

# Faraday Law (of Induction), (Maxwell-)Faraday Law & Lenz Law

Lecture 22

# Announcements

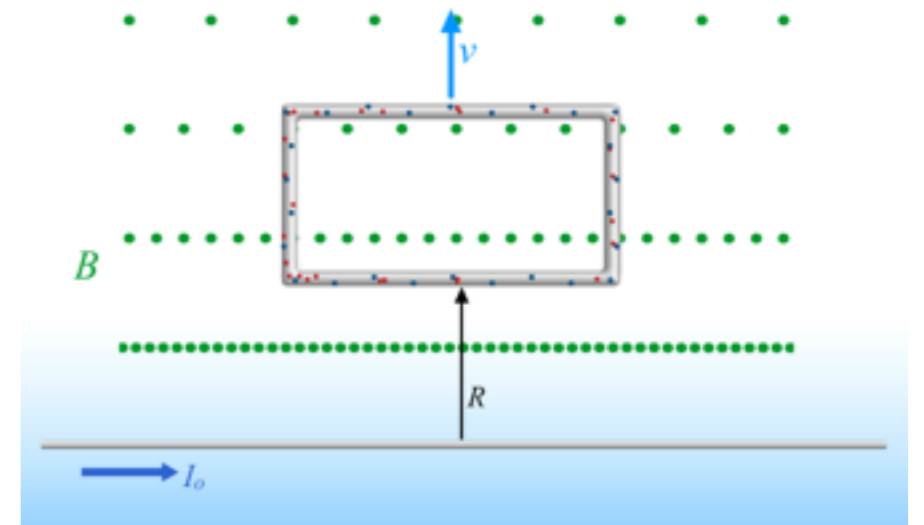
- Reading for Monday: 28-6 to 28-8
- Exam Thursday next week (3/5).
- Today's material is last new material on MT3.

# Faraday Law (of Induction)

- Changes in the Magnetic Flux through a loop induces an EMF:

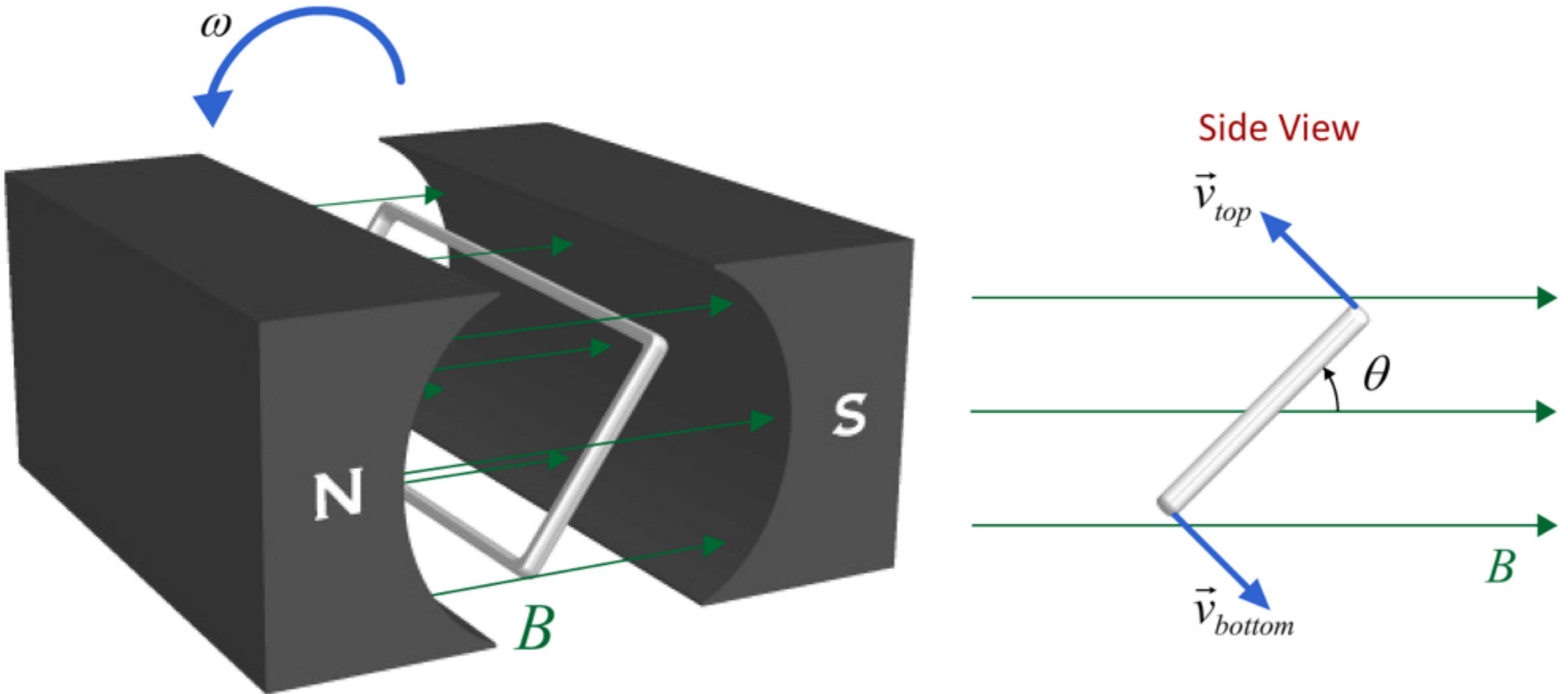
$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

