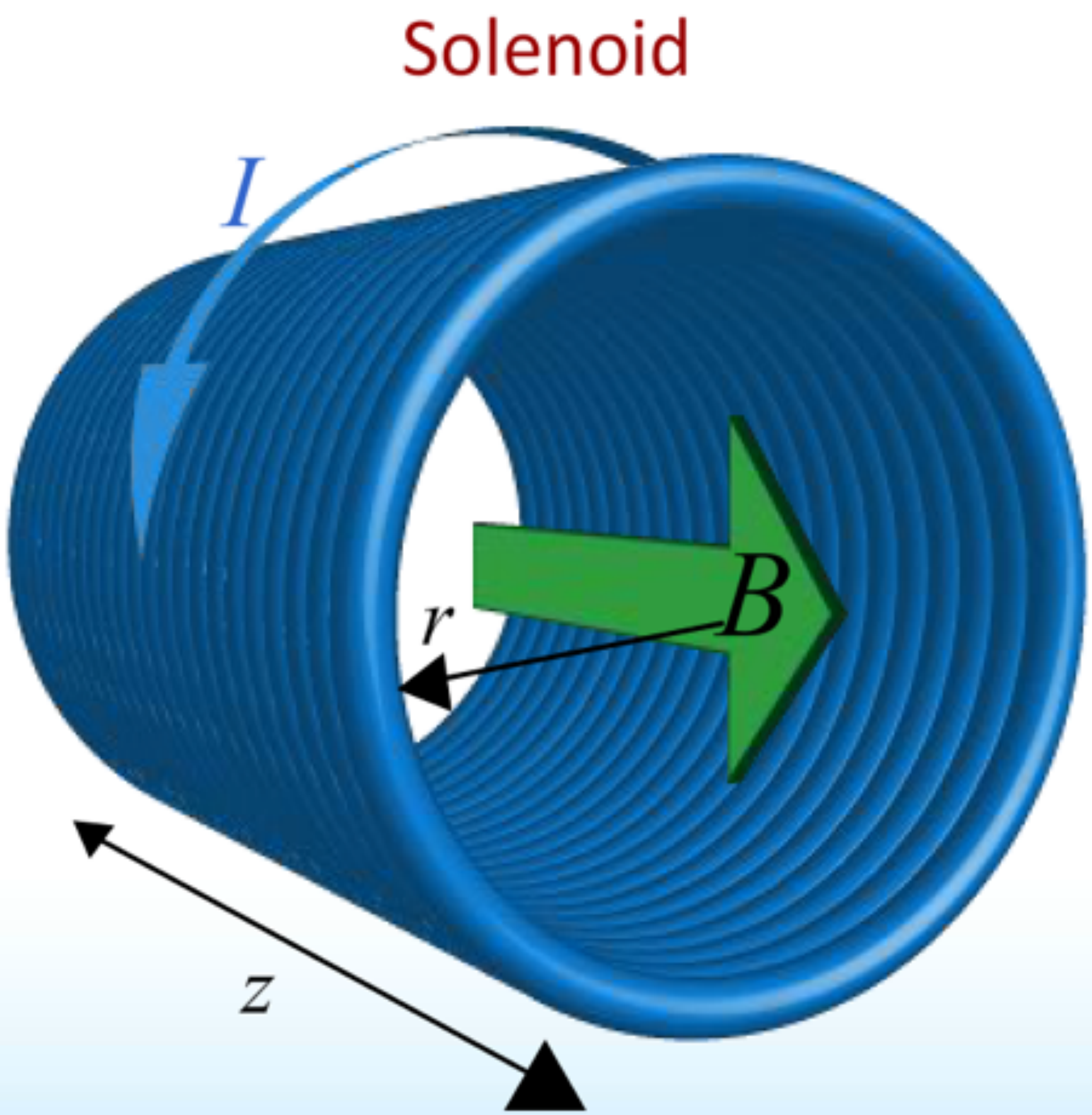
SI Unit

$$H = T\text{-}m^2/A$$

Inductor Voltage

$$\mathcal{E} = -L\frac{dI}{dt}$$

Calculation of inductance:



