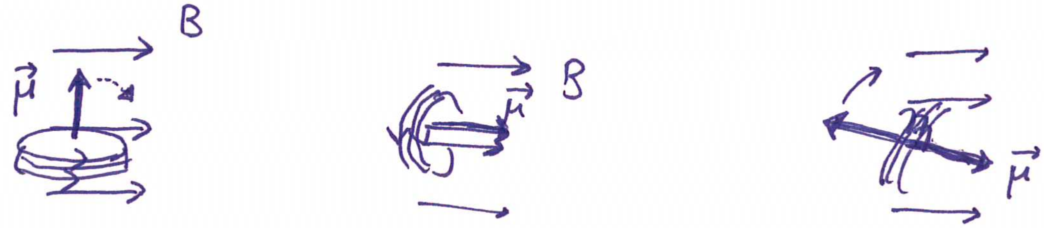
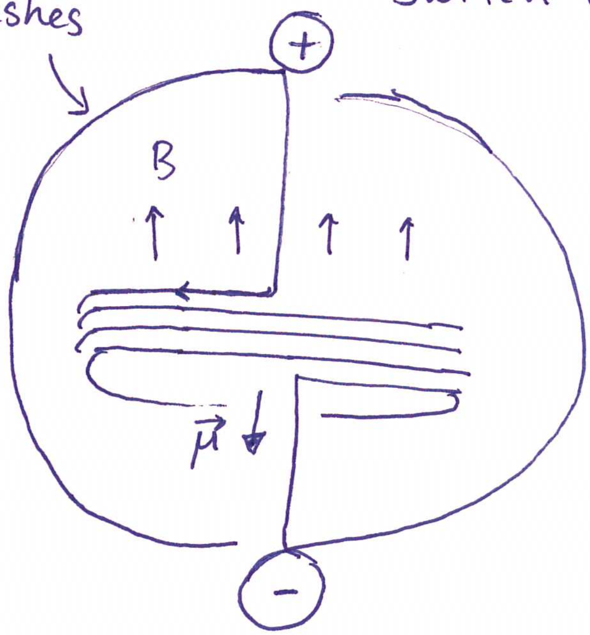
Making an electric motor (DC)



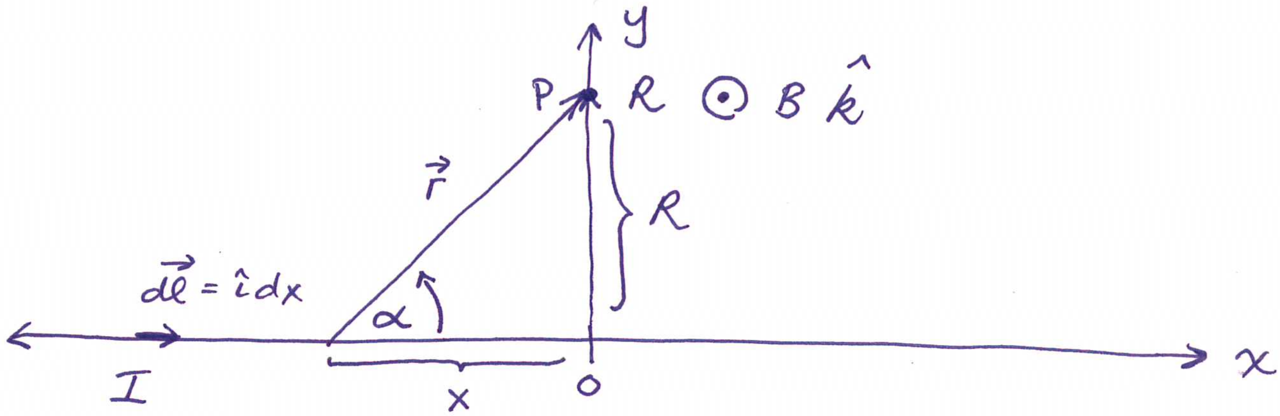
Commutator
Brushes

switch current direction



B field from an infinite wire

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$



$R = \text{const}$

$$dB = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} dx \, \frac{\sin\alpha(x)}{r^2}$$

$$\frac{x}{R} = \frac{x}{r}\frac{r}{R} = \cancel{\frac{\sin\alpha}{\cos\alpha}}$$

$$= \frac{-\cos\alpha}{\sin\alpha}$$

$$= \frac{\mu_0 I}{4\pi} \int_0^{\pi} d\alpha \, \cancel{\frac{R}{\sin^2\alpha}} \, \frac{\sin\alpha \, \cancel{\sin^2\alpha}}{R^{\cancel{2}}}$$

$$\frac{dx}{R} = \frac{\sin\alpha}{\sin\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha}$$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}$$