$$\Phi_B \equiv \int_{\mathcal{M}} d^2 A \hat{n} \cdot \vec{B}$$



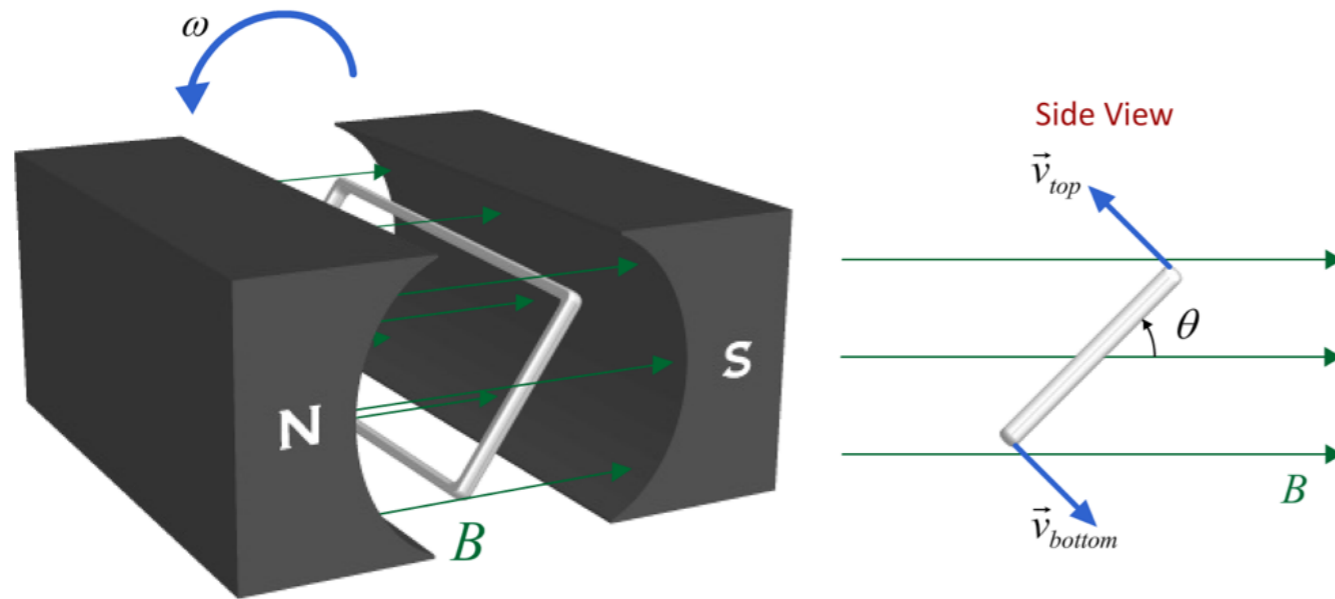
- **Transformer EMF:** Changes in B generate E
- **Motional EMF:** Changes in M generate  $F_B$

# Motional EMF: Generator



Result:  $\mathcal{E} = 2vLB \cos \theta$

# Using magnetic flux...



- Magnetic Flux:

$$\Phi_B \equiv \int_{\mathcal{M}} d^2 A \hat{n} \cdot \vec{B}$$

- EMF:

$$\mathcal{E} = -\frac{d}{dt} \Phi_B$$

- Generator:

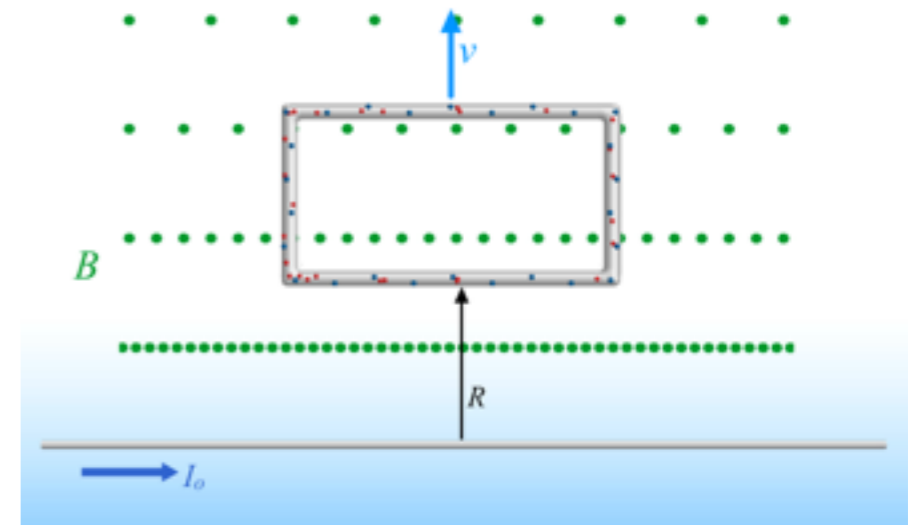
$$\mathcal{E} = -AB \frac{d}{dt} \cos \phi(t) = -\frac{d}{dt} A \hat{n} \cdot \vec{B}$$

# Faraday Law of Induction

- Changes in the Magnetic Flux through a loop induces an EMF:

$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

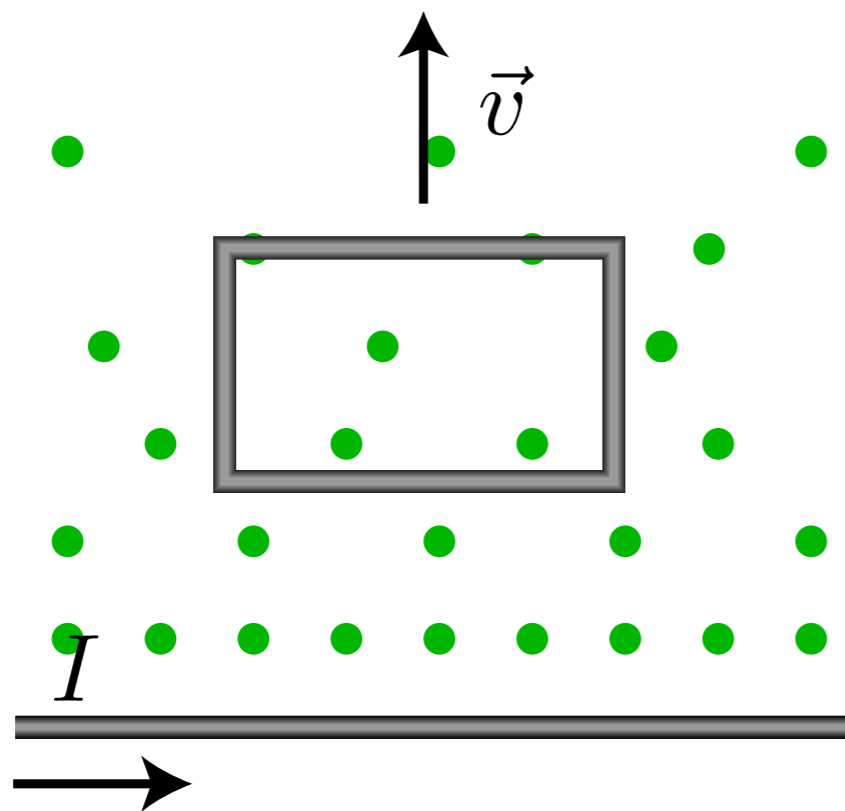
$$\Phi_B \equiv \int_{\mathcal{M}} d^2 A \hat{n} \cdot \vec{B}$$



