RC Circuits!

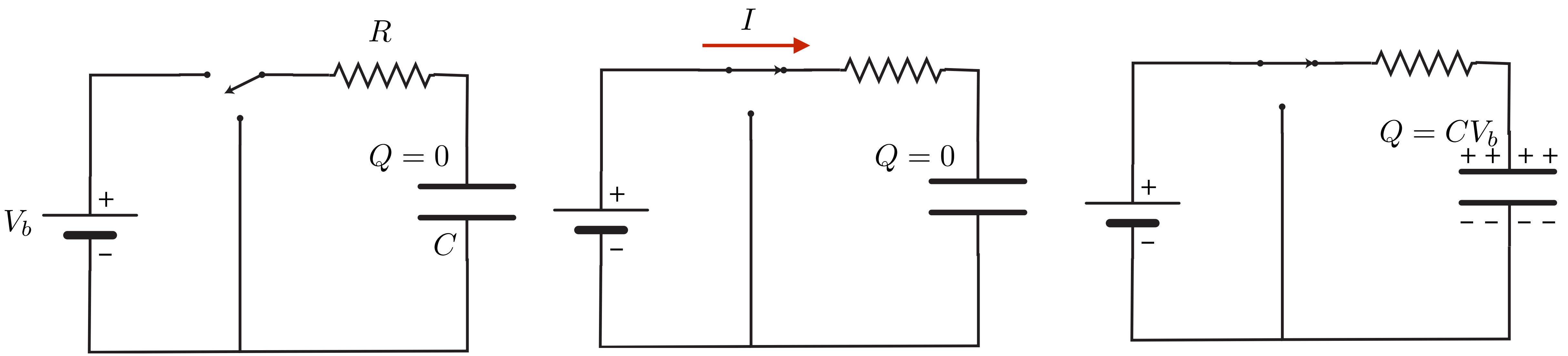
Lecture 15

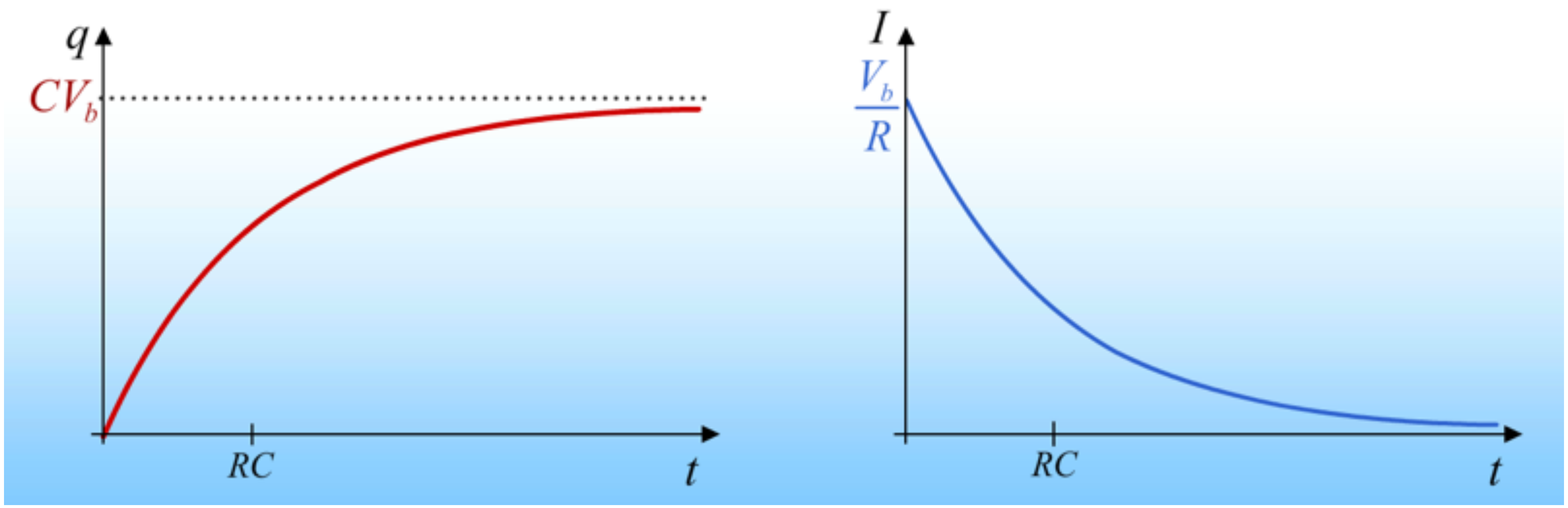
Announcements

- Exam on Thursday.
    - Check the website
        - Practice exam & solutions
        - Seating chart
        - Study hard!