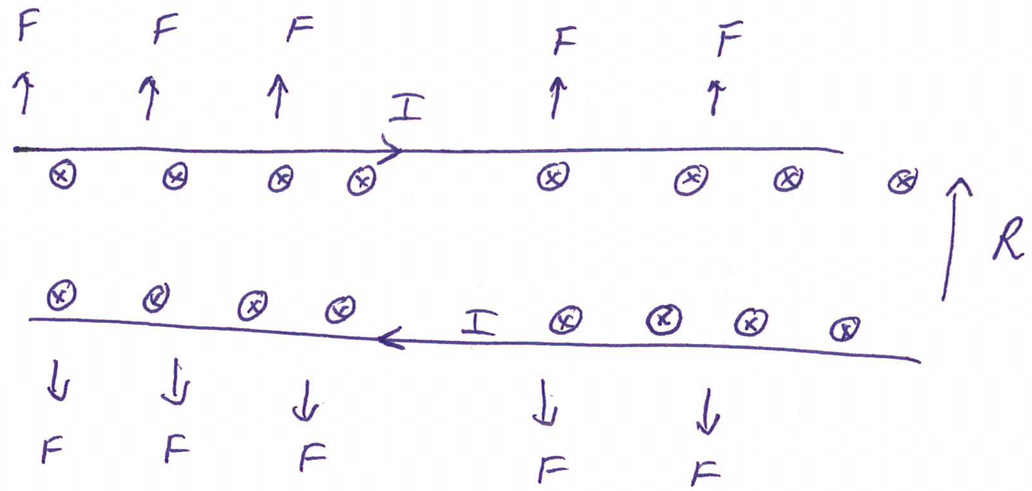
$$= \frac{1}{\sin^2\alpha}$$

$$= \frac{\mu_0 I}{4\pi R} \int_0^{\pi} d\alpha \, \sin\alpha$$

$$= \frac{\mu_0 I}{4\pi R} \left[-\cos\alpha\right]_0^{\pi}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

Pinch Wire Demo

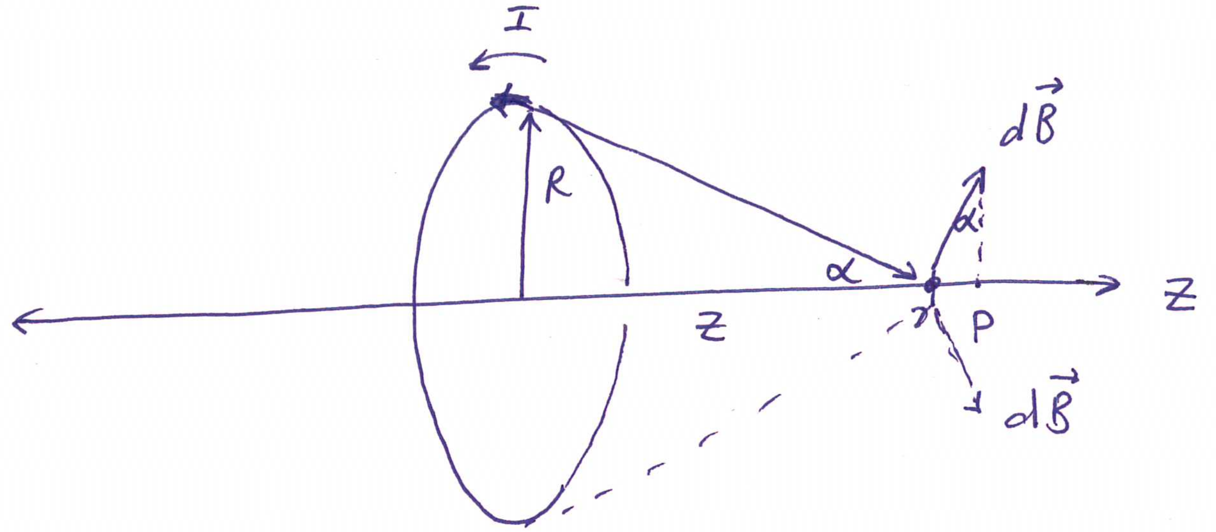


$$\vec{F} = I \vec{L} \times \vec{B} \qquad \vec{B} = \frac{\mu_0}{2\pi} \frac{I}{R}$$

$$F = \frac{I^2 L}{R} \frac{\mu_0}{2\pi}$$

B field from a loop

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$



By symmetry, all but z component cancels.

$$B = \hat{z} \cdot \int d\vec{B} = \frac{\mu_0}{4\pi} I \int dl \, \frac{\hat{z} \cdot (\underbrace{\hat{R} \cos\alpha}_{0} + \hat{z} \sin\alpha)}{r^2}$$

$$\sin\alpha = \frac{R}{r}$$

$$= \frac{\mu_0 I}{2 \cancel{4\pi}} \frac{\cancel{2\pi} R}{r^2} \sin\alpha$$

$$r = \sqrt{R^2 + z^2}$$

$$\boxed{B = \frac{\mu_0 I}{2 \cancel{4\pi}} \frac{R^2}{r^3} = \frac{\mu_0 I}{2 \cancel{4\pi}} \frac{R^2}{(R^2 + z^2)^{3/2}}}$$