Magnetic Field

$$B = \mu_o n I$$

Magnetic Flux

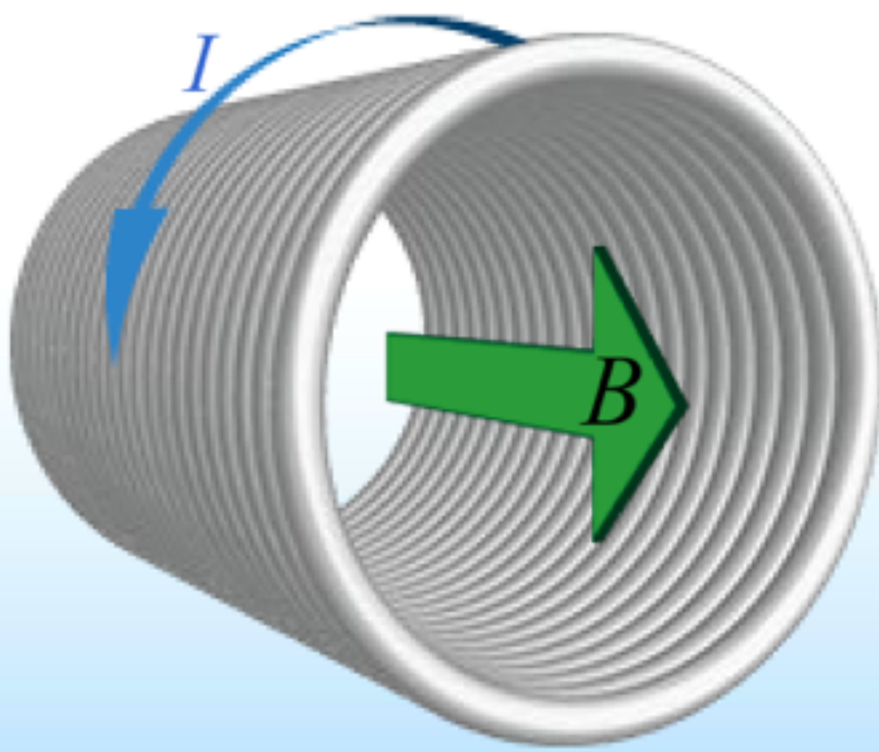
$$\Phi_B = \mu_o n^2 z \pi r^2 I$$

Self-Inductance

$$L = \mu_o n^2 z \pi r^2$$

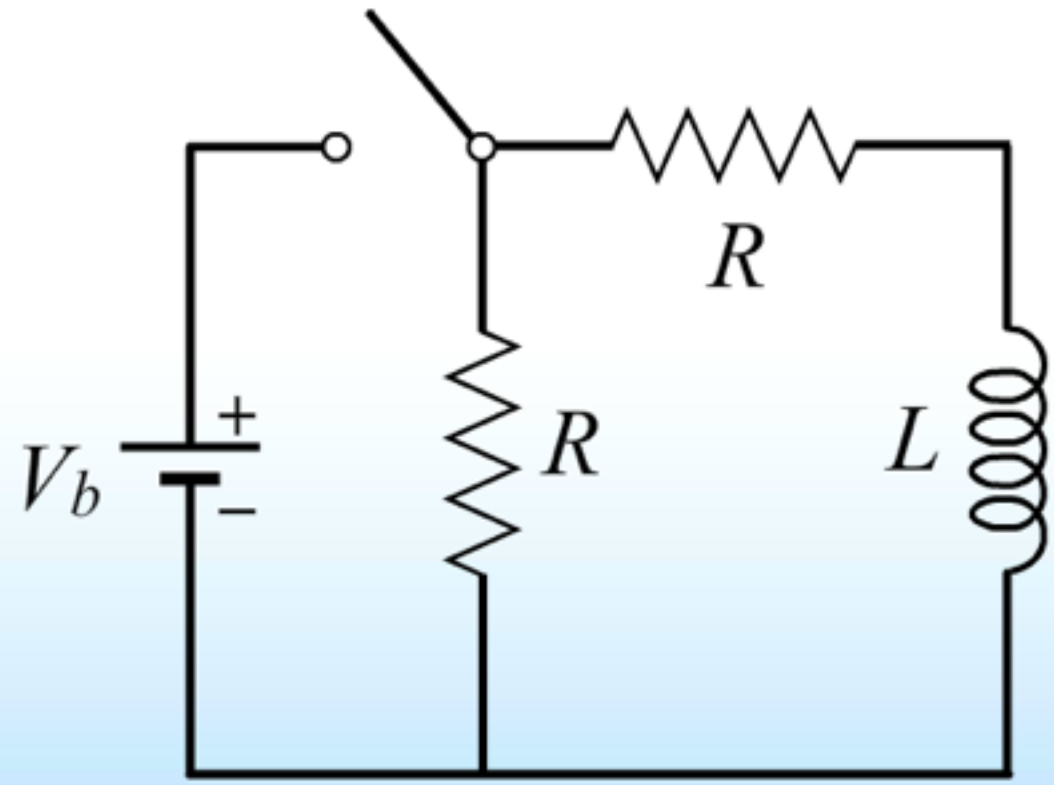
New: RL circuits

Solenoids



$$L = \Phi_B / I$$

RL Circuit



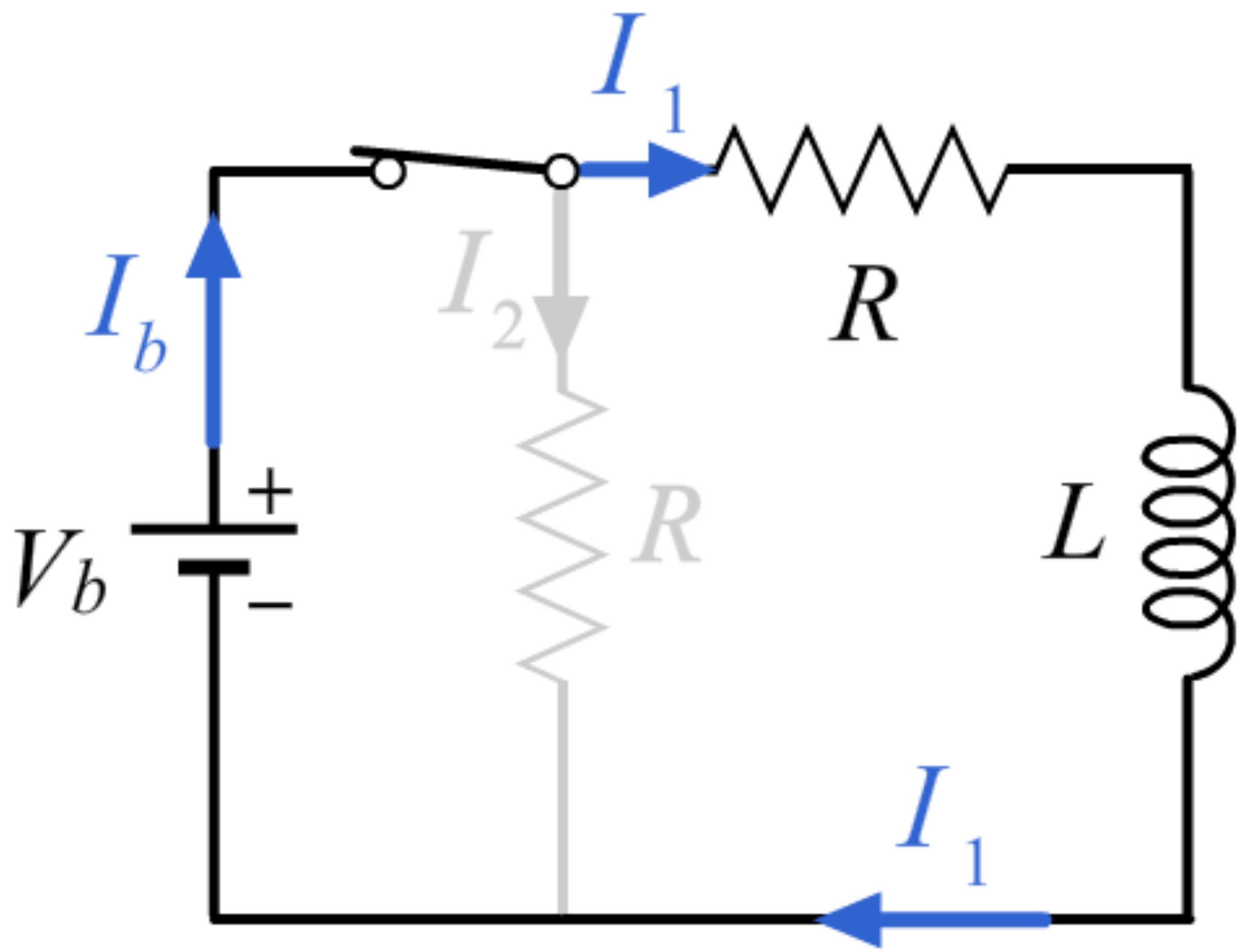
$$V_L = L\frac{dI}{dt}$$

(Reasoning) RL Circuits 1

Closing Switch

$t = 0$	
$V_R(0) =$	
$V_L(0) =$	
$I_1(0) =$	

$t \to \infty$	
$V_R(\infty) =$	
$V_L(\infty) =$	
$I_1(\infty) =$	