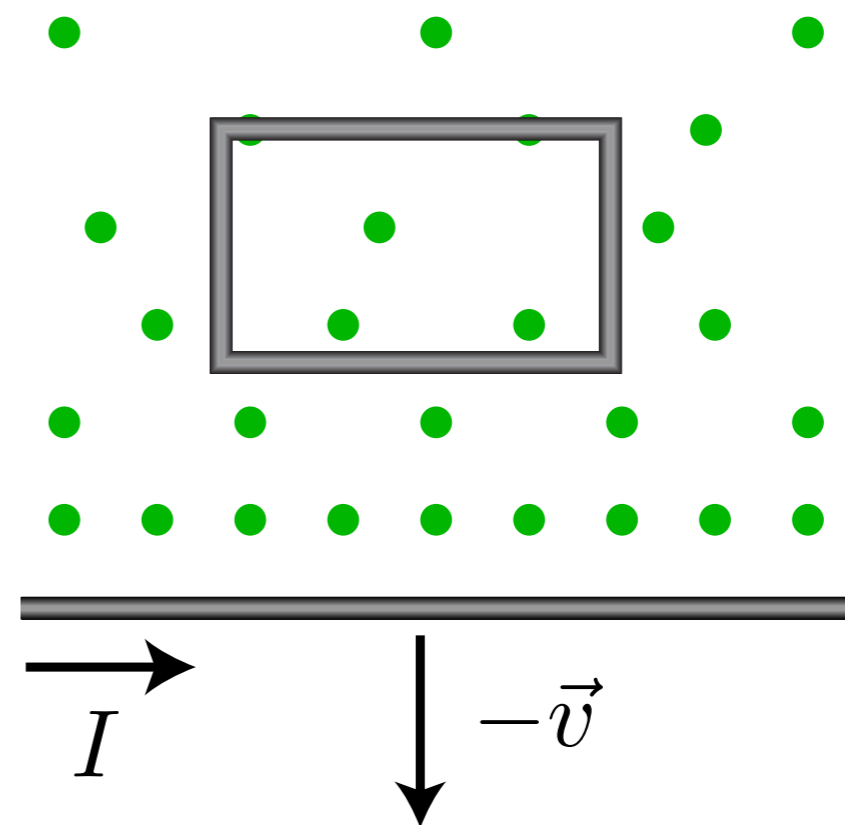
- **Transformer EMF:** Changes in B generate E
- **Motional EMF:** Changes in M generate  $F_B$

# But there are two mechanisms for changing the flux!

(A) Loop moves  
Motional EMF

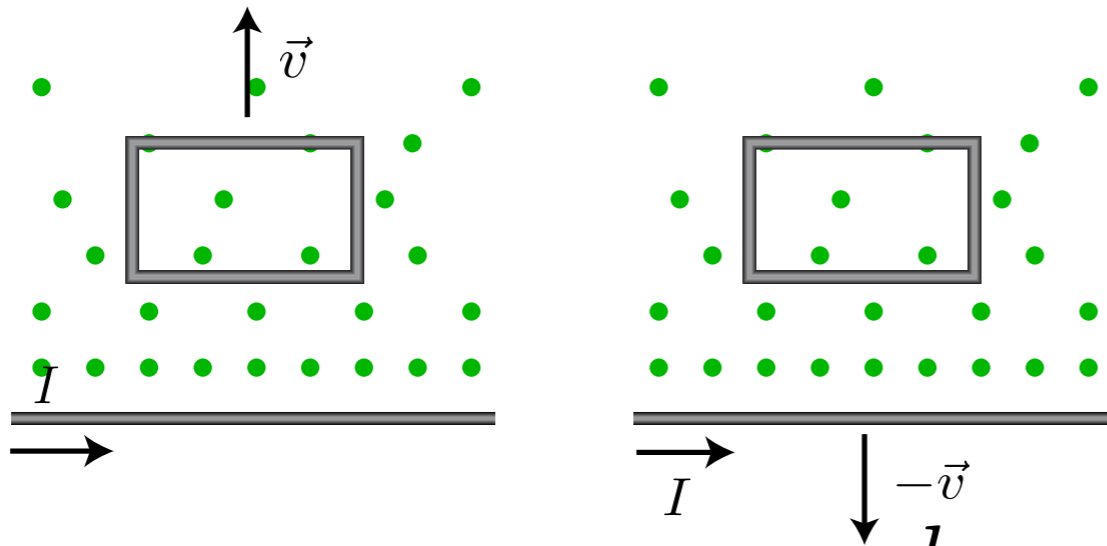


(B) B changes (wire moves)  
Transformer EMF



$$\Phi_B^{(A)}(t) = \Phi_B^{(B)}(t)$$

# But there are two mechanisms for changing the flux!



$$\Phi_B^{(A)}(t) = \Phi_B^{(B)}(t)$$

- Faraday Law:  $\mathcal{E} = -\frac{d}{dt}\Phi_B$        $\mathcal{E}^{(A)} = \mathcal{E}^{(B)}$

- EMF's are equal!

