Faraday Law (of Induction), (Maxwell-)Faraday Law & Lenz Law

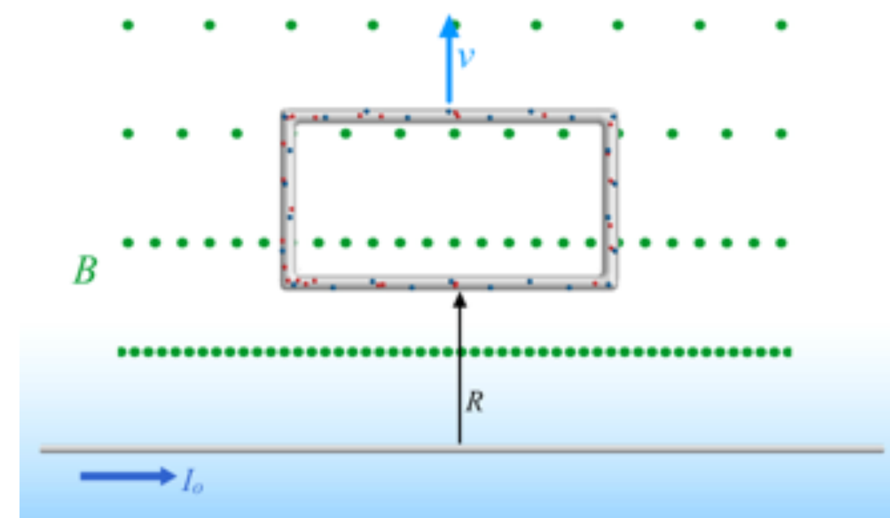
Lecture 22

Faraday Law
(of Induction)

- Changes in the Magnetic Flux through a loop induces an EMF:

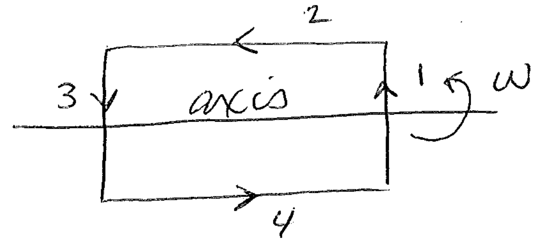
$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

$$\Phi_B \equiv \int_{\mathcal{M}} d^2A\,\hat{n}\cdot\vec{B}$$



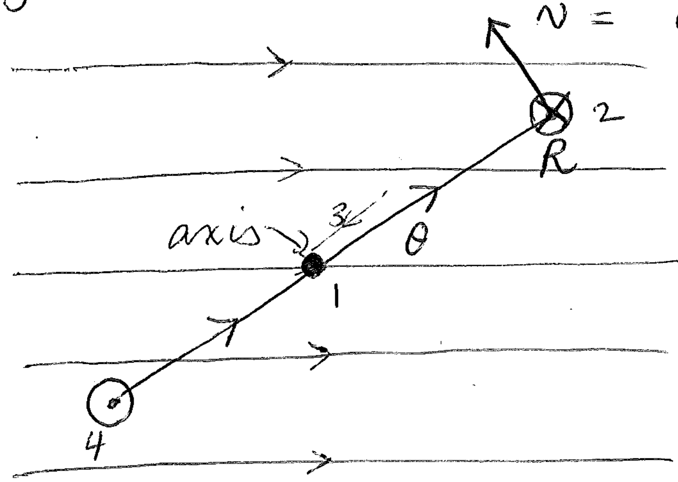
- **Transformer EMF:** Changes in B generate E
- **Motional EMF:** Changes in M generate F_B

Generator EMF Calculation



$\vec{v} \times \vec{B}$ calculation:

$$\mathcal{E} = \oint d\vec{l} \cdot (\vec{E} + \vec{v} \times \vec{B})$$



$$\vec{v} = \omega R \hat{\theta} \qquad \hat{\theta} = -\sin\theta \, \hat{x} + \cos\theta \, \hat{y}$$

$$\vec{B} = B \hat{x}$$

$$\vec{v} \times \vec{B} = -B\omega R \cos\theta \, \hat{z}$$

$$\theta = \omega t$$

$$\mathcal{E} = \oint d\vec{l} \cdot (\underbrace{\vec{E}}_{0} + \vec{v} \times \vec{B})$$

$$= \int_1 dl \, \underbrace{\hat{r} \cdot (\vec{v} \times \vec{B})}_{0} + \int_2 dl \, (-\hat{z}) \cdot (\vec{v} \times \vec{B}) + \ldots$$

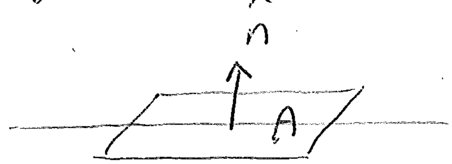
$$\int_3 dl \, \underbrace{\hat{r} \cdot (\vec{v} \times \vec{B})}_{0} + \int_4 dl \, (\hat{z}) \cdot (\vec{v} \times \vec{B})$$

$$= \underbrace{LB\omega R \cos\theta}_{2} + \underbrace{LB\omega R \cos\theta}_{4}$$

$$= \omega \underbrace{(2RL)}_{A} \cdot B \cos\theta$$

$$= \omega A B \cos\theta = \omega A B \cos \omega t$$

Faraday Law Induction:



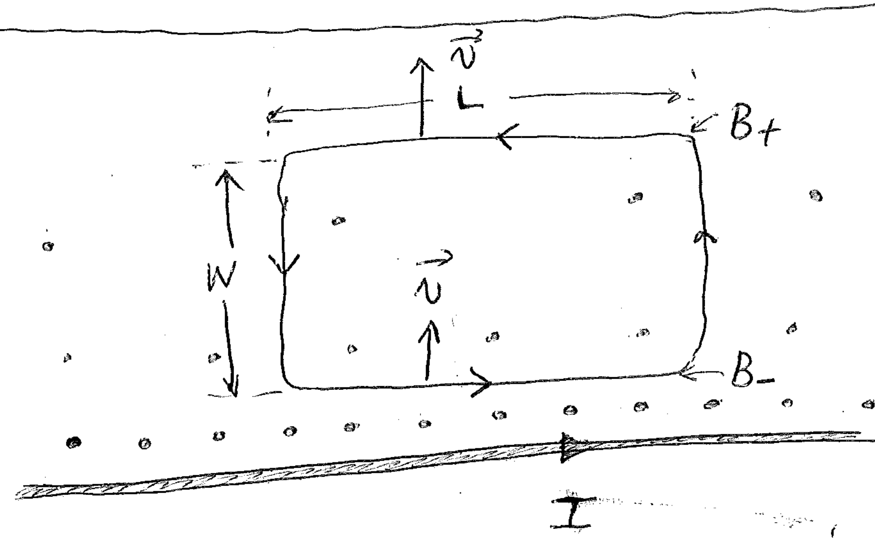
$$\hat{n} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\vec{B} = \hat{x}\,B$$

$$\Phi_B = \int d^2A\;\hat{n}\cdot\vec{B} = -AB\sin\theta = -AB\sin\omega t$$

$$\mathcal{E} = -\frac{d}{dt}\Phi_B(t) = \omega AB\cos\theta = \omega AB\cos\omega t$$

We get the same answer both ways.



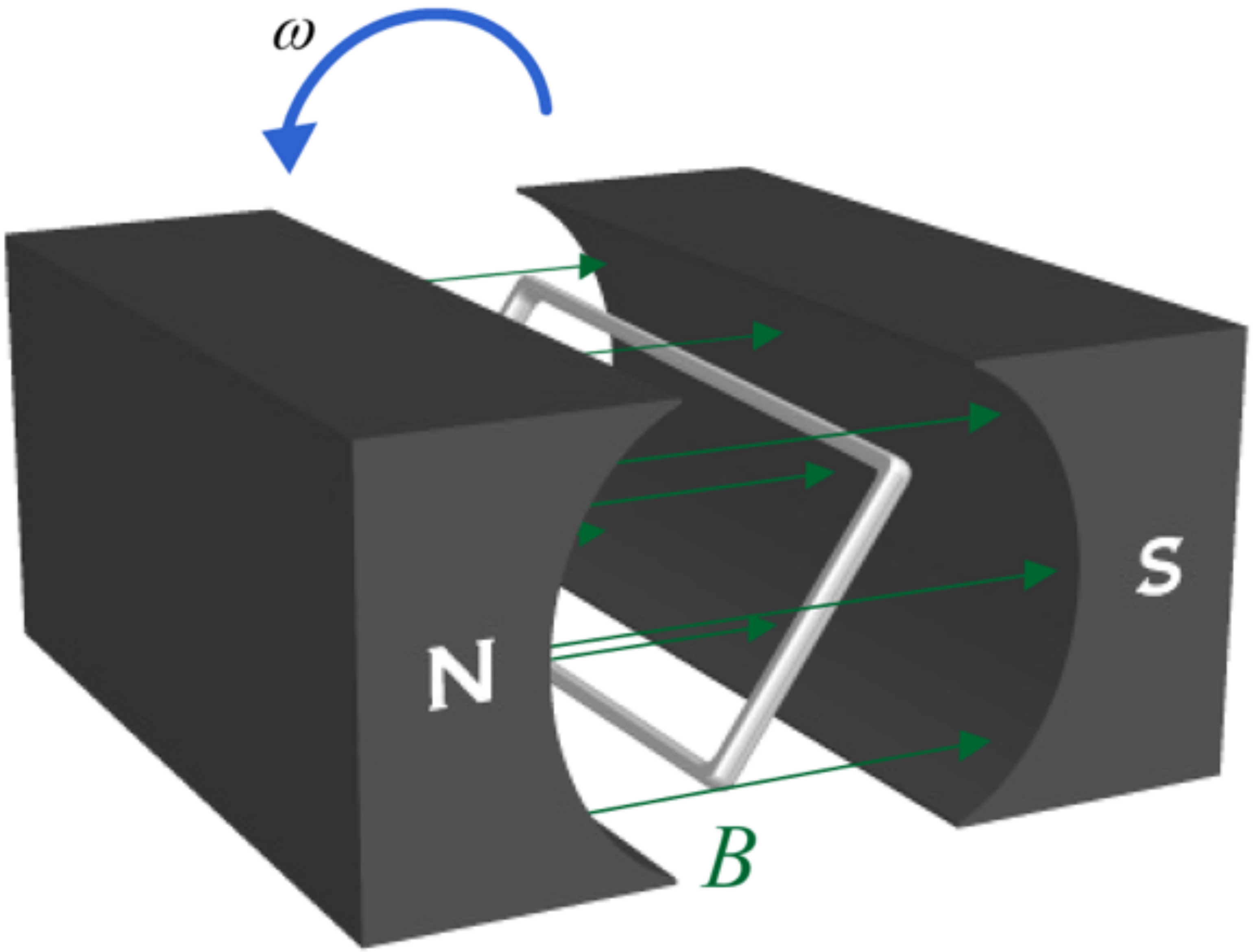
$$\Phi_B = \int d^2A\;\hat{n}\cdot\vec{B}$$