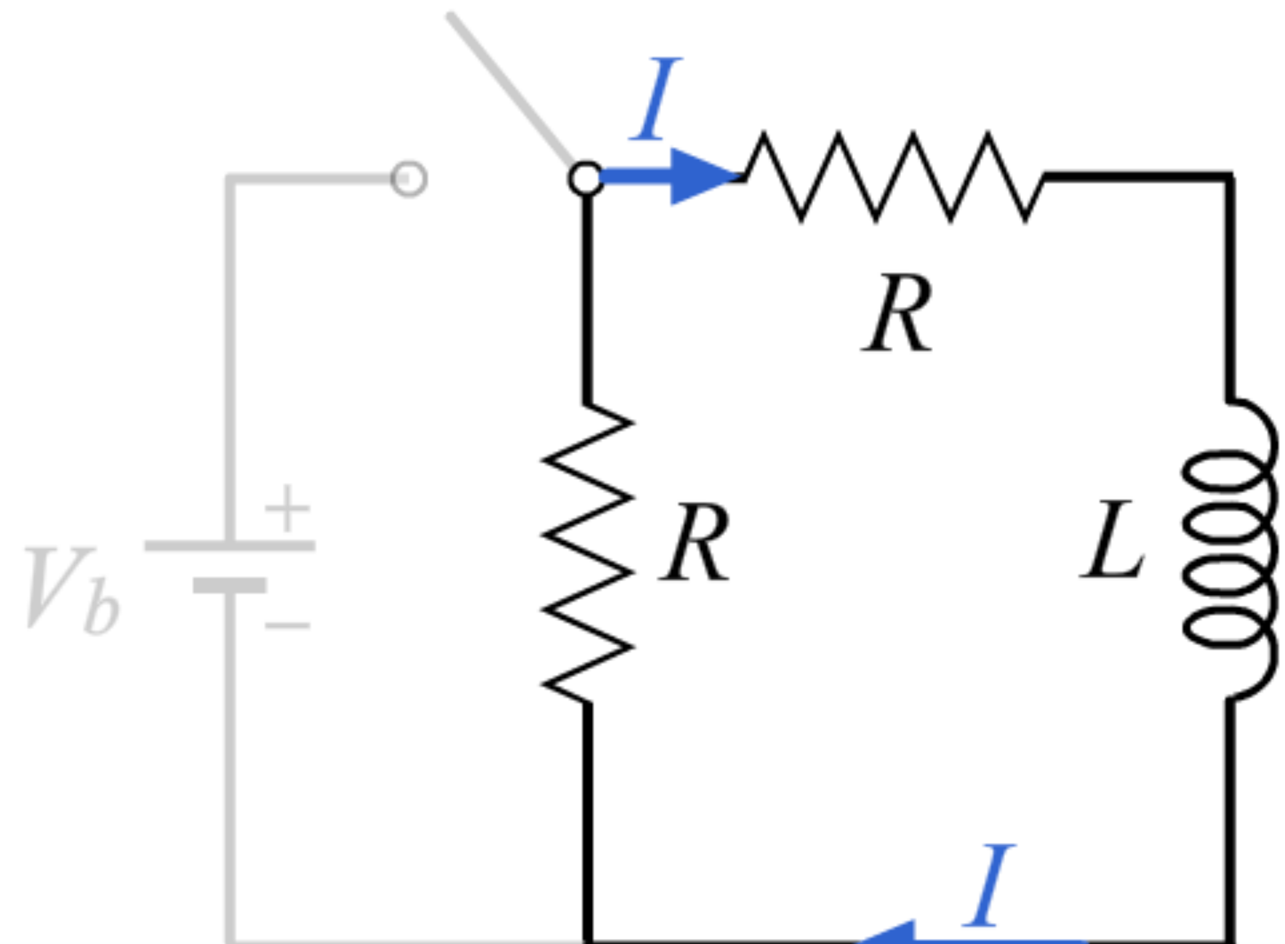
$$V_L = L\frac{dI}{dt}$$

(Reasoning) RL Circuits 1

Opening Switch

$t = 0$
$V_R(0)=$
$V_L(0)=$
$I(0)=$

$t \to \infty$
$V_R(\infty)=$
$V_L(\infty)=$
$I(\infty)=$



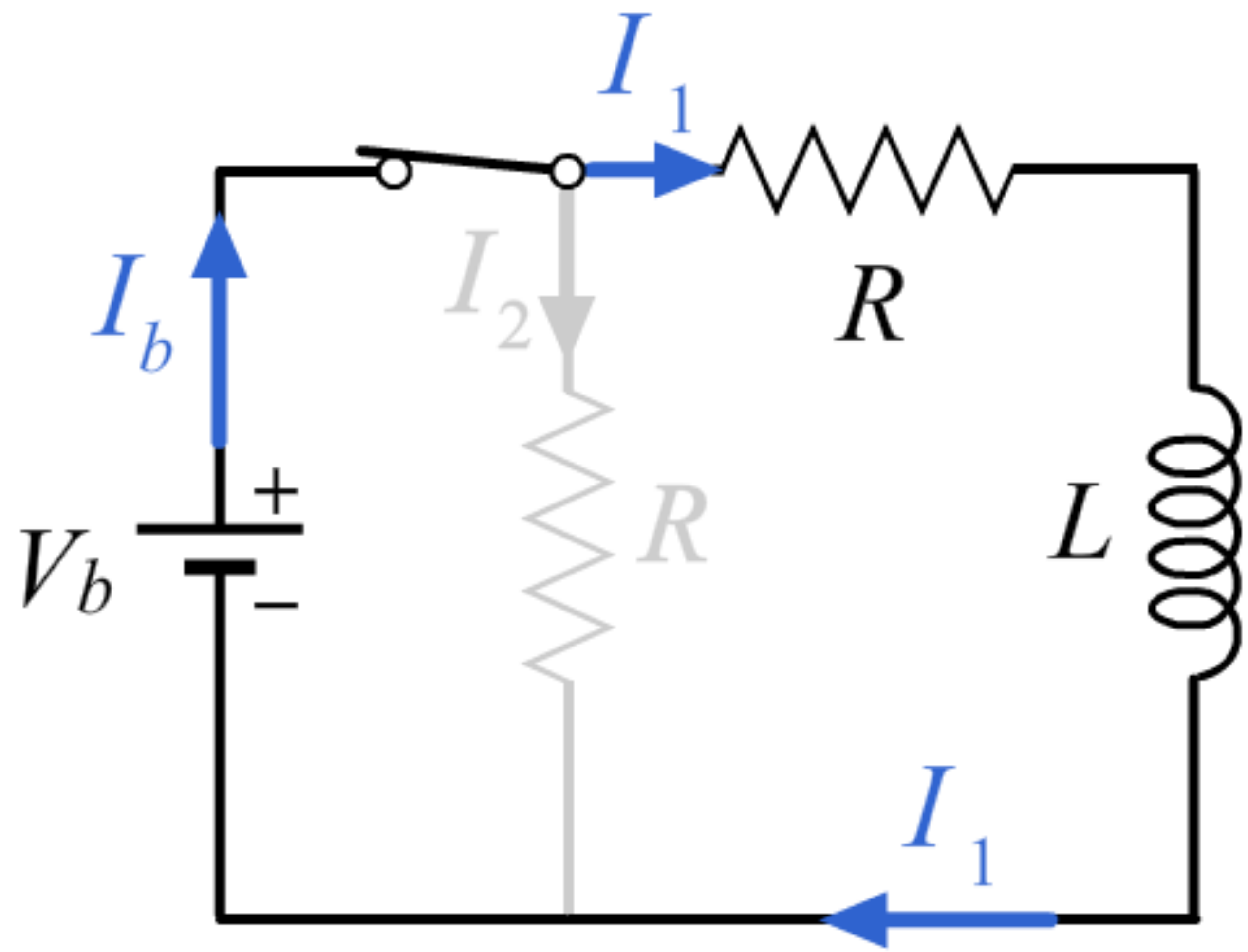
$$V_L = L\frac{dI}{dt}$$

RL Circuits: Detailed Analysis

Closing Switch

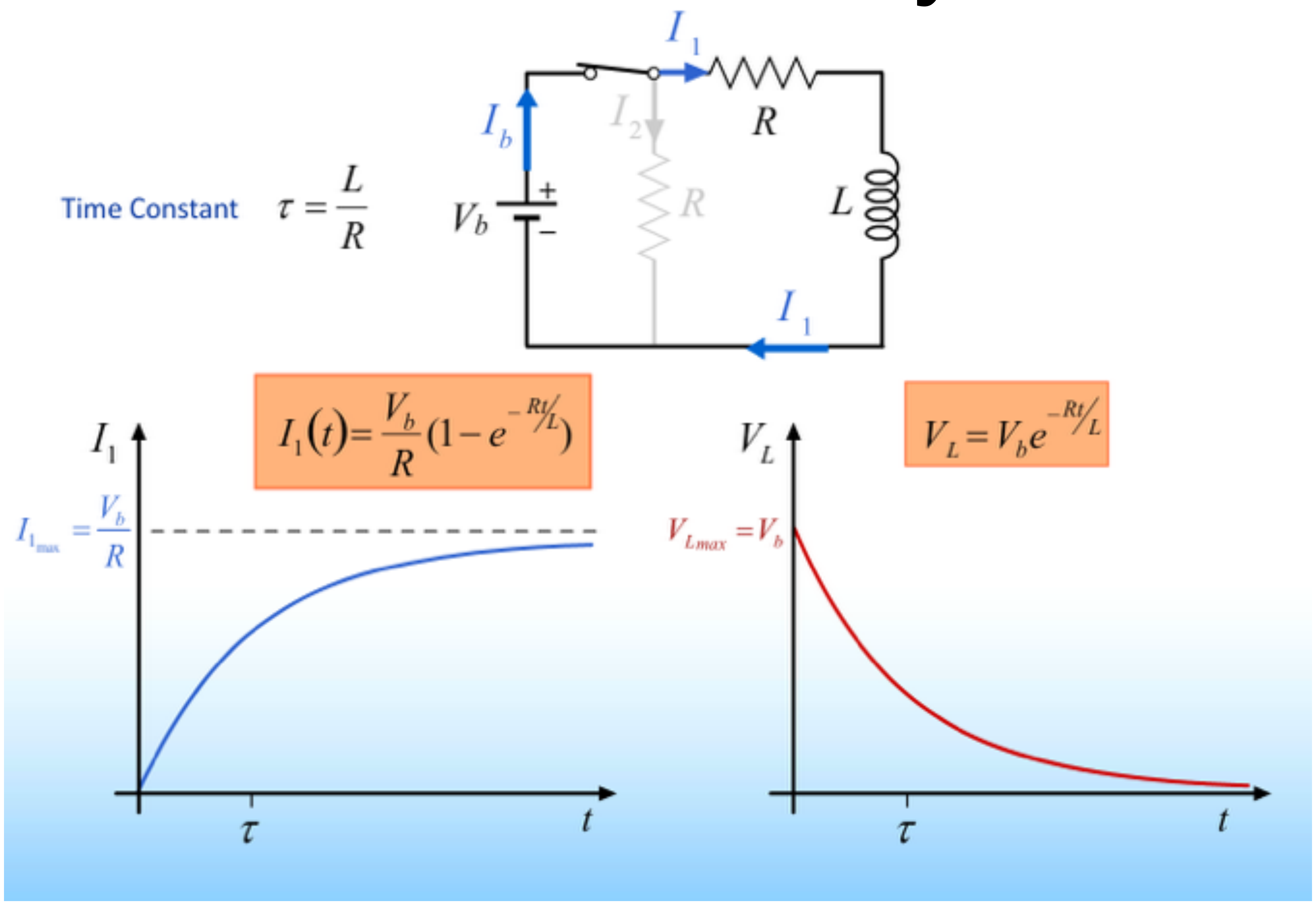
$t = 0$
$V_R(0) = 0$
$V_L(0) = V_b$
$I_1(0) = 0$

$t \to \infty$
$V_R(\infty) = V_b$
$V_L(\infty) = 0$
$I_1(\infty) = \frac{V_b}{R}$