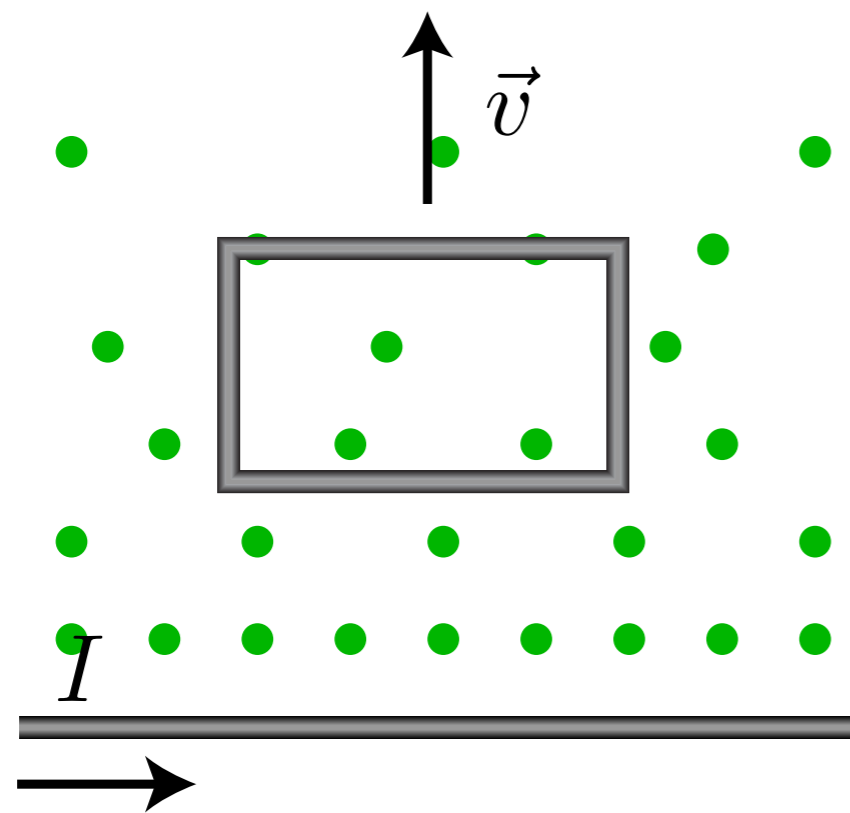
- How could this be?  $\mathcal{E} \equiv \oint_{\partial\mathcal{M}} d\vec{\ell} \cdot (\vec{E} + \vec{v} \times \vec{B})$

- $v = 0$ , Must be E — Not conservative!

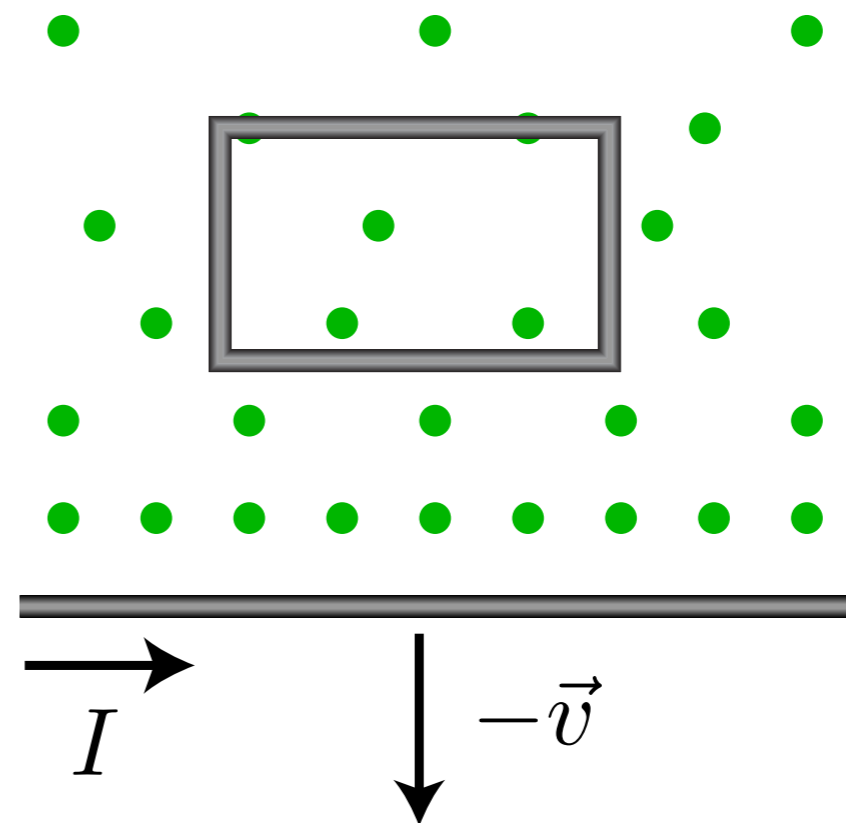


# Demo: Run experiment to test hypothesis!

(A) Loop moves



(B) B changes (wire moves)



$$\mathcal{E}^{(A)} \stackrel{?}{=} \mathcal{E}^{(B)}$$

# (Maxwell) Faraday Law:

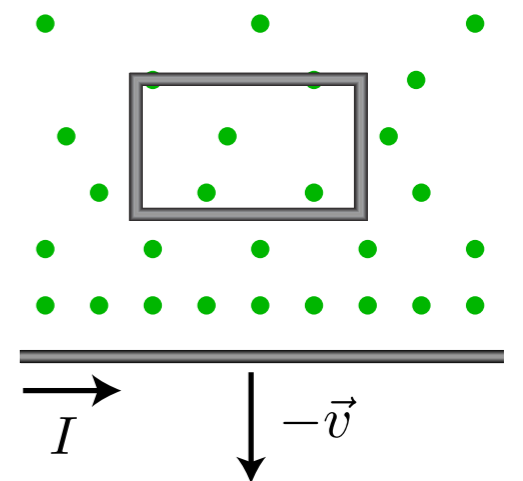
- **Faraday Law of Induction** for a stationary loop

$$\mathcal{E} = \oint_{\partial\mathcal{M}} d\vec{\ell} \cdot \left( \vec{E} + \vec{v} \times \vec{B} \right) = -\frac{d}{dt} \int_{\mathcal{M}} d^2A \hat{n} \cdot \vec{B}$$

- **(Maxwell-)Faraday Law** (Maxwell Equations)

$$\oint_{\partial\mathcal{M}} d\vec{\ell} \cdot \vec{E} = - \int_{\mathcal{M}} d^2A \hat{n} \cdot \frac{\partial \vec{B}}{\partial t}$$

- **New** law of physics! Describes E and B field, not just loops!