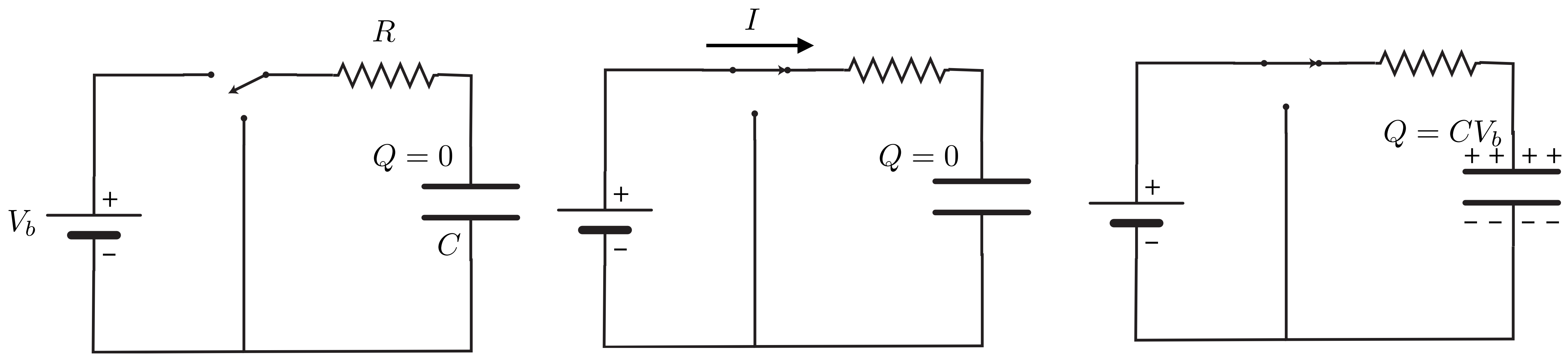
RC Demo

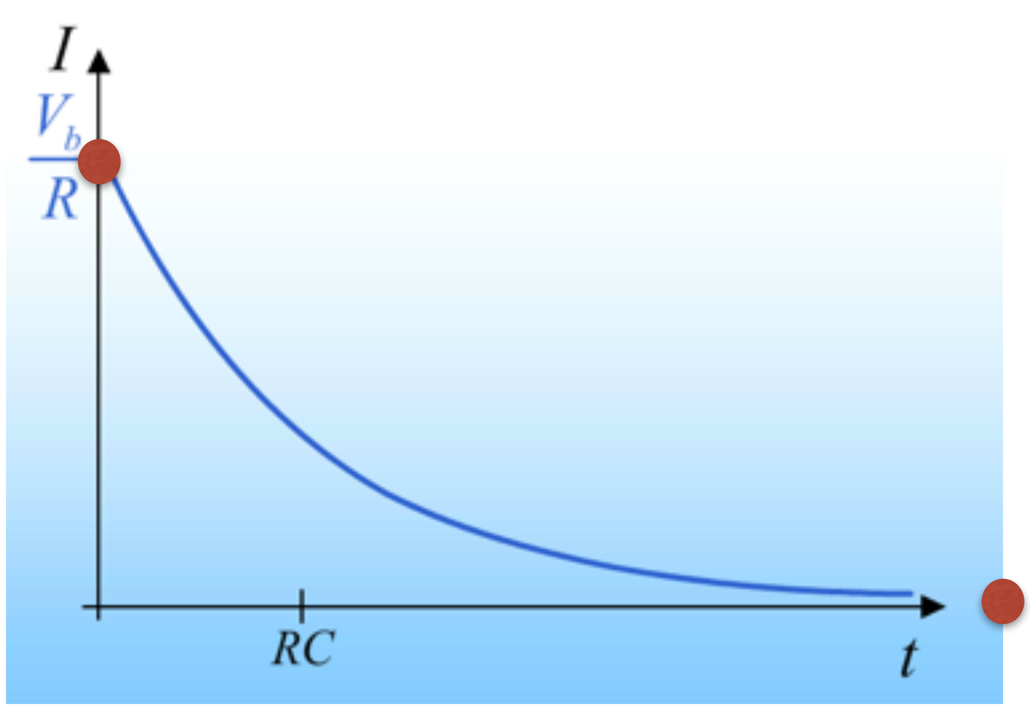
Charging:
Overview of the action.





Charging: Overview of the action.

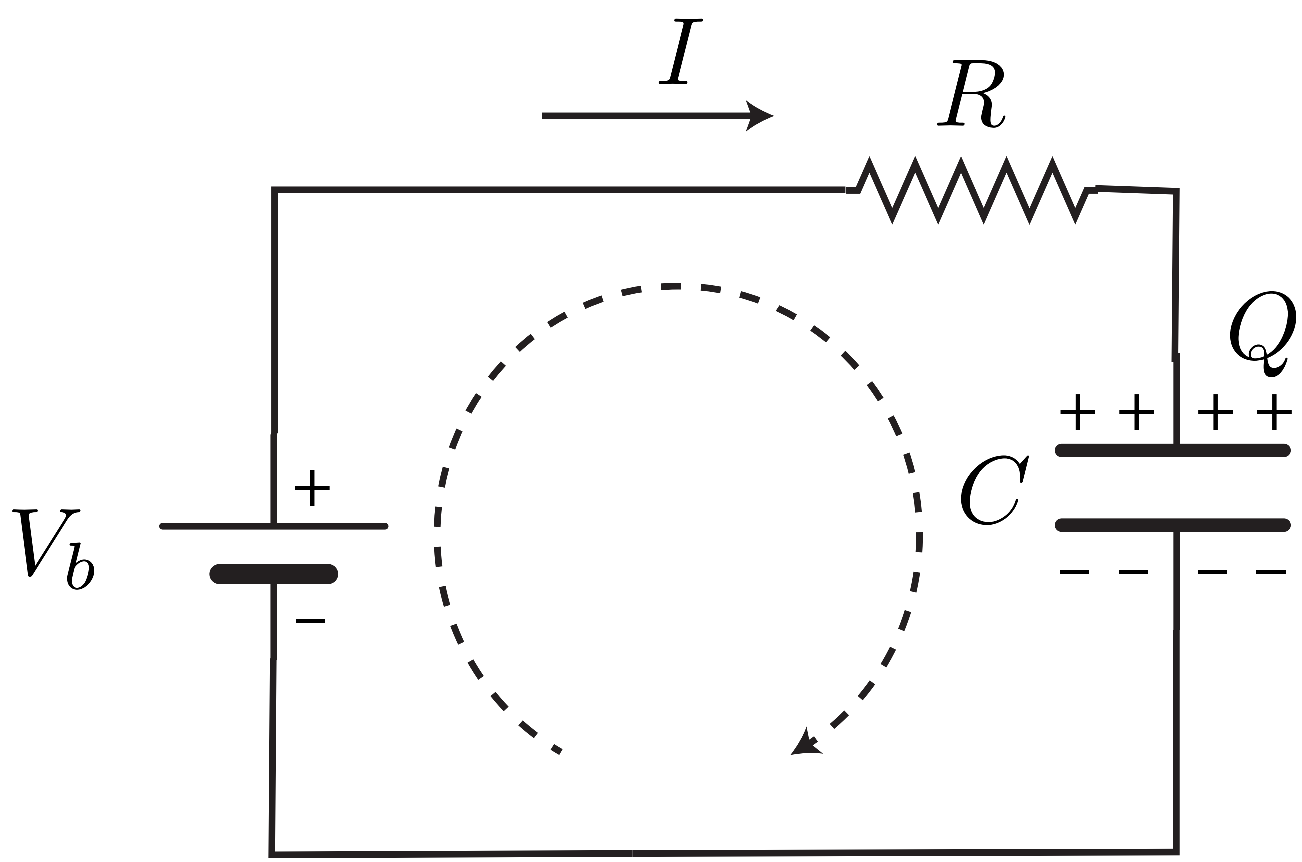




My capacitor heuristic:

- Small t (large v) →
  C is a short circuit
- Large t (small v) →
  C is an open circuit