$$V_L = L\frac{dI}{dt}$$

RL Circuits: Detailed Analysis



- Results

RL Circuits: Detailed Analysis

Opening Switch

$$t = 0$$

$$V_R(0) = V_b$$

$$V_L(0) = -2V_b$$

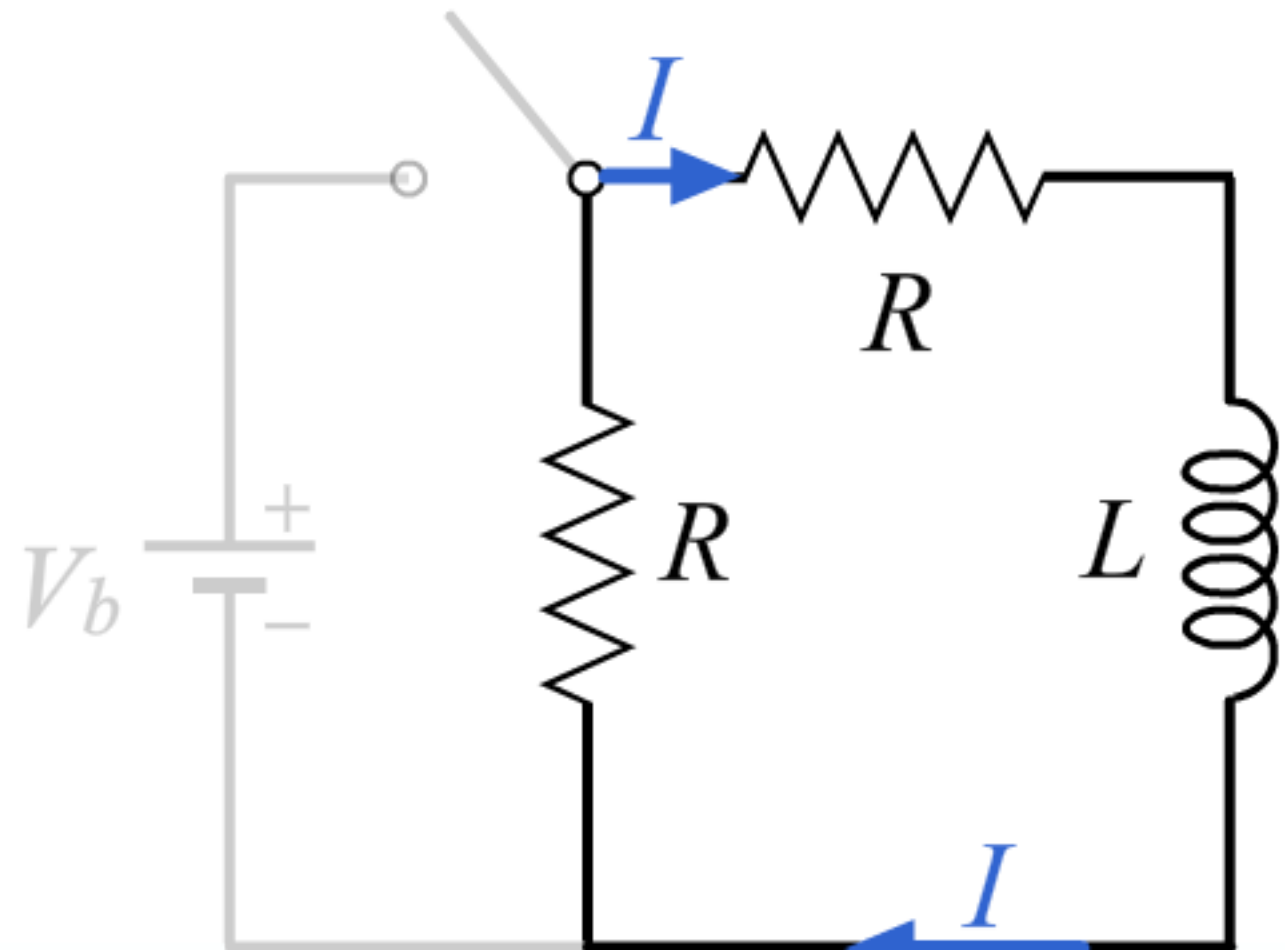
$$I(0) = \frac{V_b}{R}$$

$$t \to \infty$$

$$V_R(\infty) = 0$$