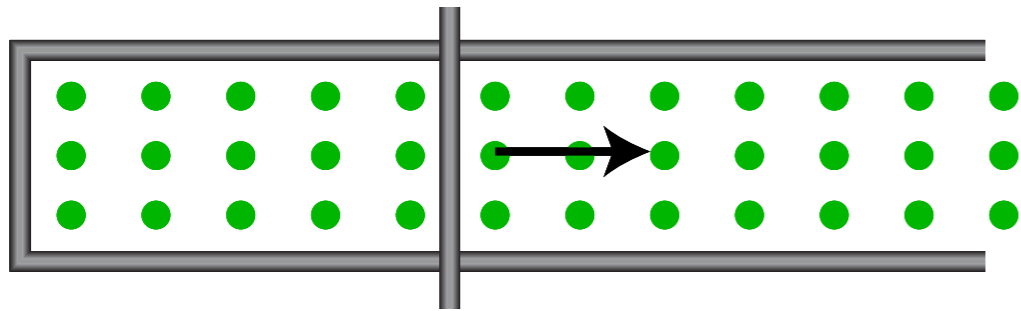
# Electrodynamics & the Maxwell Equations

- Gauss Law (E):  $\oint_{\mathcal{M}} d^2A \hat{n} \cdot \vec{E} = Q_{\text{inside}}/\epsilon_0$
- Gauss Law (B):  $\oint_{\mathcal{M}} d^2A \hat{n} \cdot \vec{B} = 0$
- Ampère Law:  
 $\mathcal{M} = \text{const}$   $\oint_{\partial\mathcal{M}} \vec{d}\ell \cdot \vec{B} = \mu_0 I + \mu_0\epsilon_0 \frac{d}{dt} \int_{\mathcal{M}} d^2A \hat{n} \cdot \vec{E}$
- Faraday Law:  
 $\mathcal{M} = \text{const}$   $\oint_{\partial\mathcal{M}} \vec{d}\ell \cdot \vec{E} = -\frac{d}{dt} \int_{\mathcal{M}} d^2A \hat{n} \cdot \vec{B}$

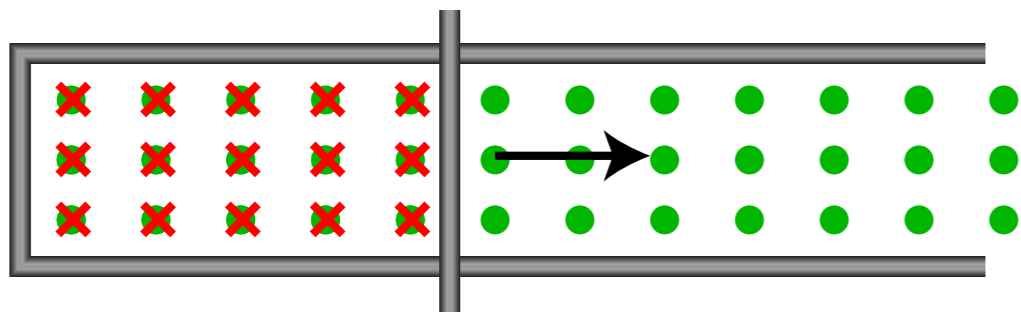
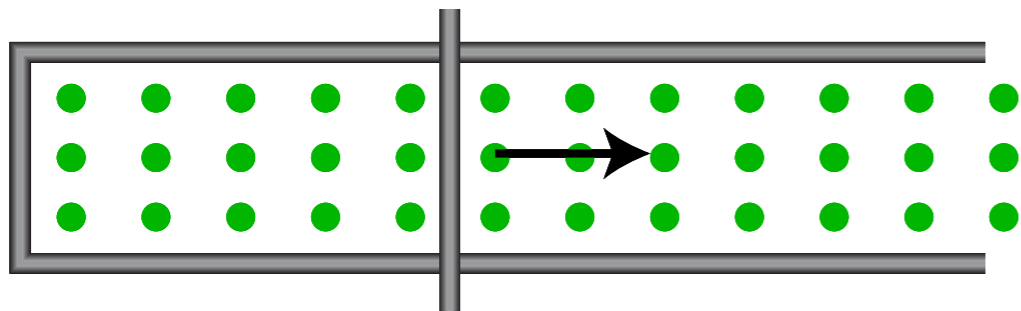
# Which way does the current flow? (Lenz's Law)

- **Lenz's Law** = “How to get the signs right!”
- **Lenz's Law** = Currents are generated to oppose the change that created it.
- **Lenz's Law** = Newton's Third Law + Energy conservation