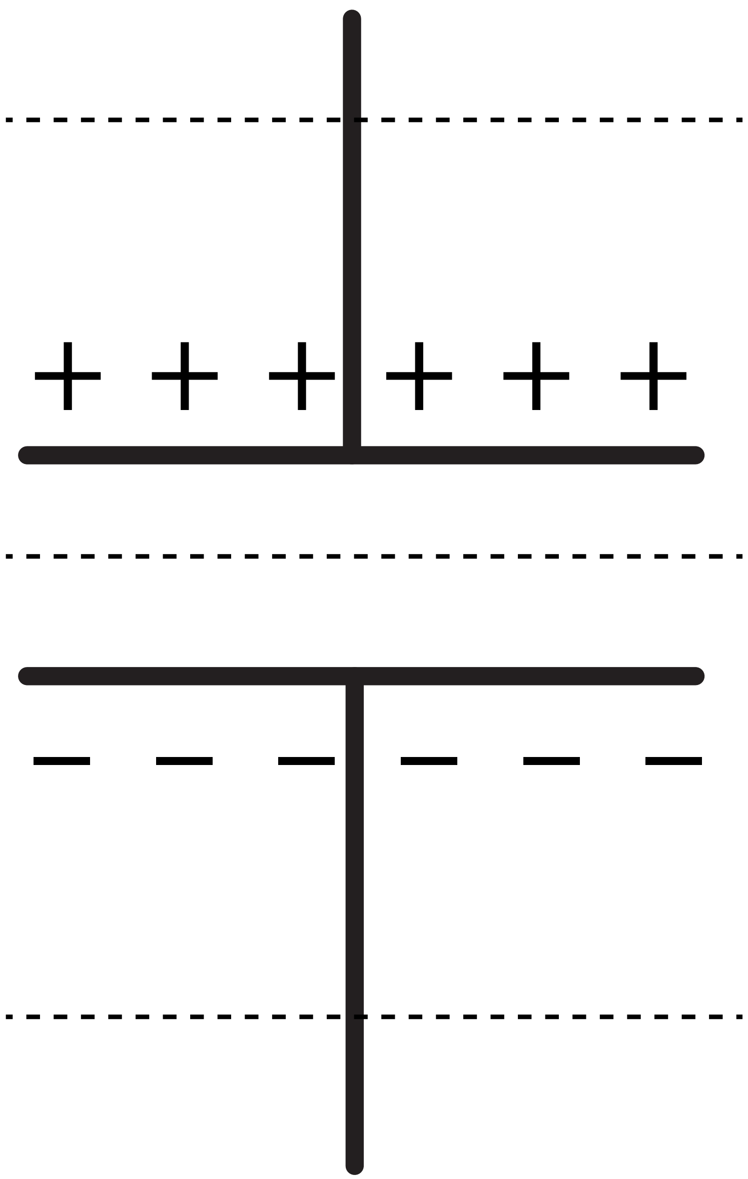
Charging:
Setting up the equations

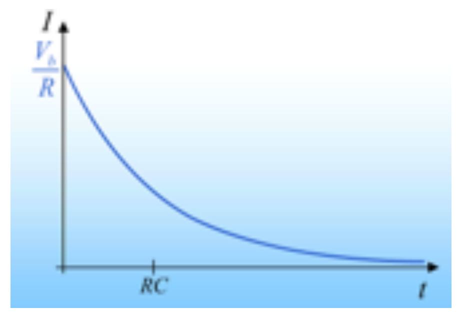


Kirchhoff Voltage Law:

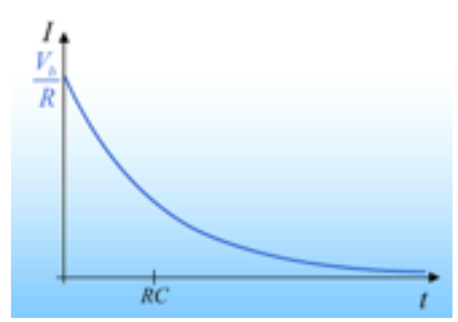
$$-V_b + IR + Q/C = 0$$

Aside: Does current really flow through a capacitor?



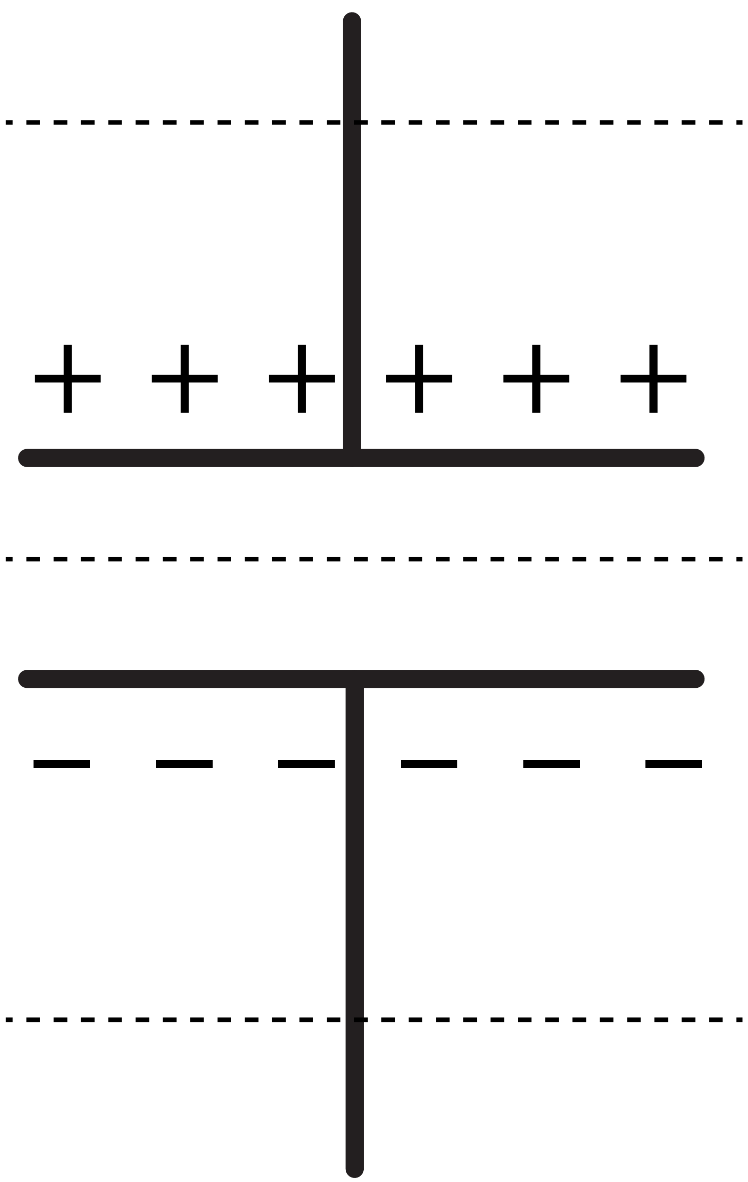


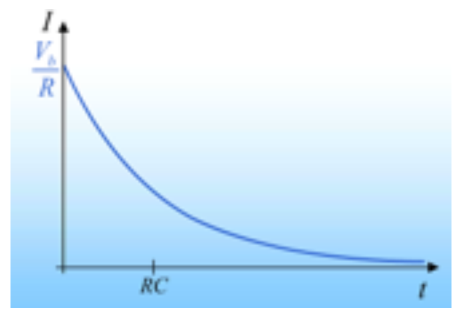
$$I(t) = 0$$



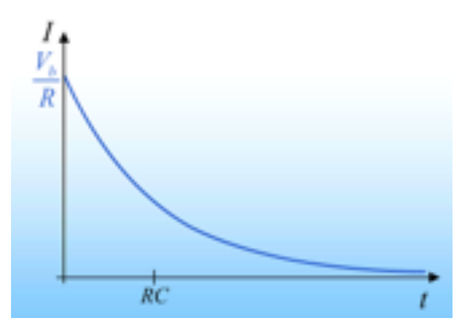
- Current through 3 surface
- The current through the gap is always 0!
- "Current through the capacitor"
  = change in Q on plates