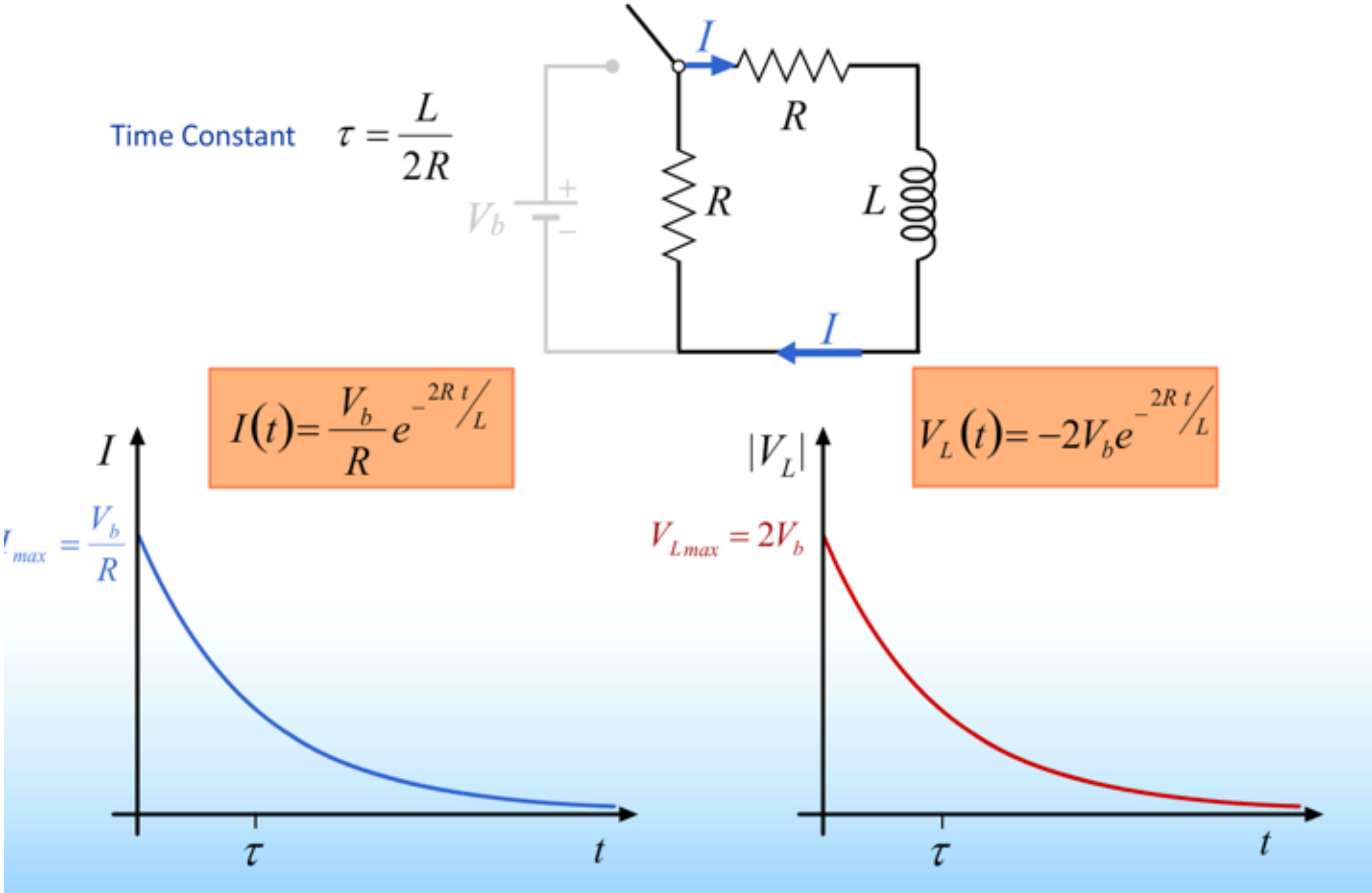
$$V_L(\infty) = 0$$

$$I(\infty) = 0$$



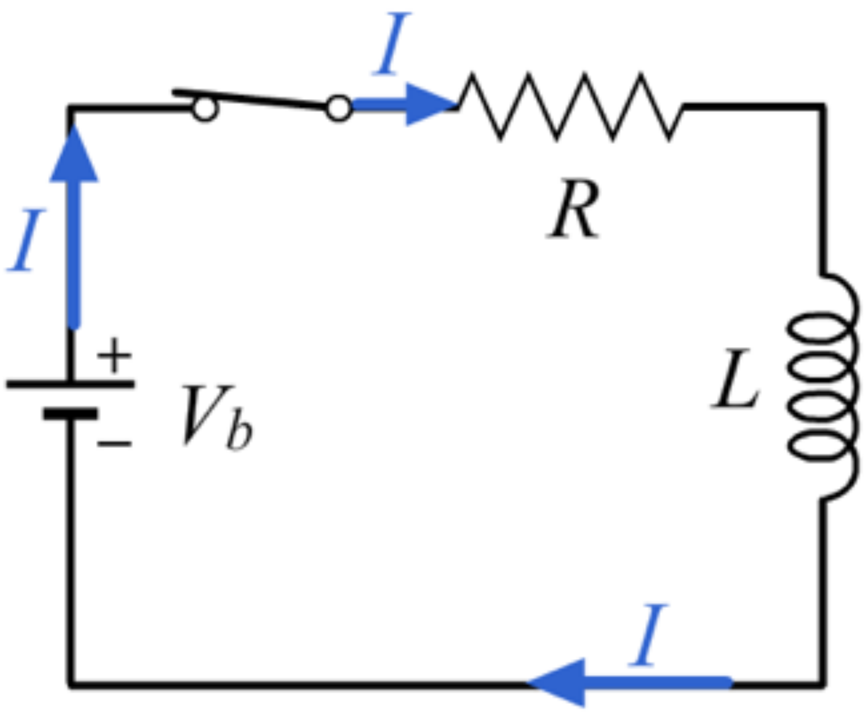
$$V_L = L\frac{dI}{dt}$$

RL Circuits: Detailed Analysis

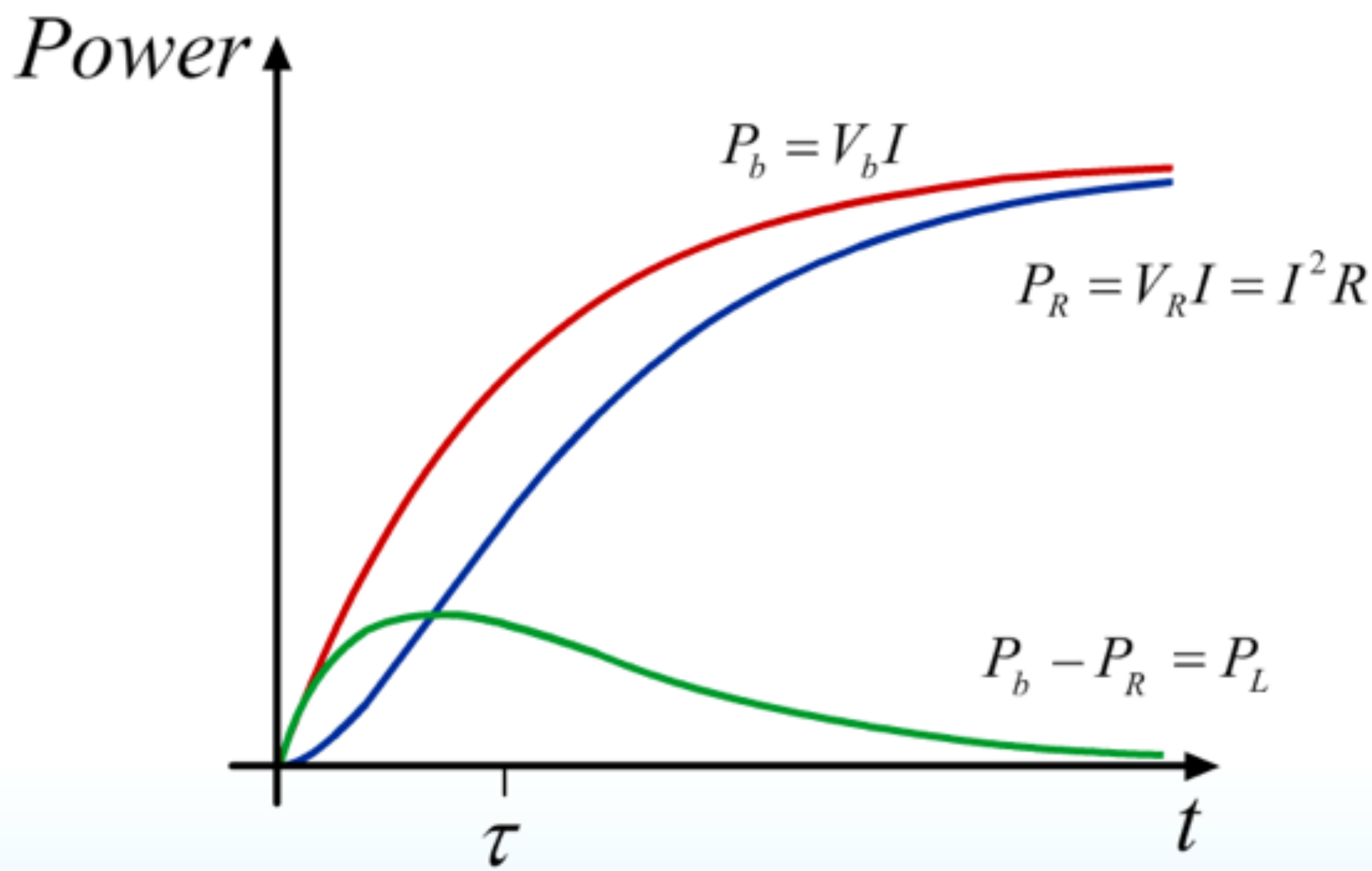


- Results

Where is the energy stored?