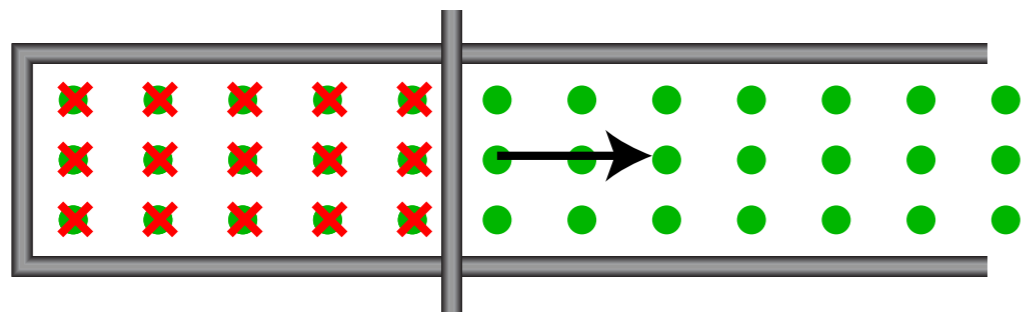
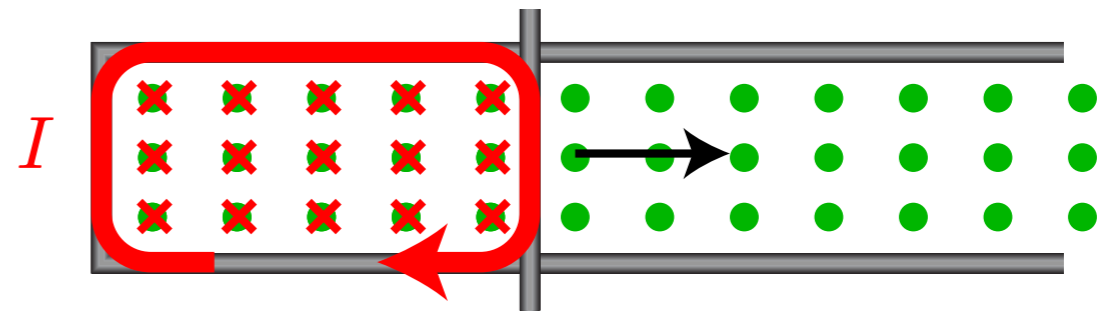
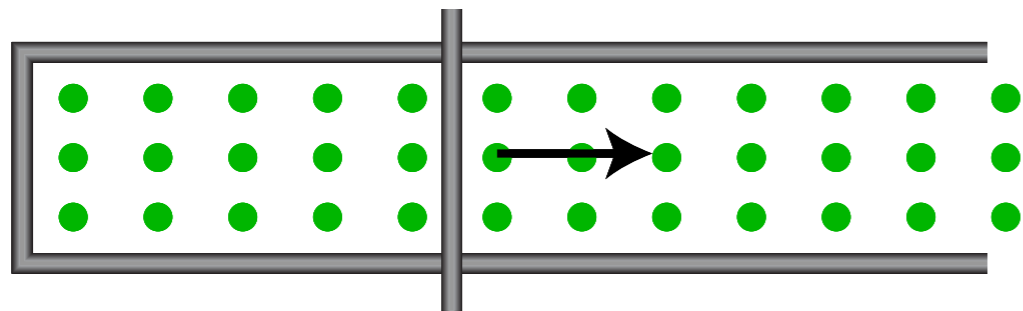
# Lenz's Law: Example 1