Aside: Does current really flow through a capacitor?





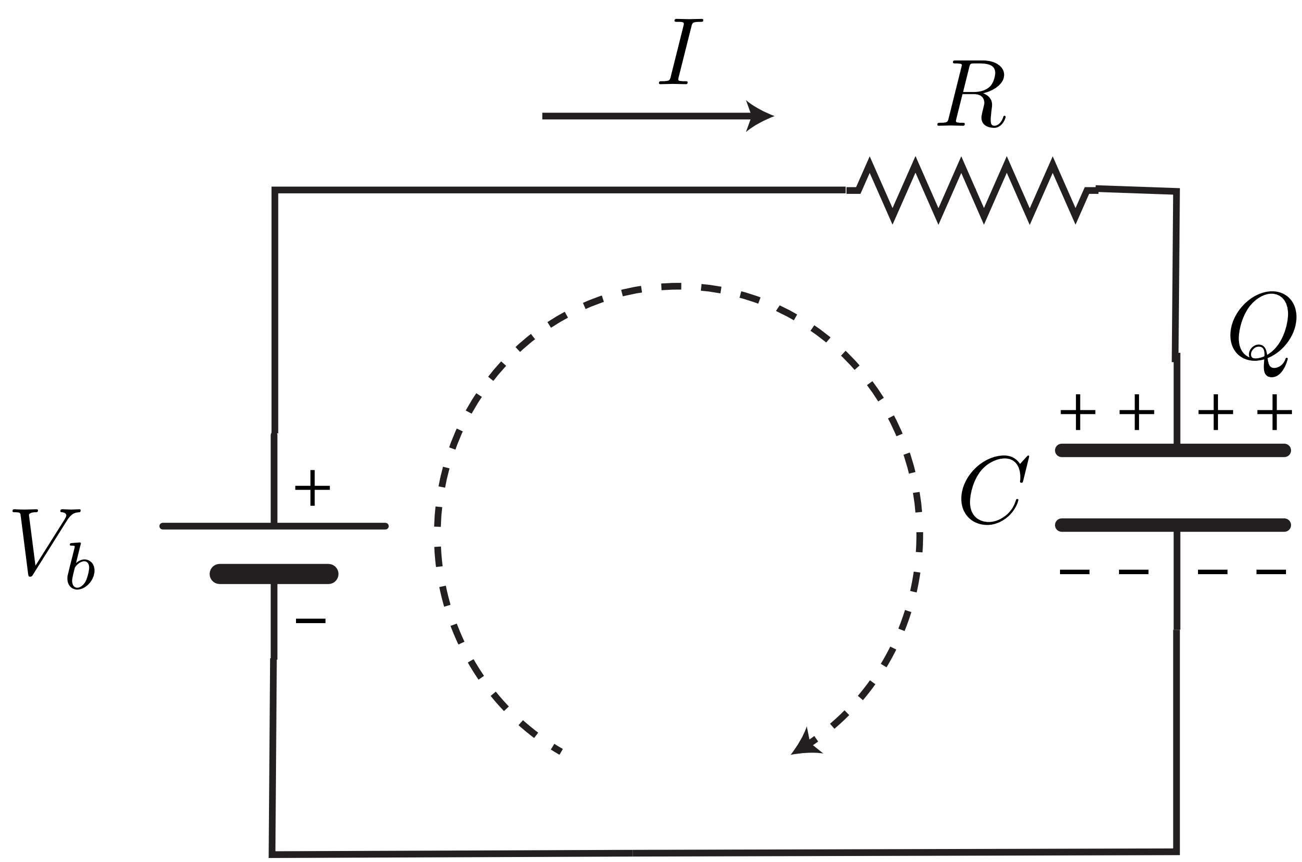
$$I(t) = 0$$



- "Current through the capacitor"
  = change in Q on plates

$$I = \frac{dQ}{dt}$$

Charging: Setting up the equations



Kirchhoff Voltage Law:

$$-V_b + R\frac{dQ}{dt} + Q/C = 0$$

Initial State:

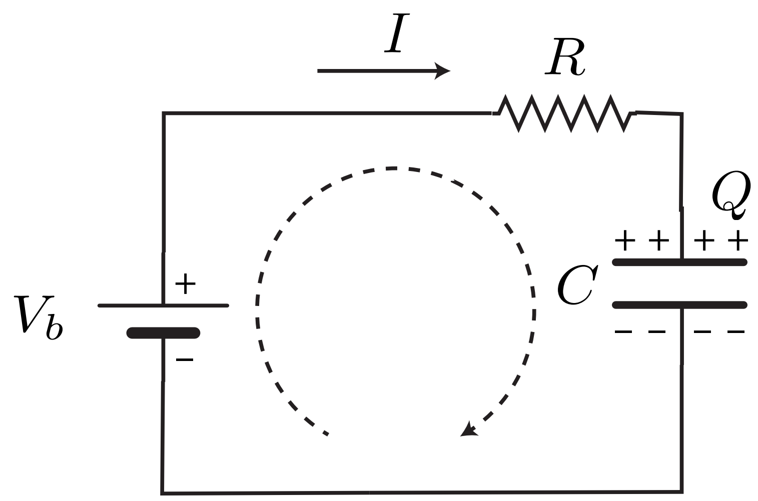
$$Q = 0$$

Charging:
Solving the equations

Overhead.