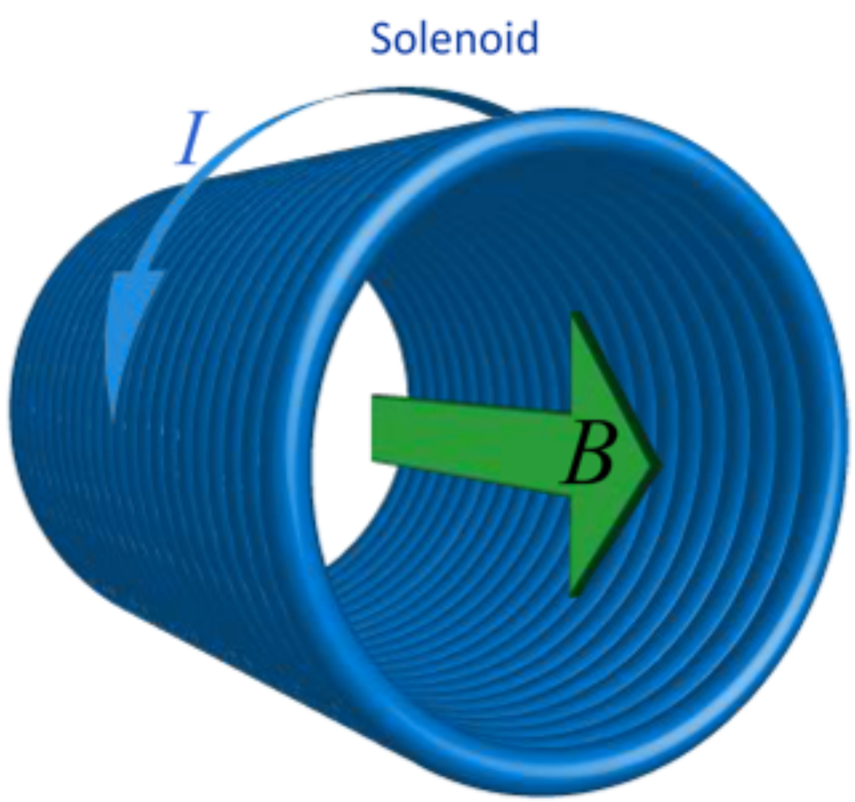


$$I(t) = \frac{V_b}{R}\left(1 - e^{-Rt/L}\right)$$



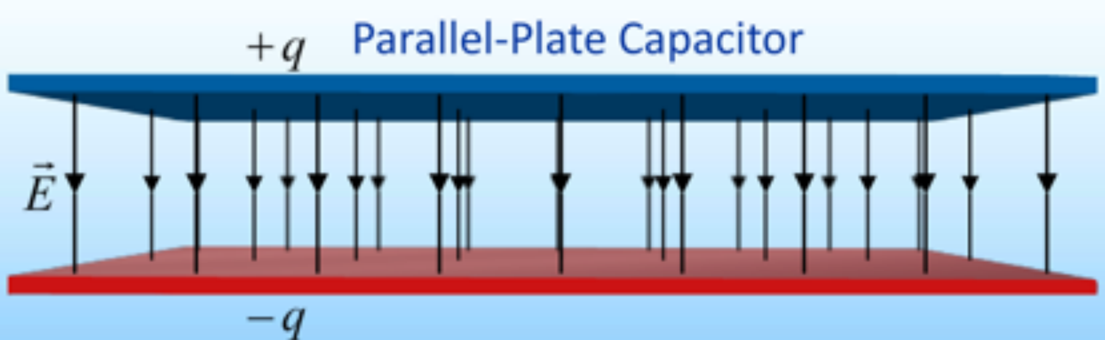
$$P_B - P_R = V_b I - I^2 R$$

Where is the energy stored?



