Forces and torques generated by the B field

Lecture 18

Announcements

- Pick up exams
- Mean: 61 (std TBA)
- Regrades due Friday
- Please check long answer score against webassign!

From last time: Unification

- **Electricity** and **Magnetism** were **unified** into a single theory!

- Current **generates** B and B **applies forces** on currents.

- Lorentz Force Law (J.J. Thomson ++):

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Ampère Law (w/o Maxwell correction):

$$\vec{\nabla} \times \vec{B} = \mu_0 \, \vec{j} \qquad \oint d\vec{\ell} \cdot \vec{B} = \mu_0 \, I_{\text{inside}}$$

Lorentz Force Law

- Force on moving charges:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

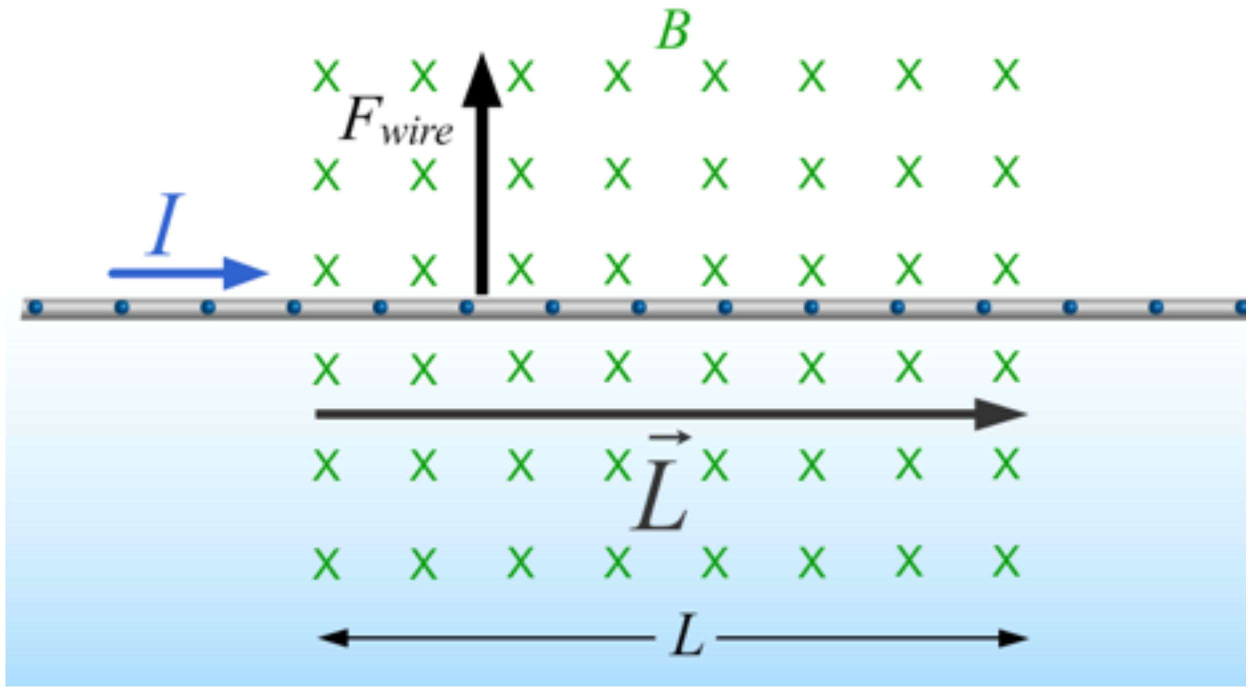
- Derive a force on wires carrying current (overhead):

- Result:

$$\vec{F} = I\vec{L} \times \vec{B}$$

Demo: Jumping Wire

$$\vec{F} = I\vec{L} \times \vec{B}$$



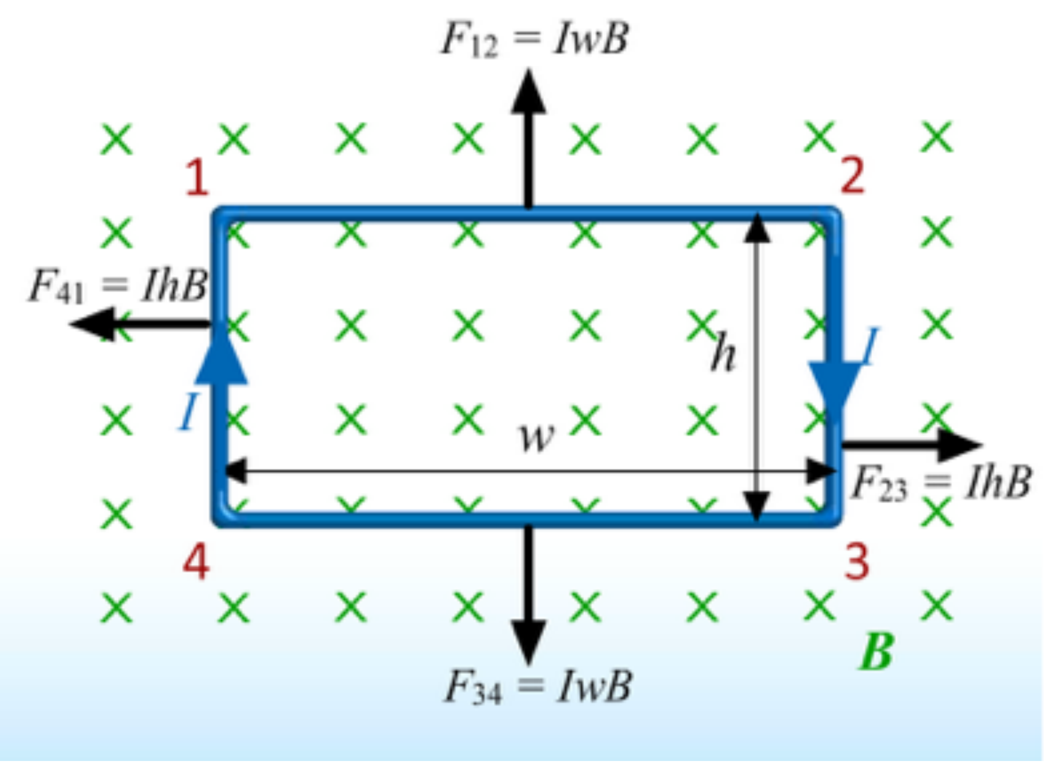
- Work out the direction of the force…

Compute the force on a closed loop

- L is the end-to-end displacement of the loop

$$\vec{F} = I\vec{L} \times \vec{B}$$

- Closed loops have L = 0
- **Physical intuition**: Cancelation between opposing sides.



Torque on a closed loop

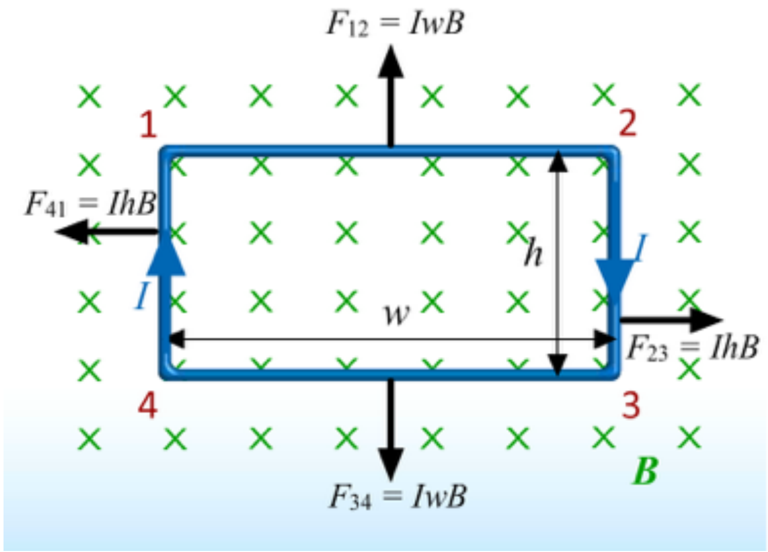
- Compute the torque on a closed loop.