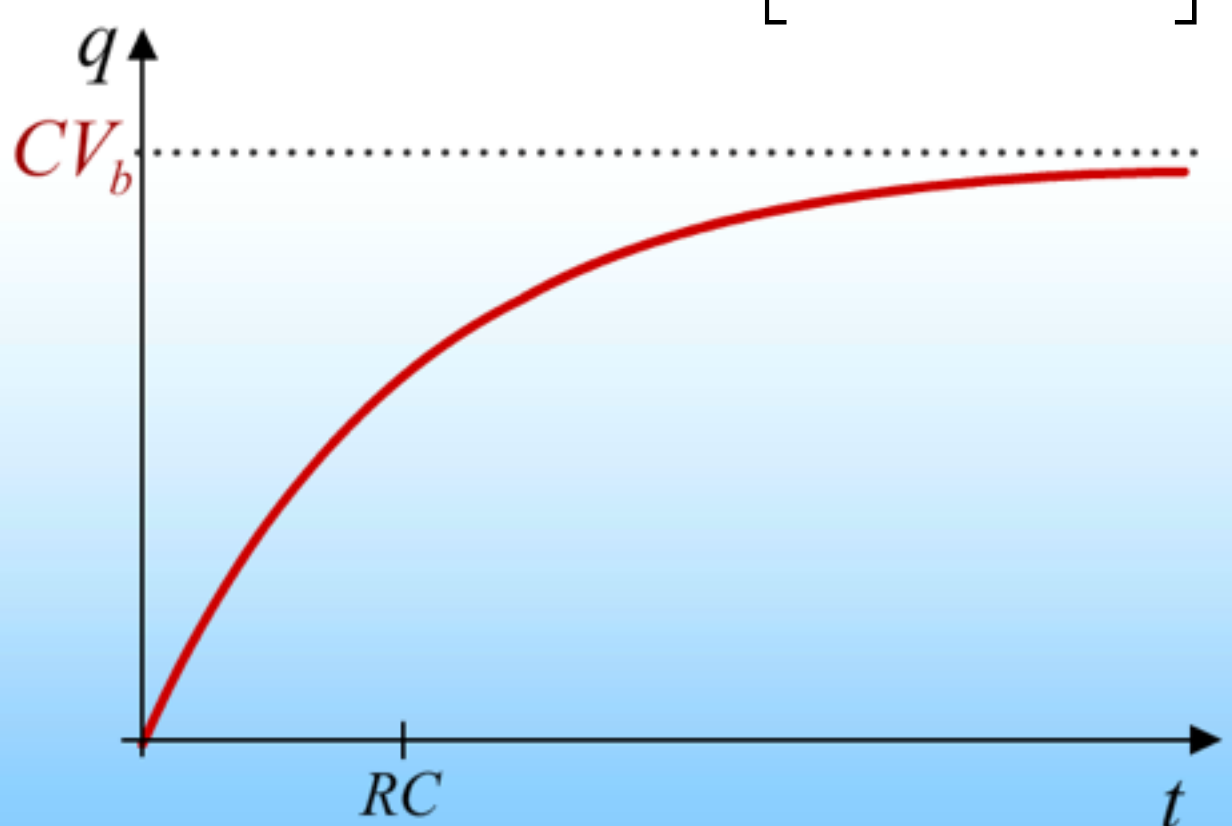
Charging: Solution

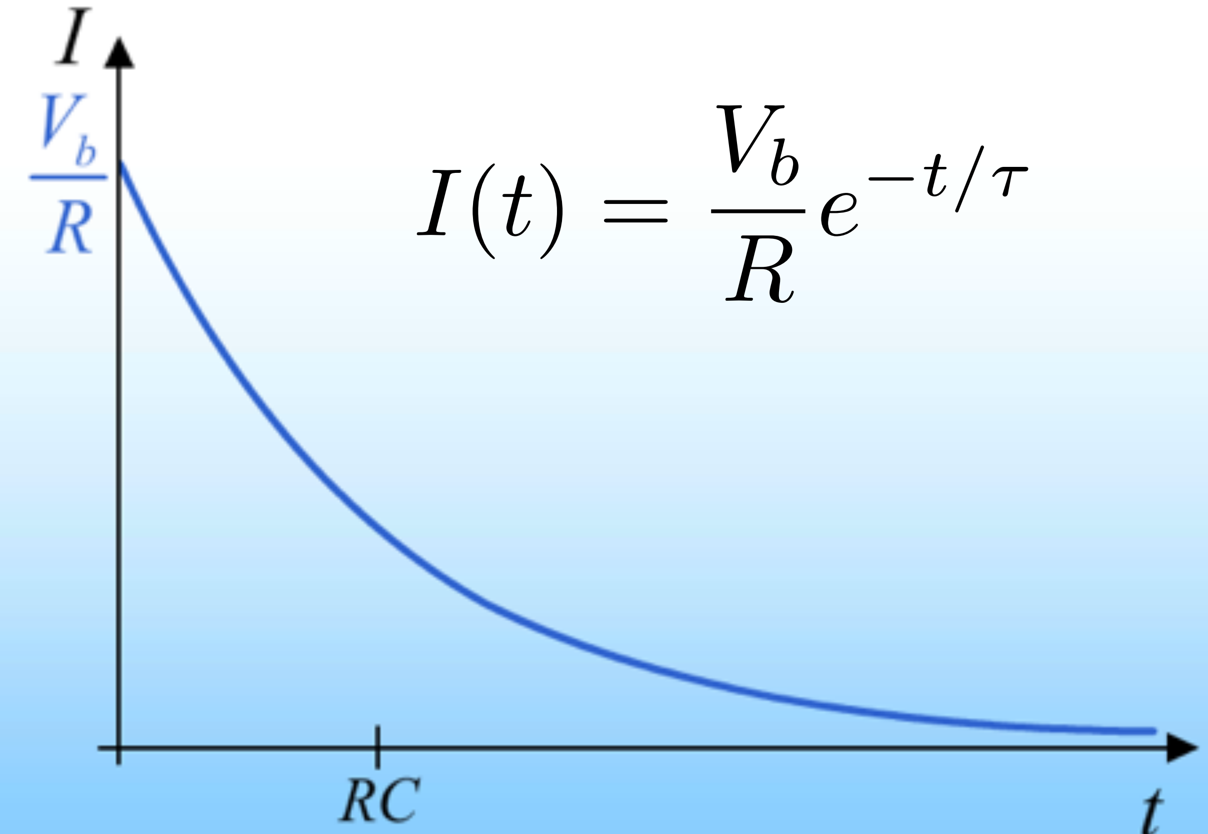
RC time constant: $\tau \equiv RC$



$$Q(t) = CV_b\left[1 - e^{-t/\tau}\right]$$

$$V_C(t) = V_b\left[1 - e^{-t/\tau}\right]$$





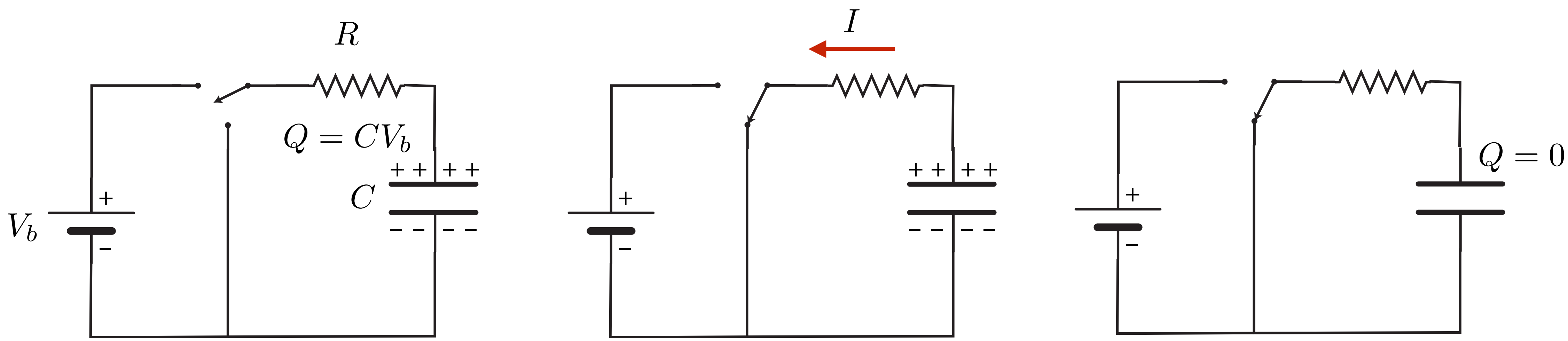