Magnetic Energy Density

$$u_B = \frac{B^2}{2\mu_o}$$



Electric Energy Density

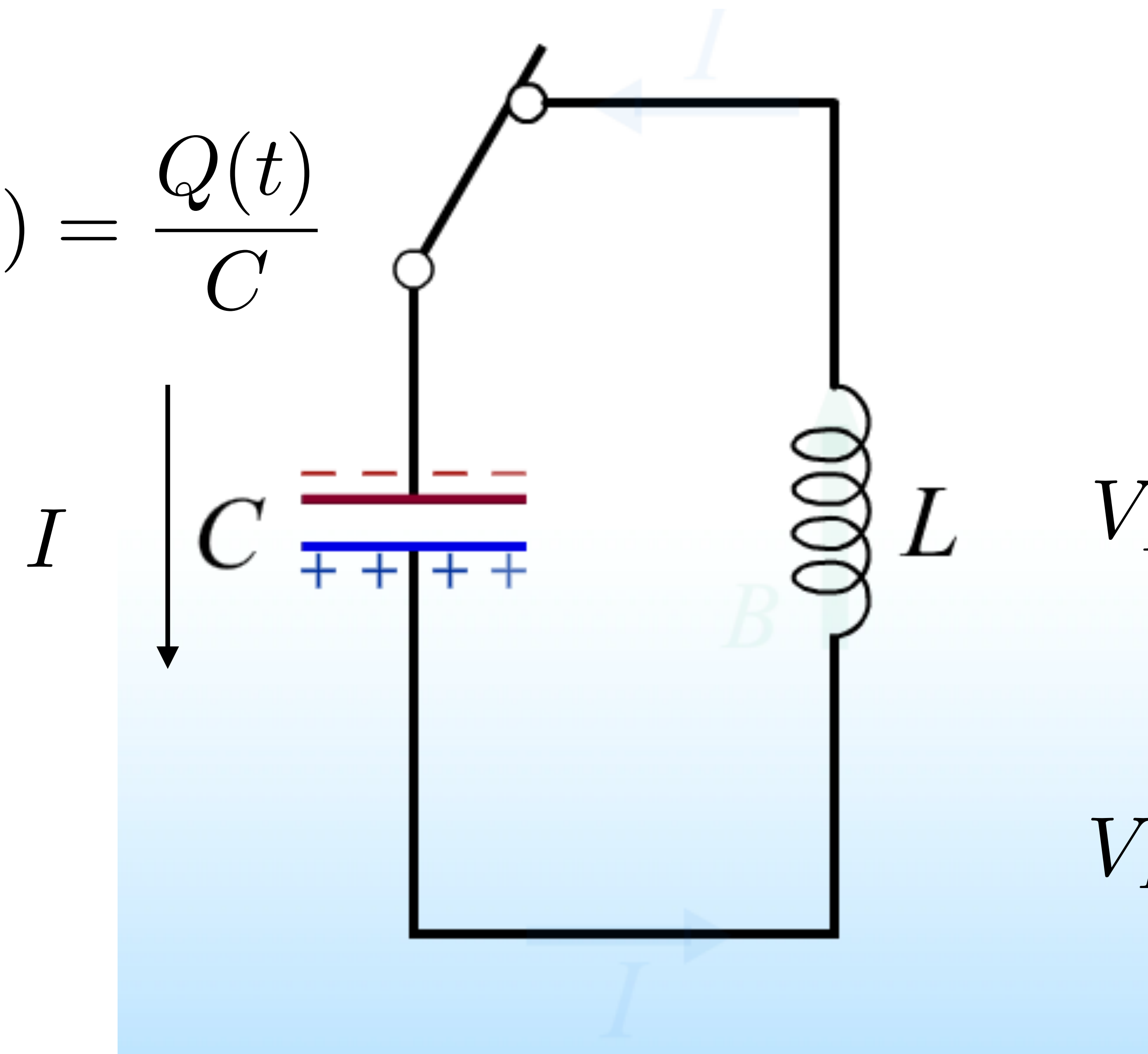
$$u_E = \frac{1}{2}\varepsilon_o E^2$$

RLC circuits

Lecture 25

LC Circuits

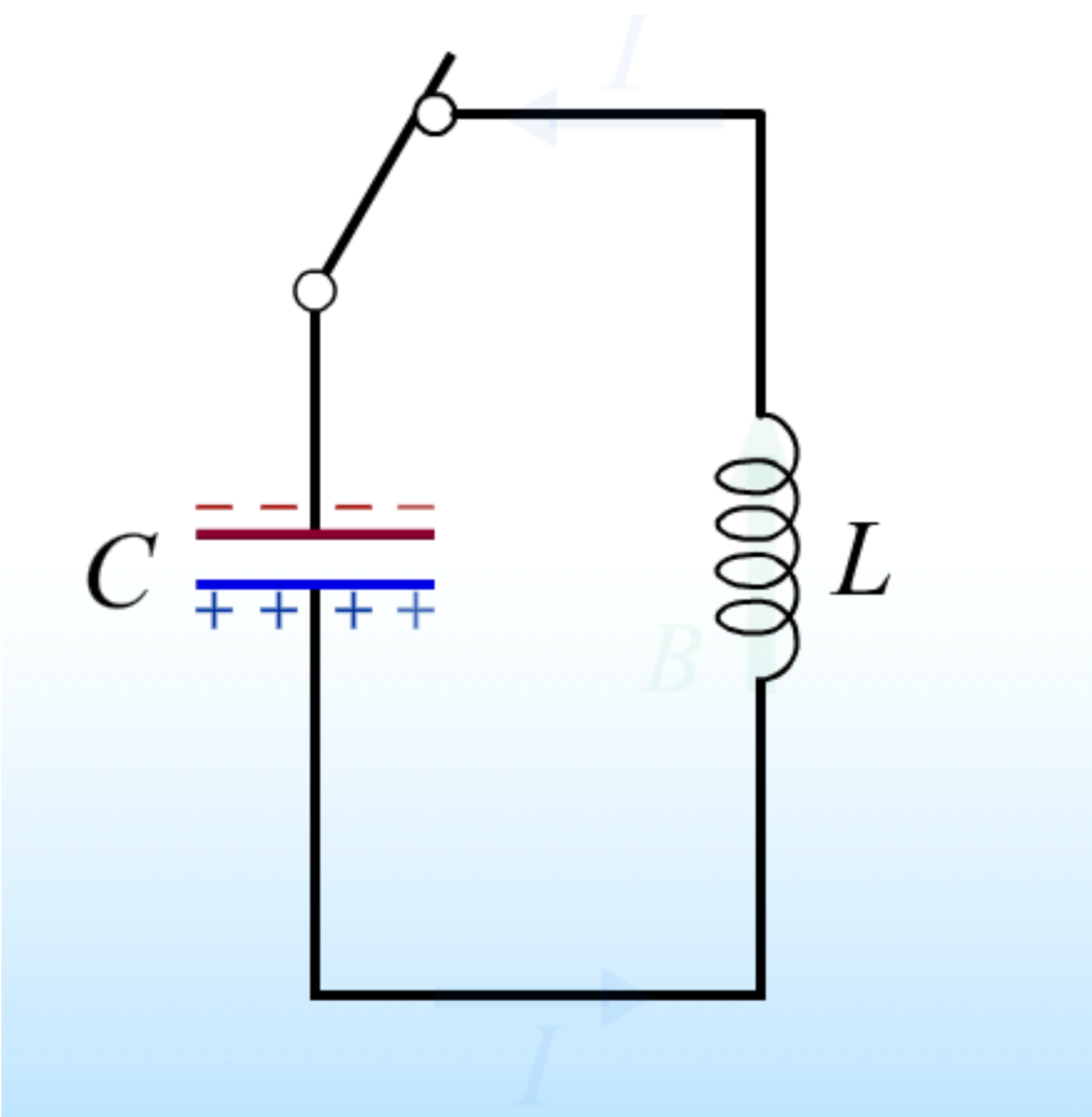
$$V_C(t) = \frac{Q(t)}{C}$$



$$V_L(t) = L\frac{dI(t)}{dt}$$

$$V_L(t) = L\frac{d^2Q(t)}{dt^2}$$

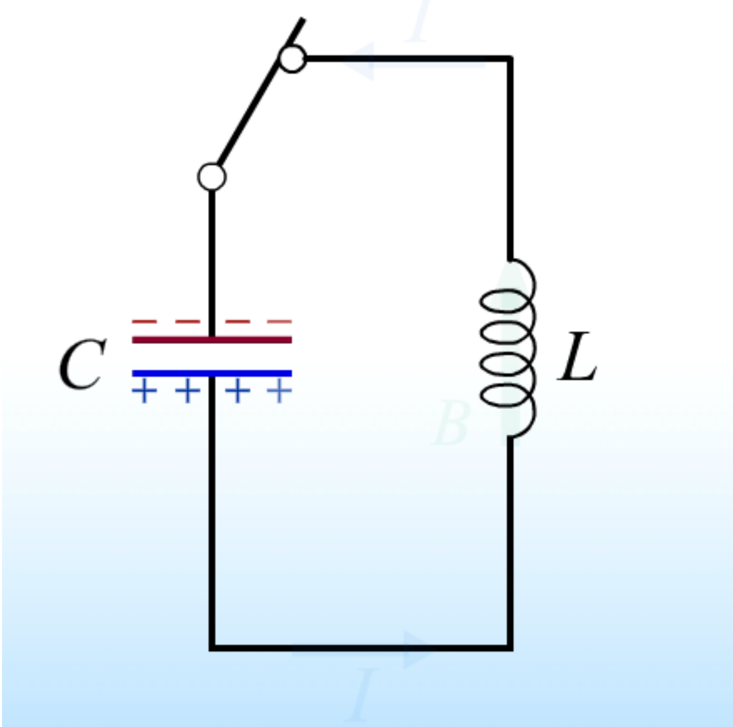
LC Circuits: Detailed Analysis

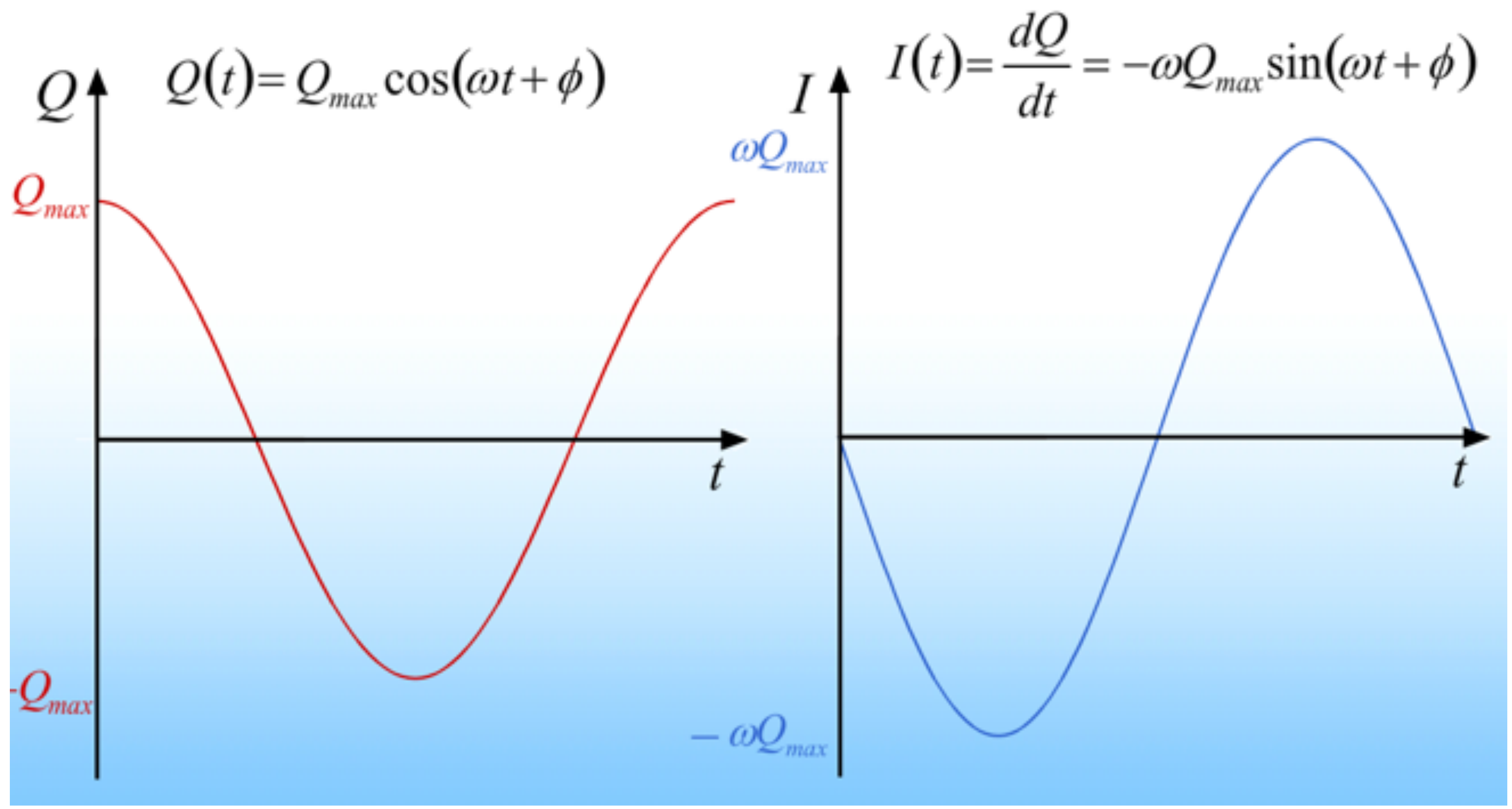


$$\text{KVL}: \quad V_L(t) + V_C(t) = 0$$

$$0 = \frac{Q(t)}{C} + L\frac{d^2Q(t)}{dt^2}$$

LC Circuits: Detailed Analysis

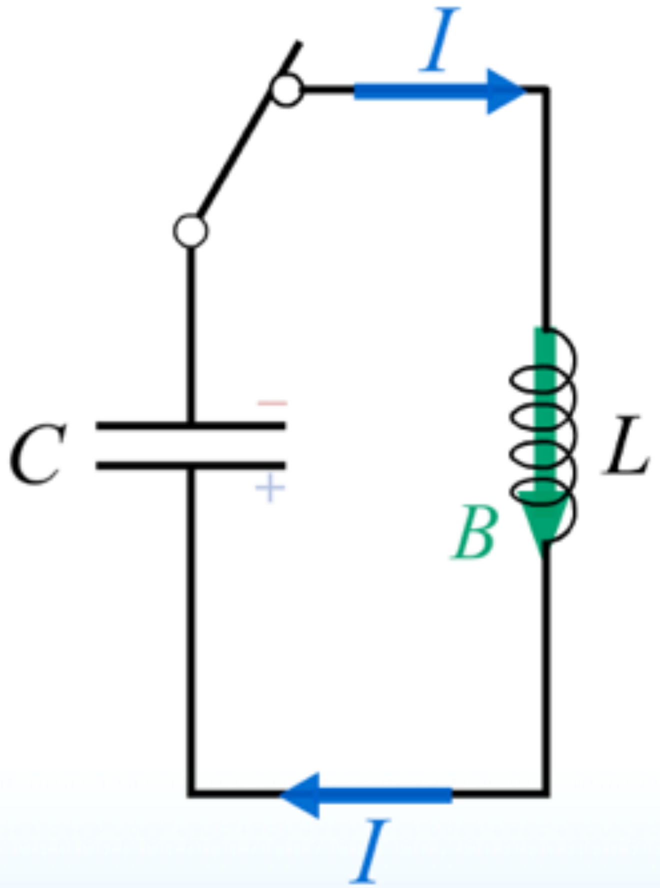




$$\omega = \sqrt{\frac{1}{LC}}$$