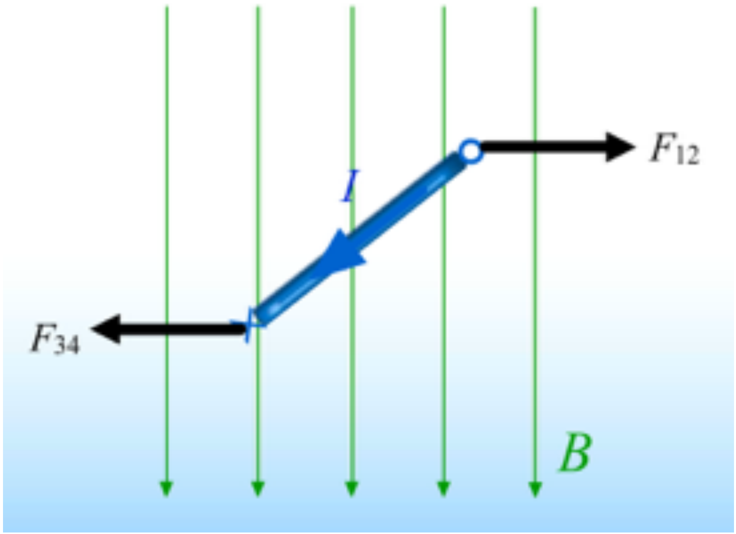
$$\vec{F} = I\vec{L} \times \vec{B} \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

- (Overhead)

- Result:

$$\vec{\tau} = N\, I\, A\, \hat{n} \times \vec{B}$$





Magnetic Dipole Moment

- Define the Magnetic Dipole Moment:

$$\vec{\mu} \equiv N\, I\, A\, \hat{n}$$

- Torque is now:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad\qquad \vec{\tau} = \vec{p} \times \vec{E}$$

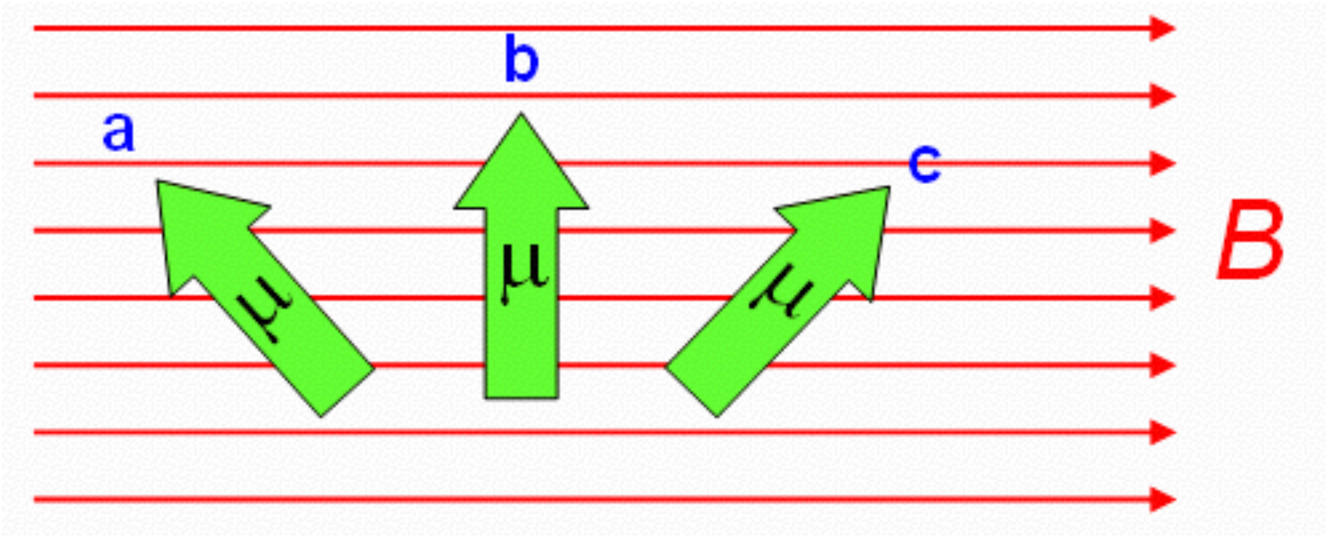
- Compute the Potential Energy:

(Overhead)

Potential energy for MDM

- Potential energy for a magnetic dipole moment
  Result:

$$U = -\vec{\mu} \cdot \vec{B} \qquad\qquad U = -\vec{p} \cdot \vec{E}$$



- Lowest energy configuration: Aligned

Demo: Electric motor