$$I(t) = \frac{V_b}{R}e^{-t/\tau}$$

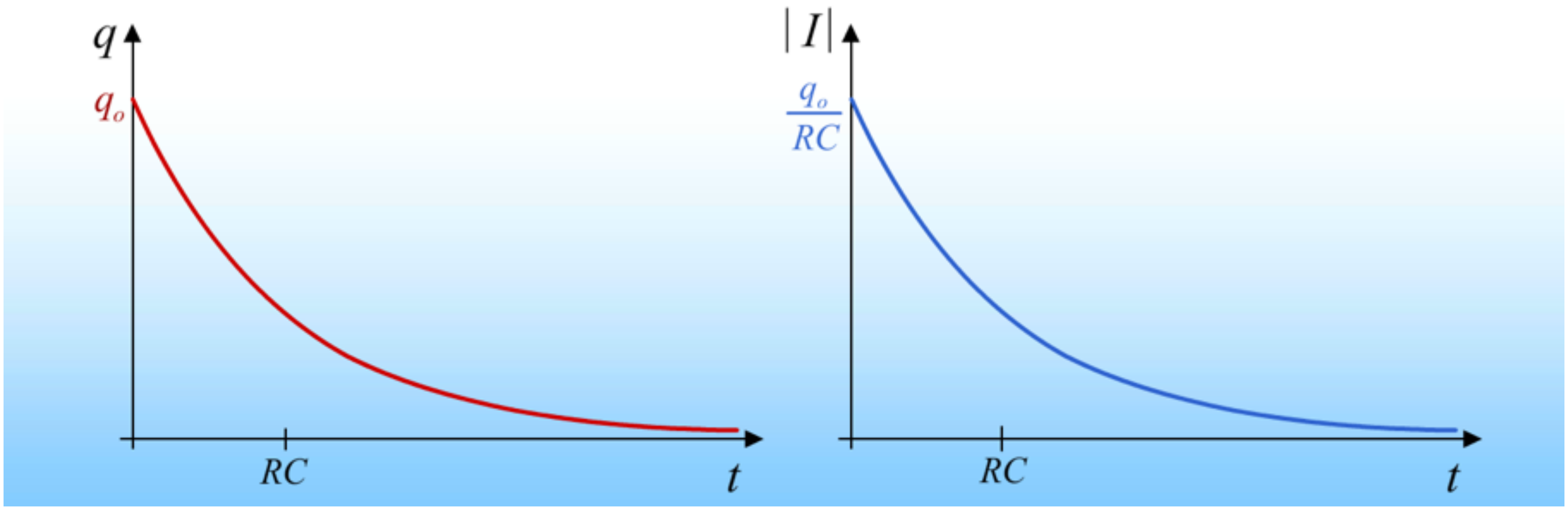
Quick check of the units….

- RC time constant units:

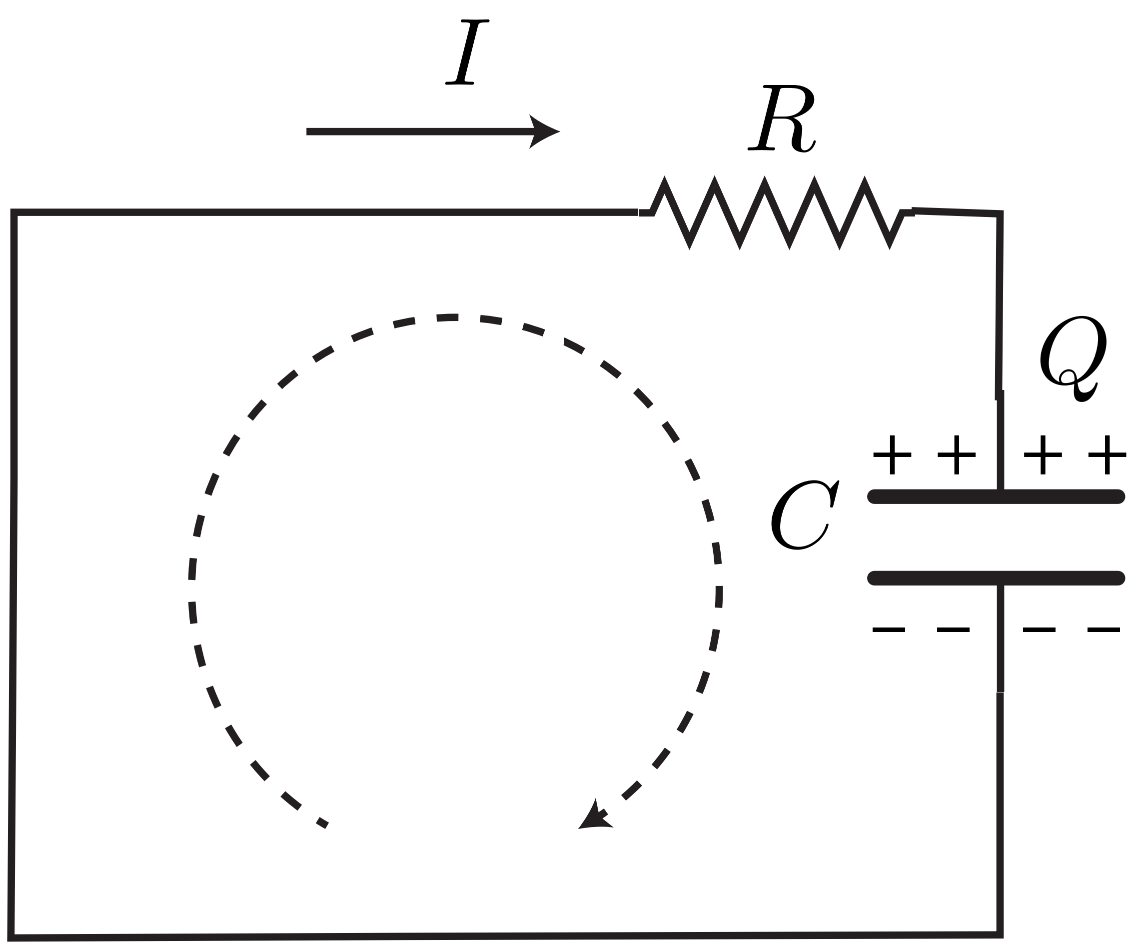
$$[\tau] = [R][C] = [V][I]^{-1}[Q][V]^{-1} = [dQ/dt]^{-1}[Q] = s$$