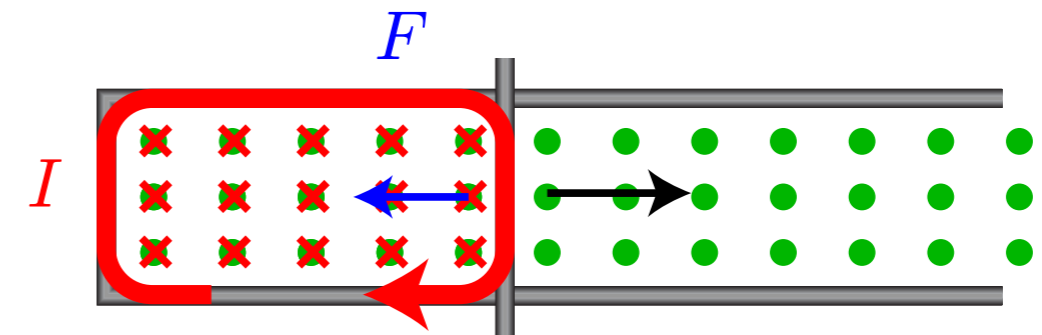
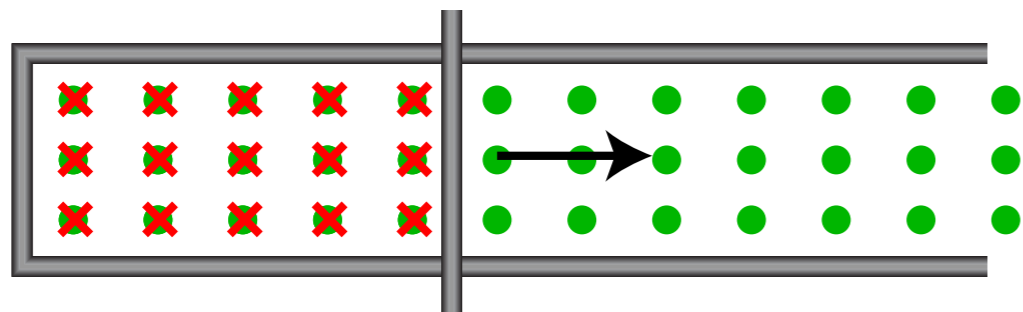
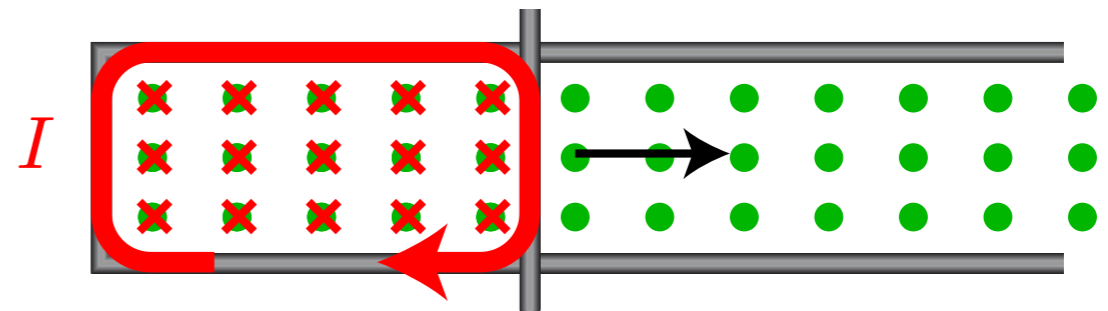
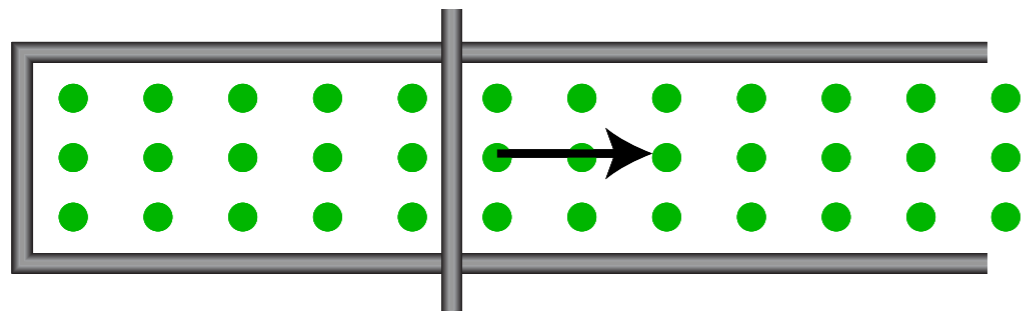
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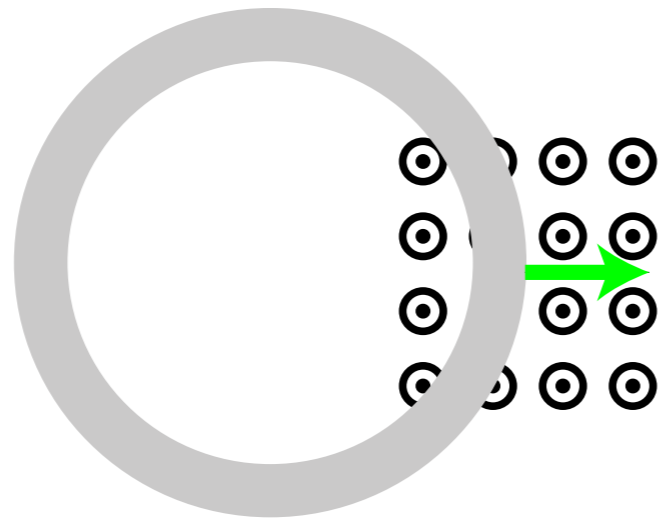
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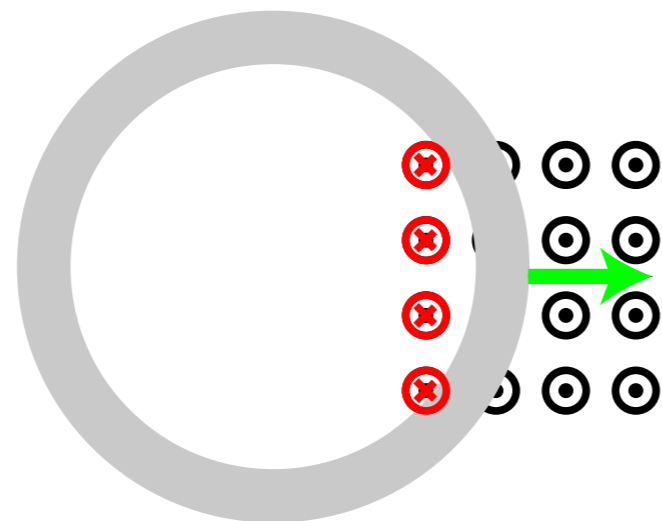
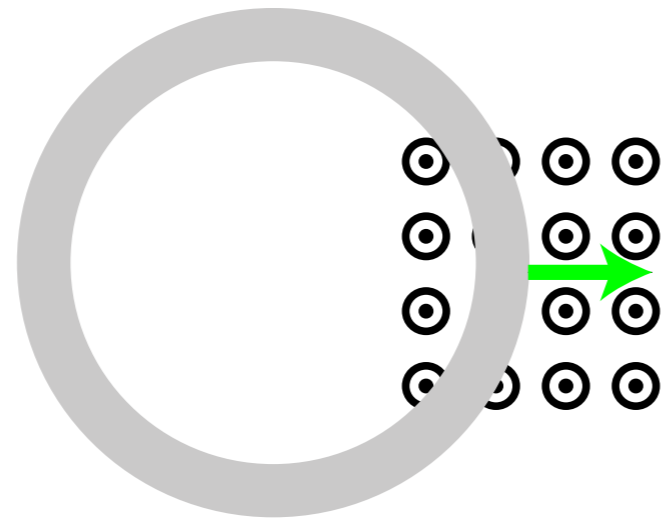