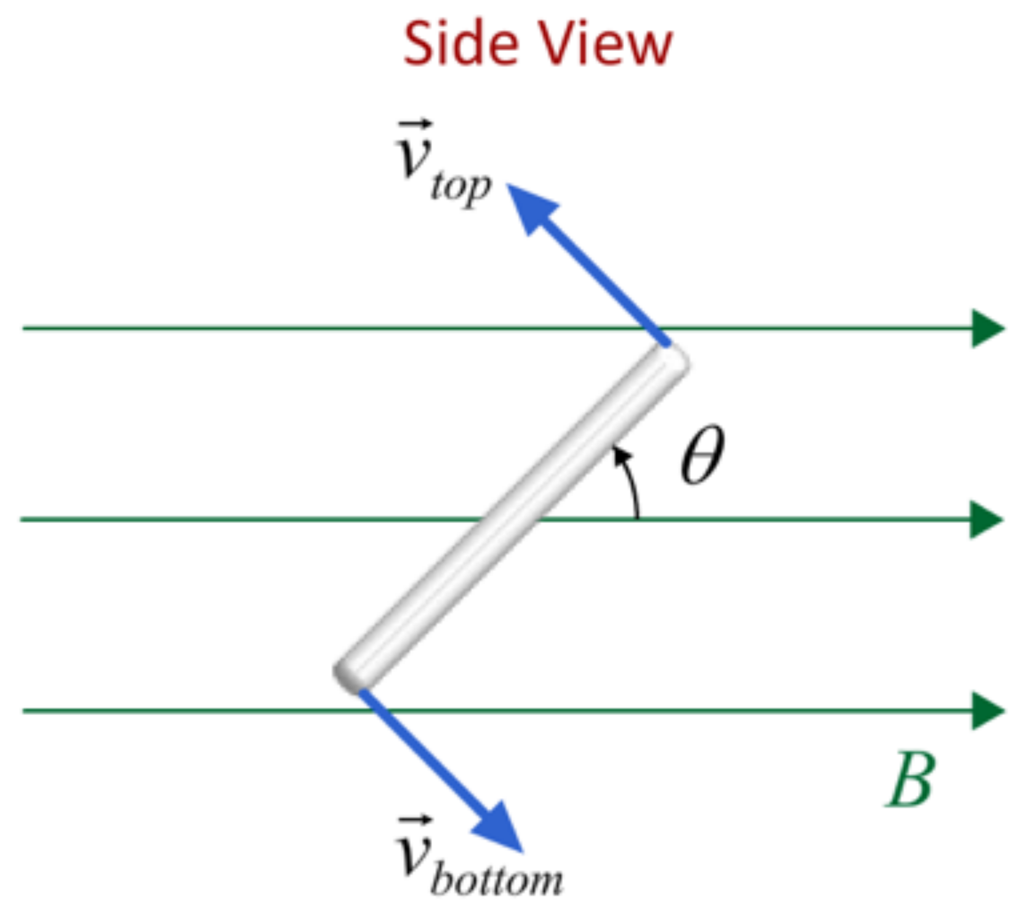
$\hat{n} = \hat{z}$, $\vec{B} = \hat{z}\,\frac{\mu_0 I}{2\pi R}$ Don't compute this!

$$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\left[L v B_+ - L v B_-\right] = L v (B_- - B_+)$$

(positive)