Discharging: Overview of the action.





Discharging: Setting up the equations



Kirchhoff Voltage Law:

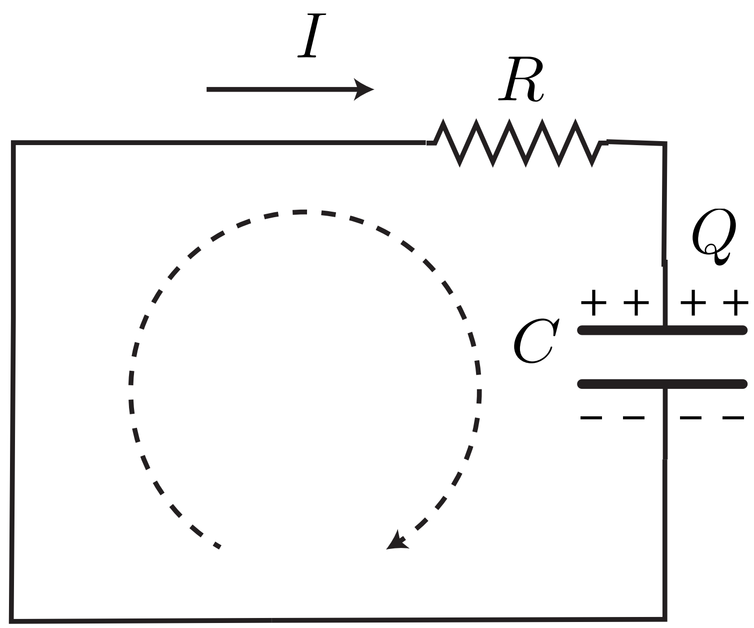
$$RI + Q/C = 0$$

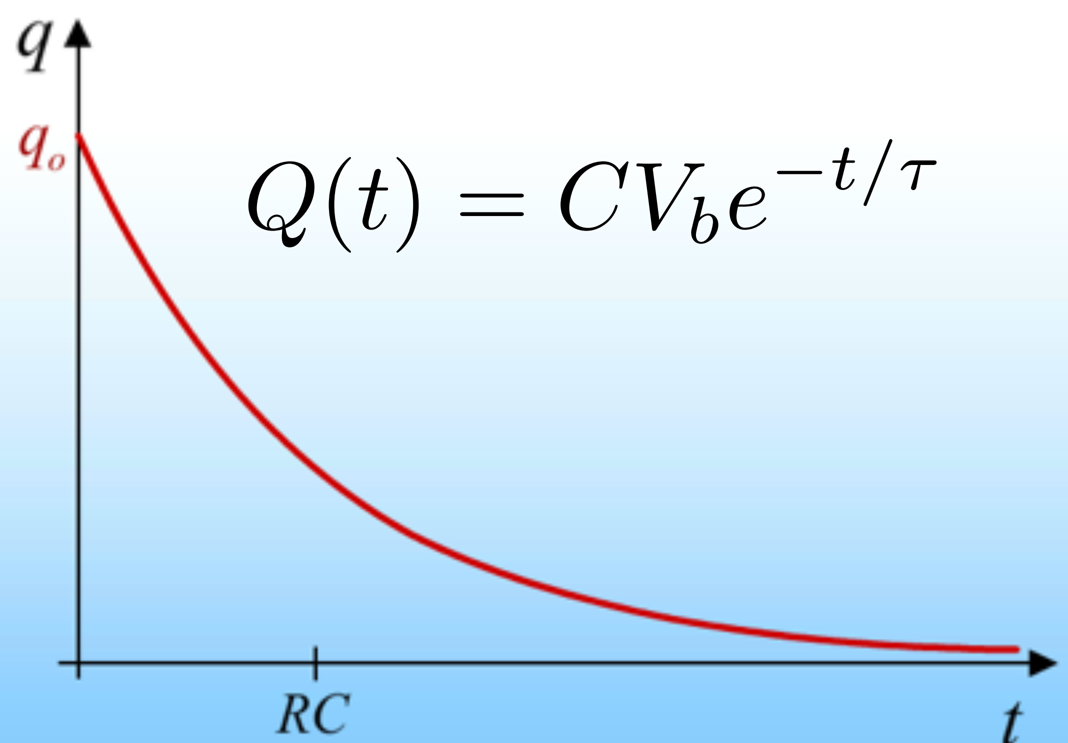
Initial State:

$$Q(0) = CV_b$$

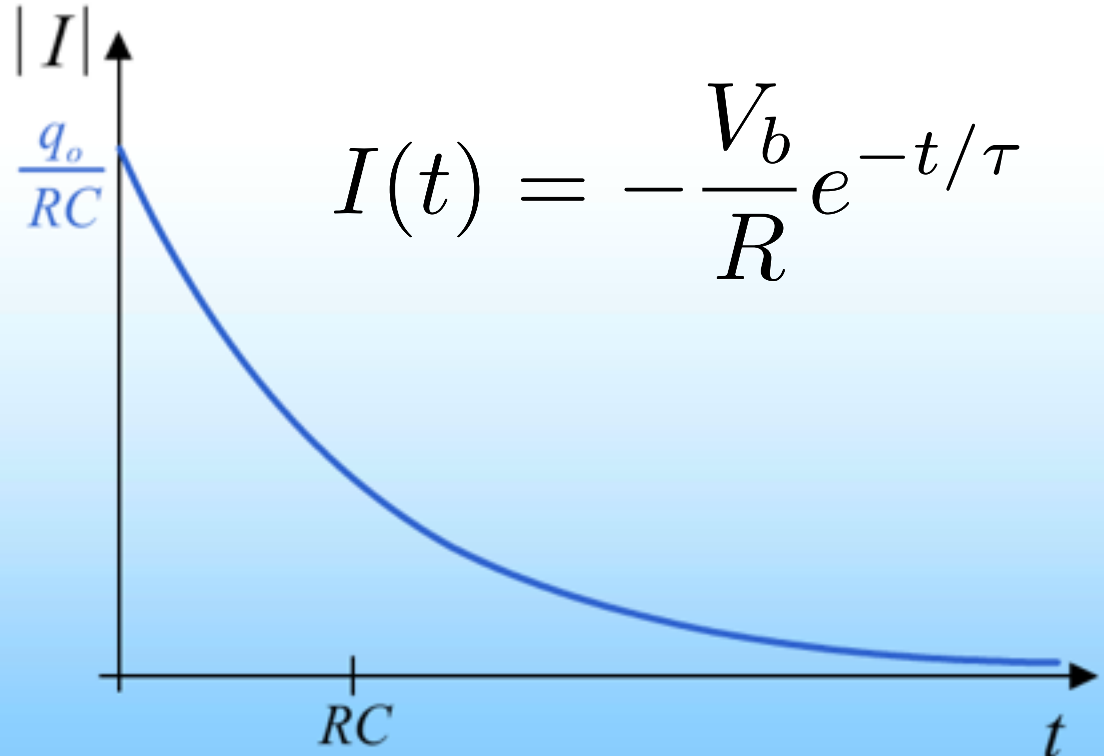
Discharging: Solution

RC time constant: $\tau \equiv RC$





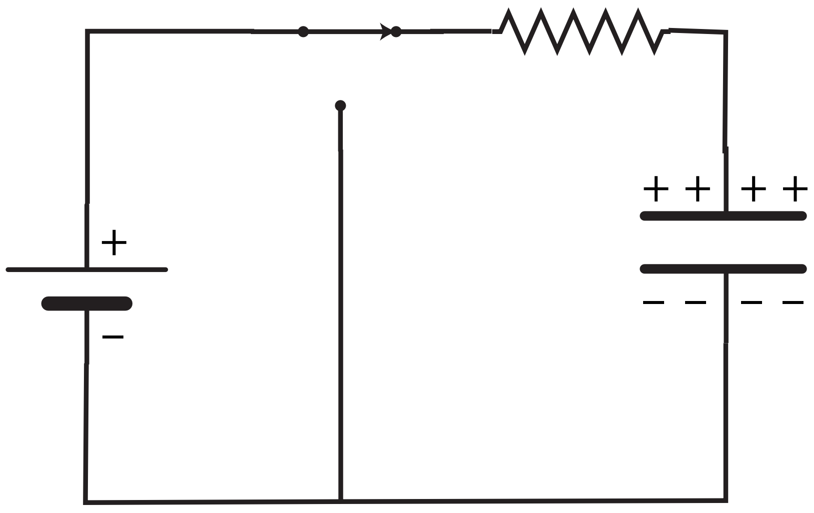
$$Q(t) = CV_b e^{-t/\tau}$$

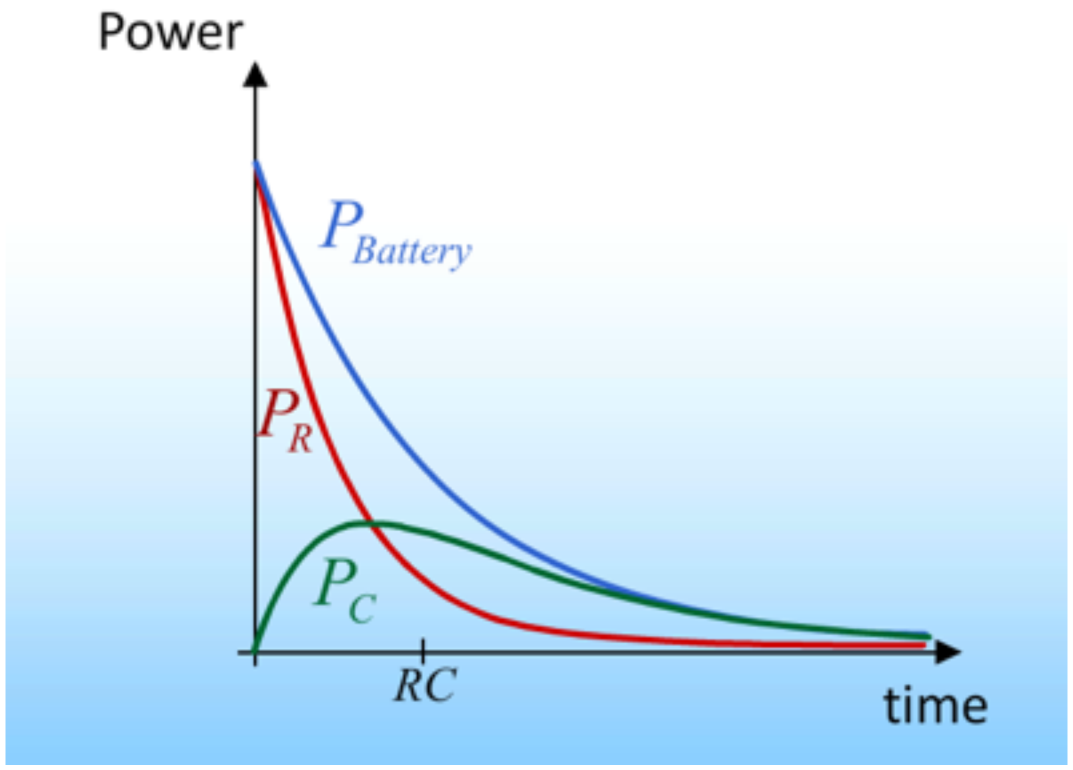


$$I(t) = -\frac{V_b}{R} e^{-t/\tau}$$

Energy flow in an RC circuit.

- Power: $P = IV$
- Energy stored: $U = \frac{1}{2}CV^2$





$$P_{Battery}(t) = V_b I_o e^{-t/RC}$$

$$P_R(t) = R\, I_o^2 e^{-2t/RC}$$

$$P_C(t) = \left[\frac{q_o}{C}\left(1 - e^{-t/RC}\right)\right]\left[I_o e^{-t/RC}\right]$$