# Lenz's Law: Example 2

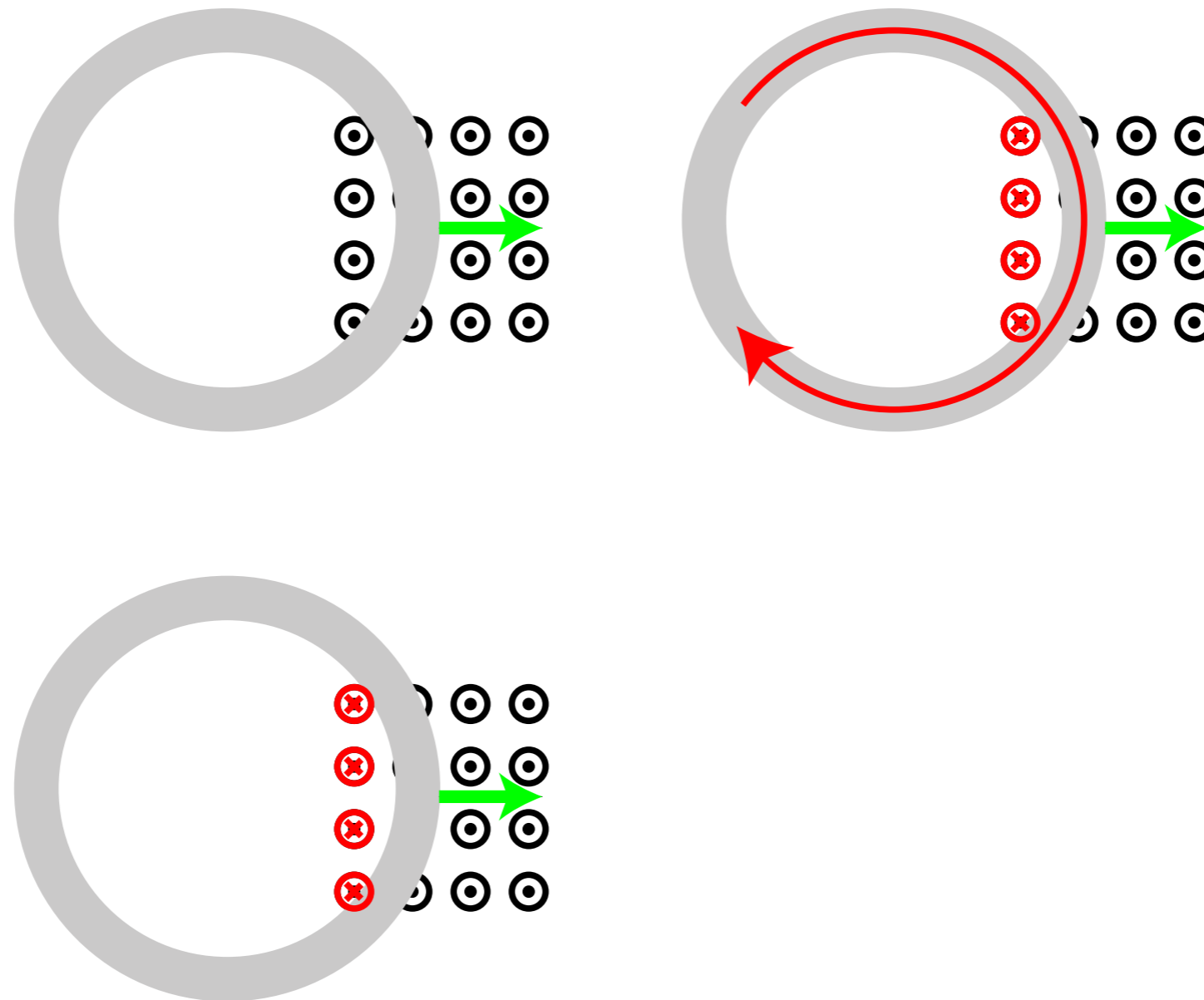
# Lenz's Law: Example 2



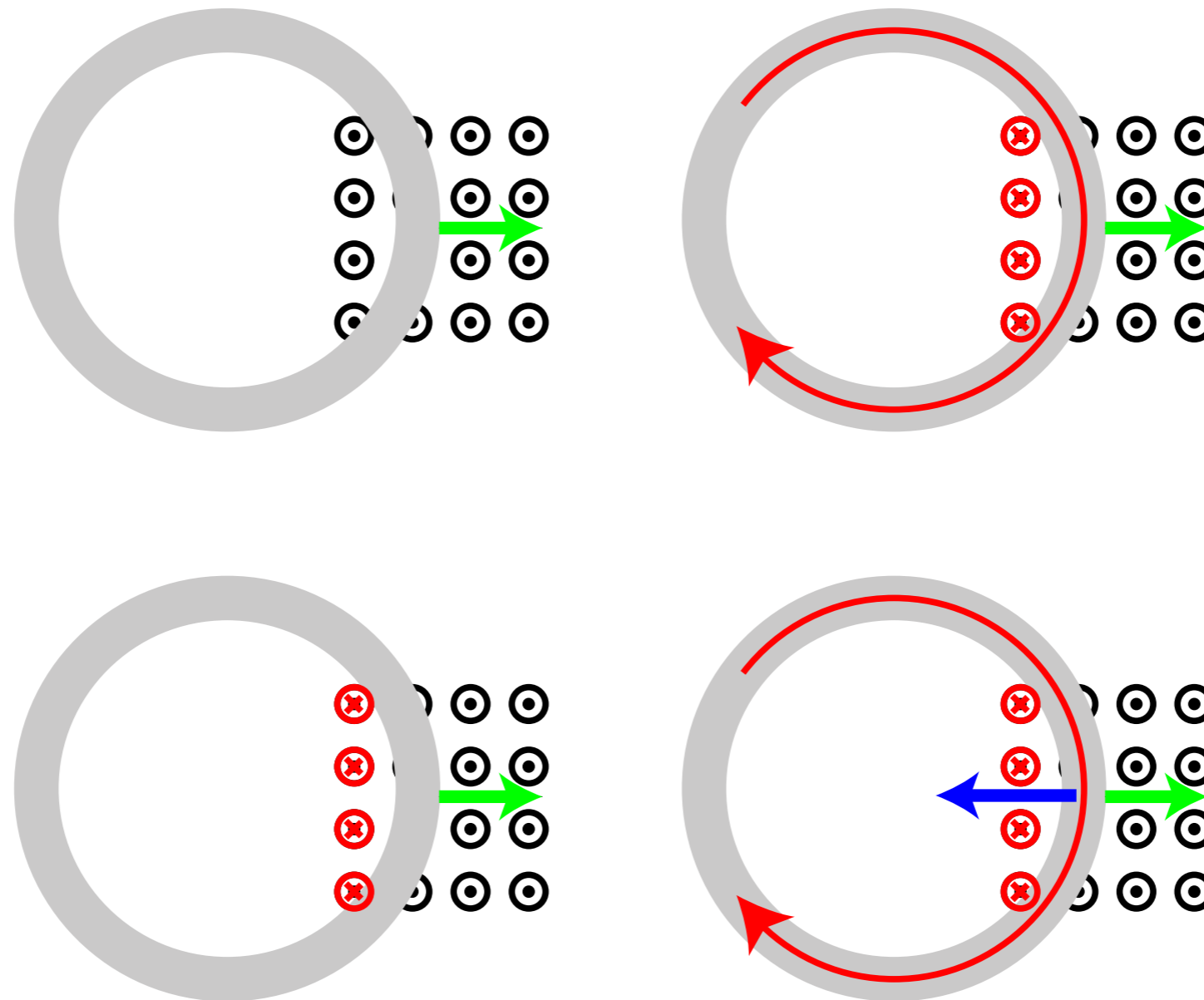
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# Eddy Current Demos