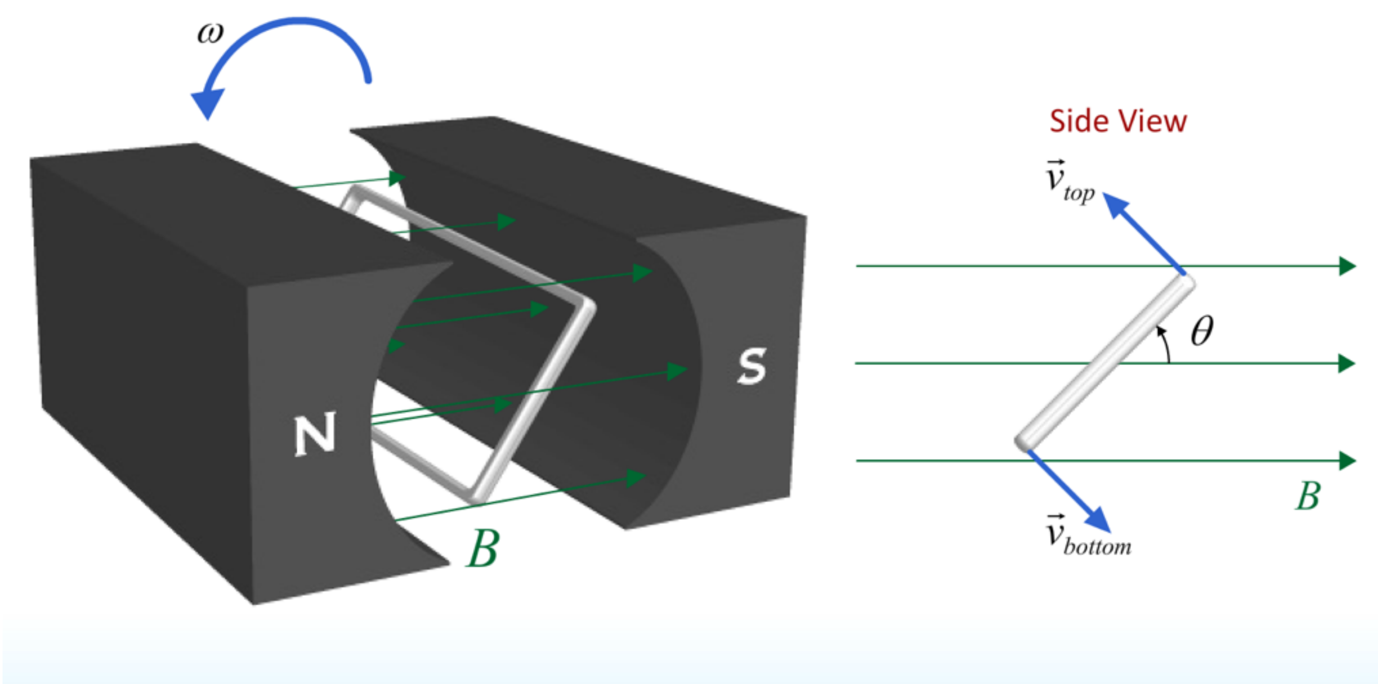
Motional EMF: Generator





Result: $\mathcal{E} = 2vLB\cos\theta$

Using magnetic flux…



- Magnetic Flux:

$$\Phi_B \equiv \int_{\mathcal{M}} d^2 A\, \hat{n} \cdot \vec{B}$$

- EMF:

$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

- Generator:

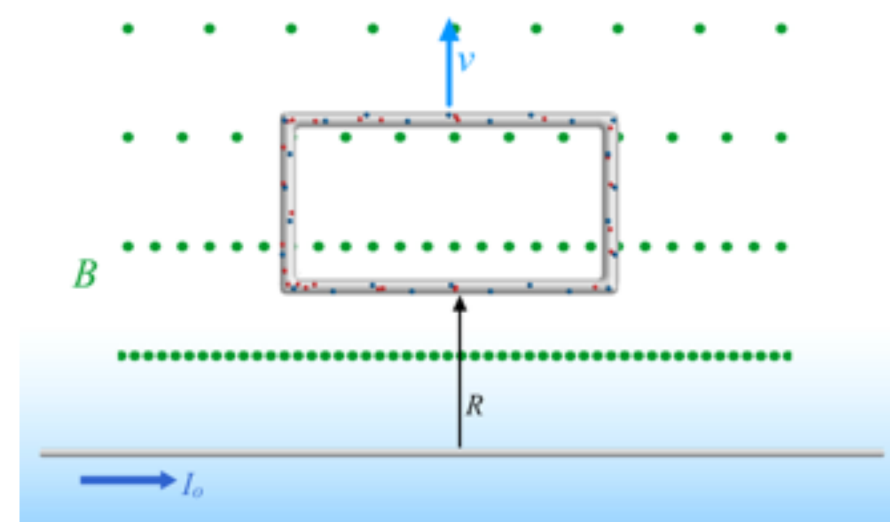
$$\mathcal{E} = -AB\frac{d}{dt}\cos\phi(t) = -\frac{d}{dt}A\hat{n} \cdot \vec{B}$$

Faraday Law of Induction

- Changes in the Magnetic Flux through a loop induces an EMF:

$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

$$\Phi_B \equiv \int_{\mathcal{M}} d^2A\, \hat{n} \cdot \vec{B}$$

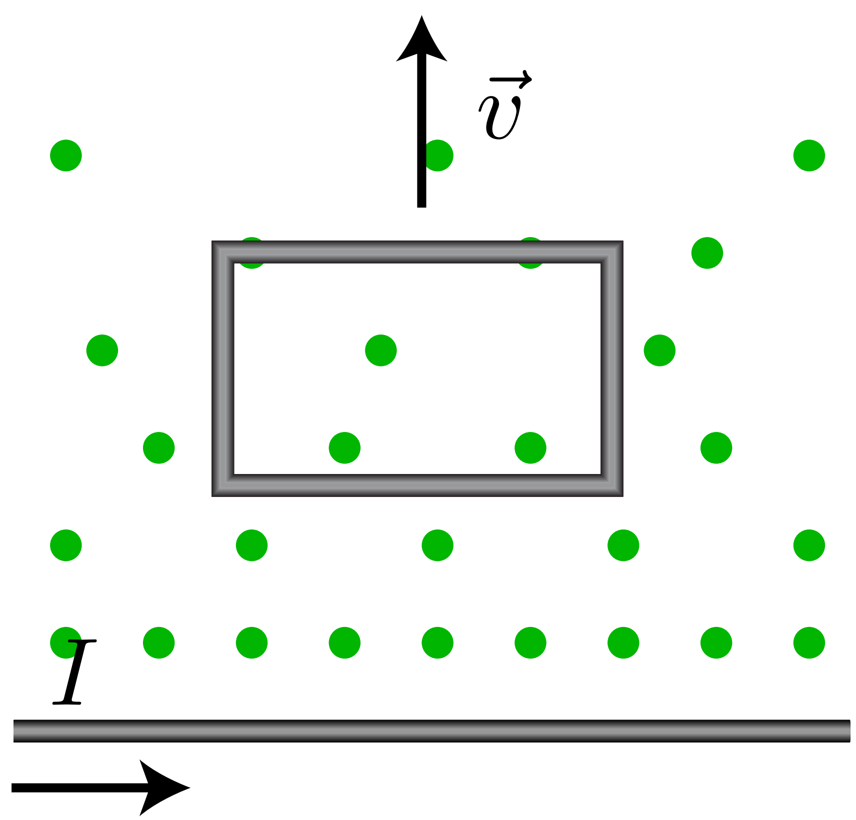


- **Transformer EMF:** Changes in B generate E

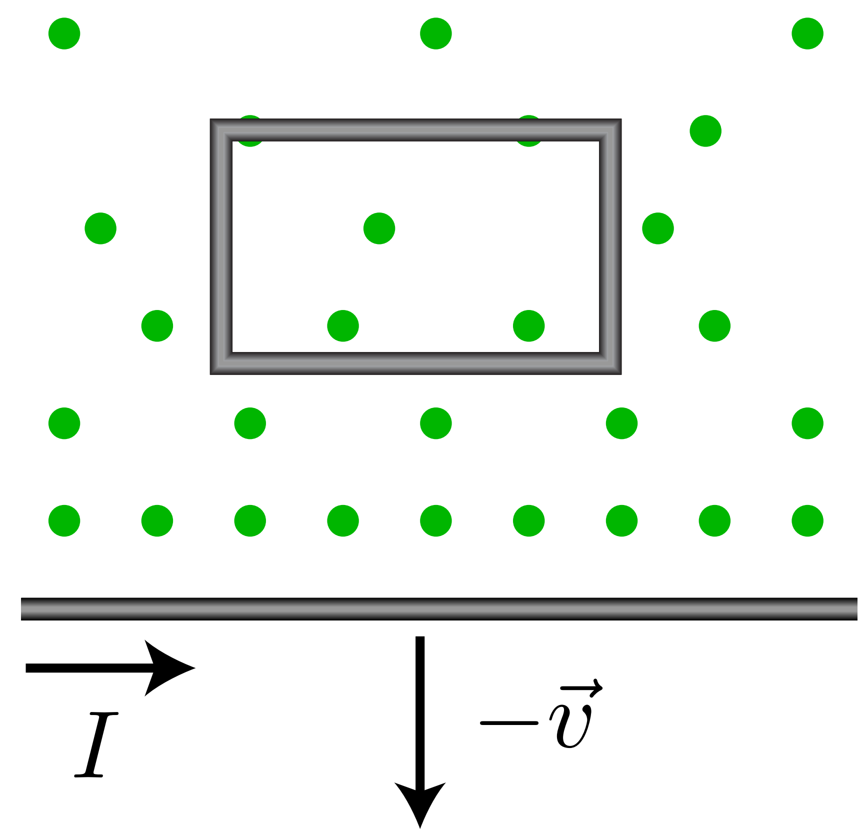
- **Motional EMF:** Changes in M generate F_B

But there are two mechanisms for changing the flux!