LC Circuits:
Energy conservation

LR Circuit



Inductor Energy

$$U_L = \frac{1}{2} L I^2$$

→

Kinetic Energy

$$K = \frac{1}{2} m v^2$$

Capacitor Energy

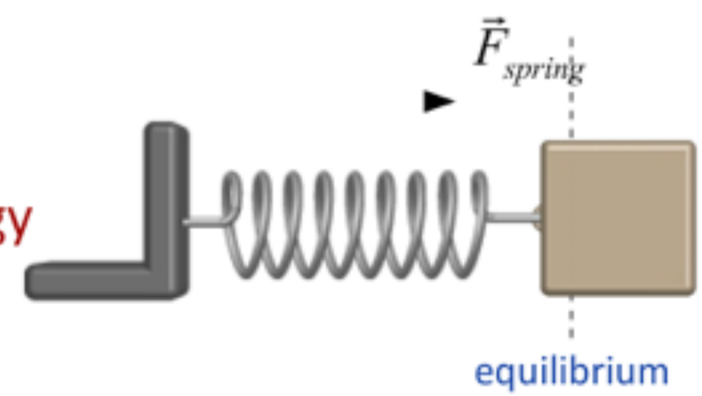
$$U_C = \frac{1}{2} \frac{Q^2}{C}$$

→

Spring Potential Energy

$$U_{spring} = \frac{1}{2} k x^2$$

Oscillating Block



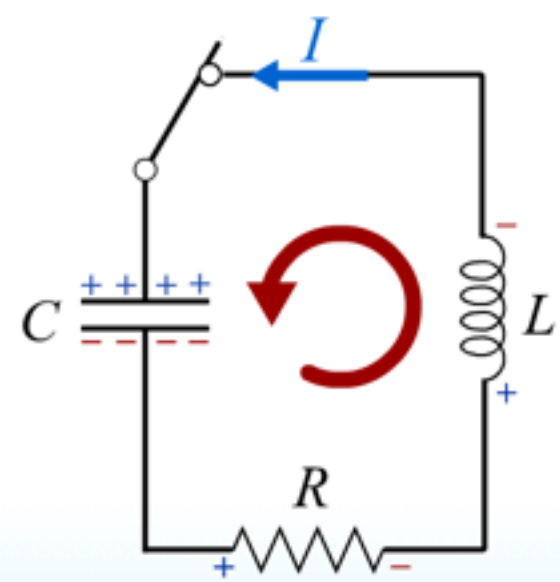
LRC Circuits

KVR

$$\frac{Q}{C} + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} = 0$$

Solution

$$Q(t) = A\,e^{-\beta t}\cos(\omega' t + \phi)$$



Damping Factor	$\beta = \frac{R}{2L}$
Oscillation Frequency	$\omega'^2 = \omega_o^2 - \beta^2$
Natural Frequency	$\omega_o = \frac{1}{\sqrt{LC}}$