(A)Loop moves
Motional EMF

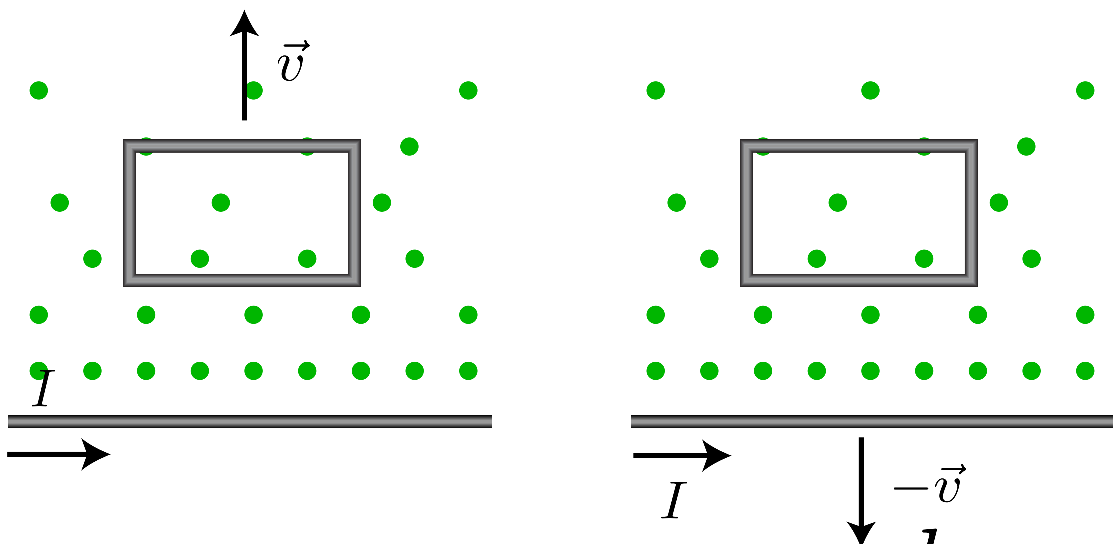


(B) B changes (wire moves)
Transformer EMF



$$\Phi_B^{(A)}(t) = \Phi_B^{(B)}(t)$$

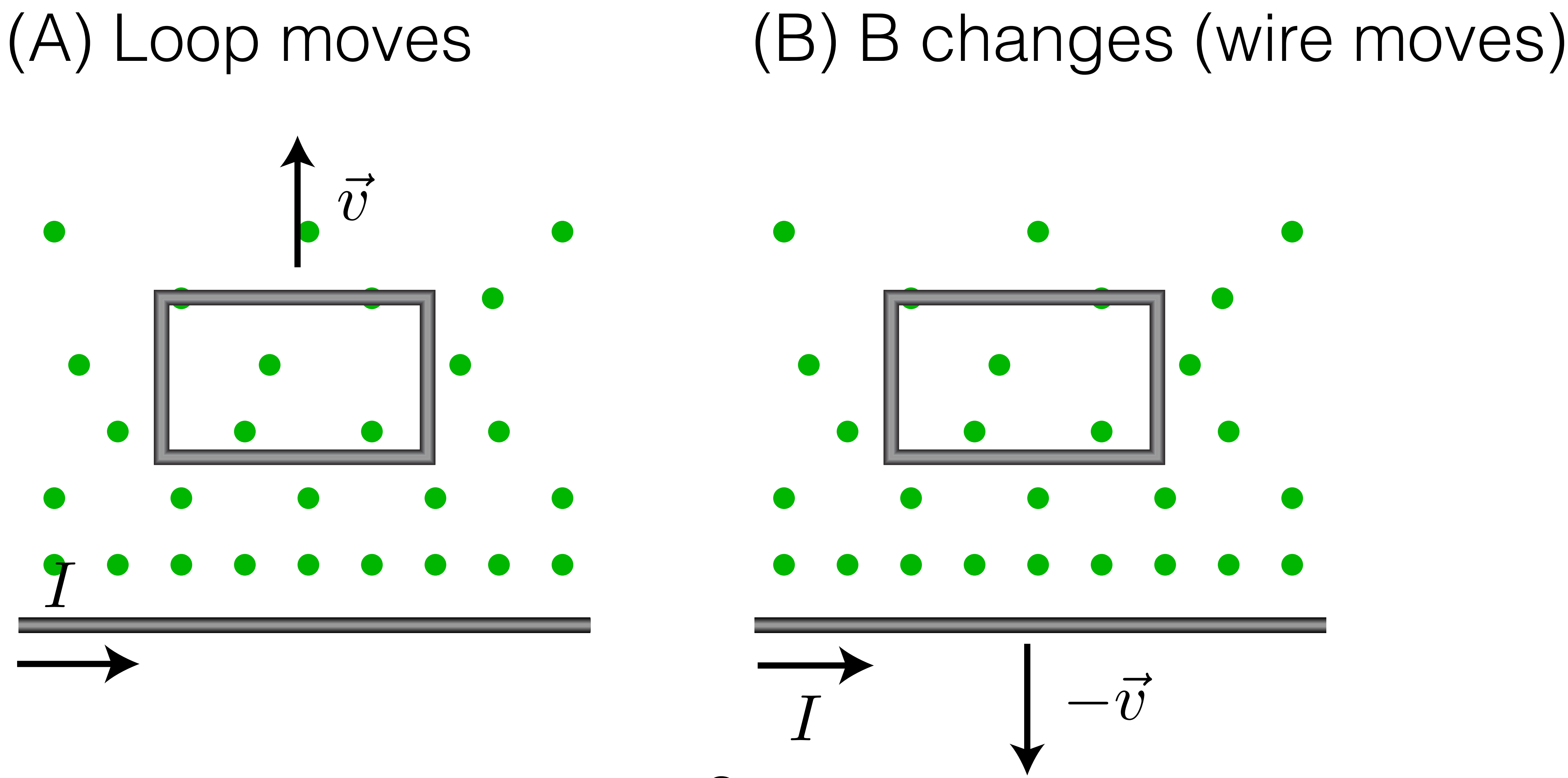
But there are two mechanisms for changing the flux!



$$\Phi_B^{(A)}(t) = \Phi_B^{(B)}(t)$$

- Faraday Law: $\mathcal{E} = -\frac{d}{dt}\Phi_B$ $\qquad \mathcal{E}^{(A)} = \mathcal{E}^{(B)}$
- EMF's are equal!
- How could this be? $\mathcal{E} \equiv \oint_{\partial\mathcal{M}} d\vec{\ell} \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right)$
- v = 0, Must be E — Not conservative!

Demo: Run experiment to test hypothesis!



$$\mathcal{E}^{(A)} \stackrel{?}{=} \mathcal{E}^{(B)}$$

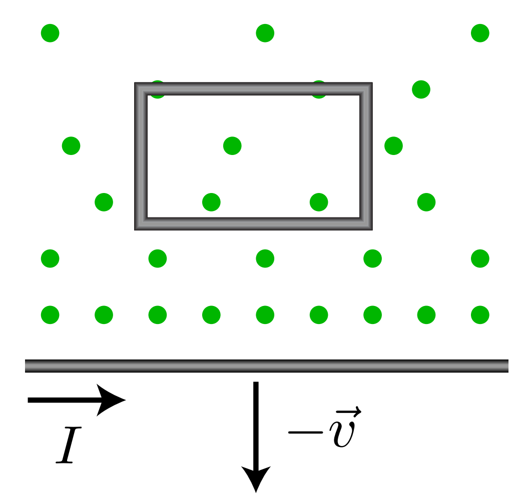
(Maxwell) Faraday Law:

- **Faraday Law of Induction** for a stationary loop

$$\mathcal{E} = \oint_{\partial\mathcal{M}} d\vec{\ell} \cdot \left(\vec{E} + \cancel{\vec{v} \times \vec{B}}\right) = -\frac{d}{dt} \int_{\mathcal{M}} d^2A\, \hat{n} \cdot \vec{B}$$

- **(Maxwell-)Faraday Law** (Maxwell Equations)

$$\oint_{\partial\mathcal{M}} d\vec{\ell} \cdot \vec{E} = -\int_{\mathcal{M}} d^2A\ \hat{n} \cdot \frac{\partial \vec{B}}{\partial t}$$



- **New** law of physics! Describes E and B field, not just loops!

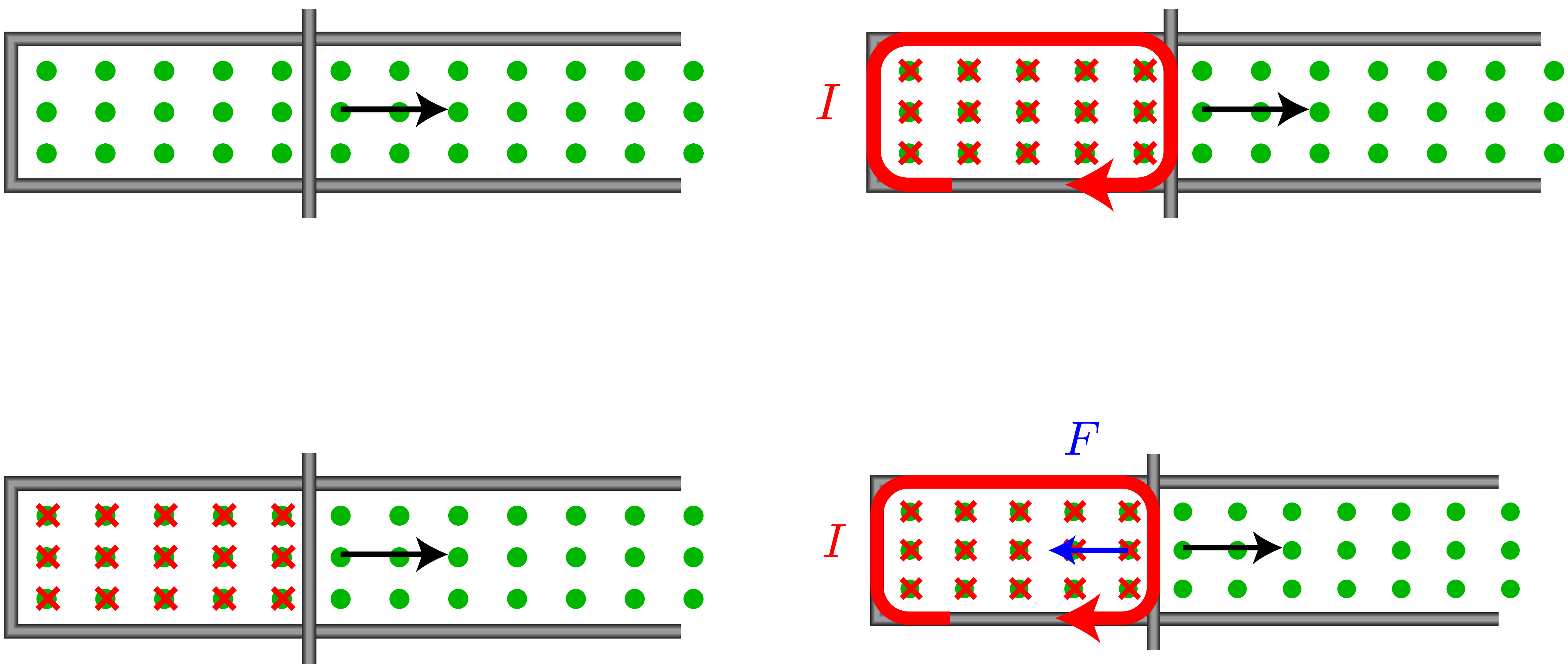
Electrodynamics & the Maxwell Equations

- Gauss Law (E): $$\oint_{\mathcal{M}} d^2A\; \hat{n}\cdot\vec{E} = Q_{\mathrm{inside}}/\epsilon_0$$
- Gauss Law (B): $$\oint_{\mathcal{M}} d^2A\; \hat{n}\cdot\vec{B} = 0$$
- Ampère Law: $\mathcal{M} = \mathrm{const}$ $$\oint_{\partial\mathcal{M}} d\vec{\ell}\cdot\vec{B} = \mu_0 I + \mu_0\epsilon_0 \frac{d}{dt}\int_{\mathcal{M}} d^2A\,\hat{n}\cdot\vec{E}$$
- Faraday Law: $\mathcal{M} = \mathrm{const}$ $$\oint_{\partial\mathcal{M}} d\vec{\ell}\cdot\vec{E} = -\frac{d}{dt}\int_{\mathcal{M}} d^2A\,\hat{n}\cdot\vec{B}$$

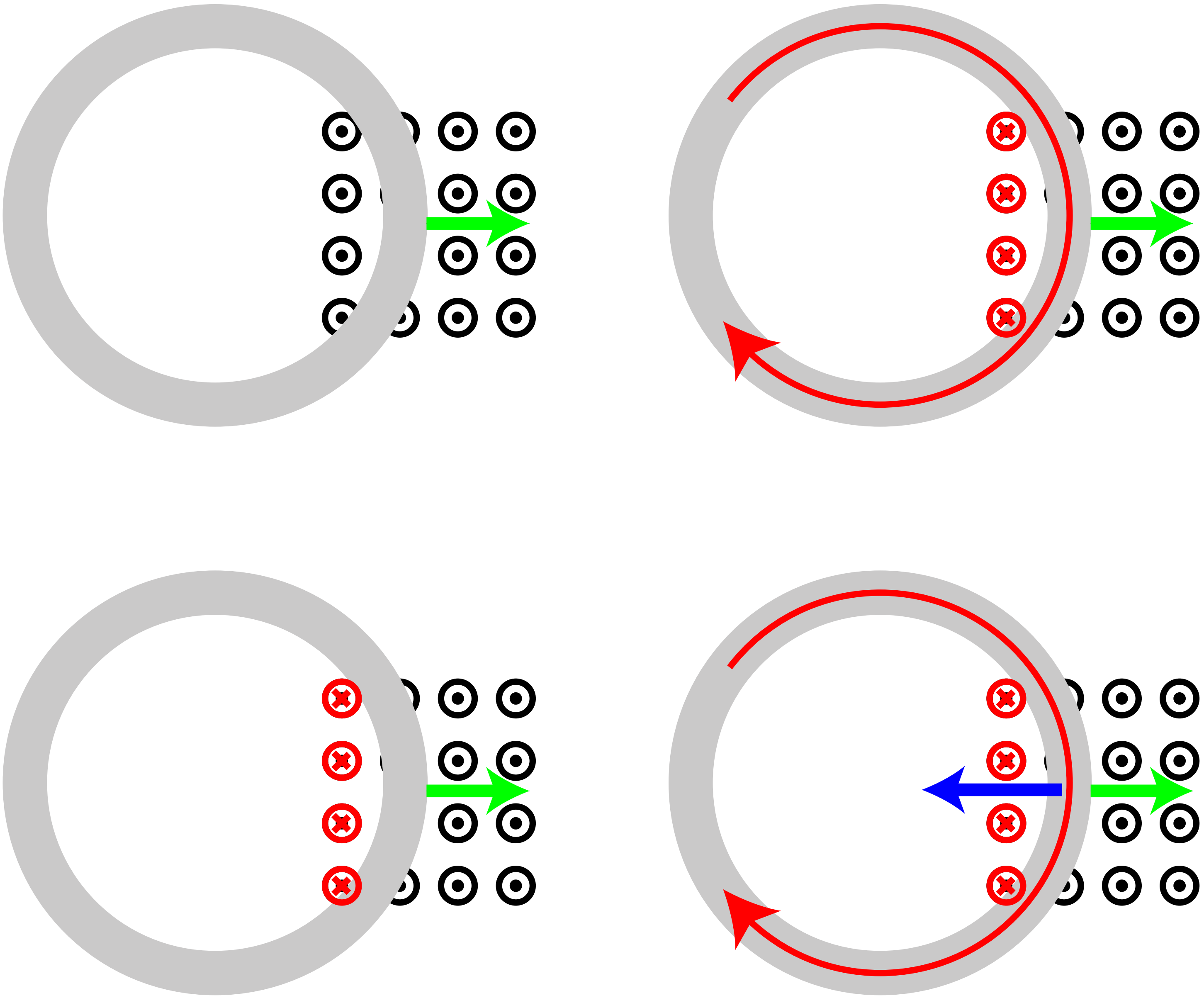
Which way does the current flow? (Lenz's Law)

- **Lenz's Law** = "How to get the signs right!"
- **Lenz's Law** = Currents are generated to oppose the change that created it.
- **Lenz's Law** = Newton's Third Law + Energy conservation

Lenz's Law: Example 1



Lenz's Law: Example 2



Eddy Current Demos