Kirchhoff Circuit Laws

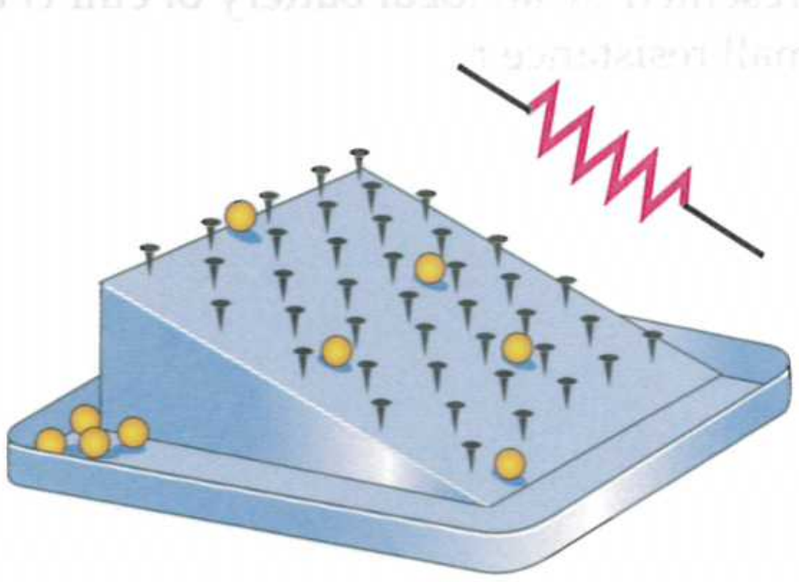
Lecture 13

Electromotive force

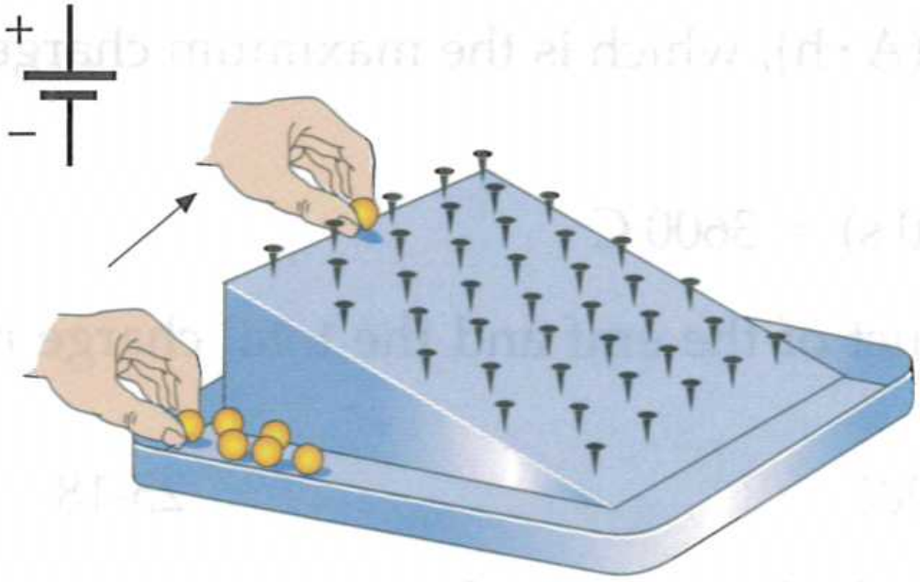
- **EMF**: Historical name for **voltage gain** in a battery
- Work done per unit charge to increase/decrease its potential

$$[\mathcal{E}] = \mathcal{V}$$

One more analogy…



(*a*)



(*b*)

Power: Supplied and Dissipated

- Power supplied by a source of emf (voltage supply)

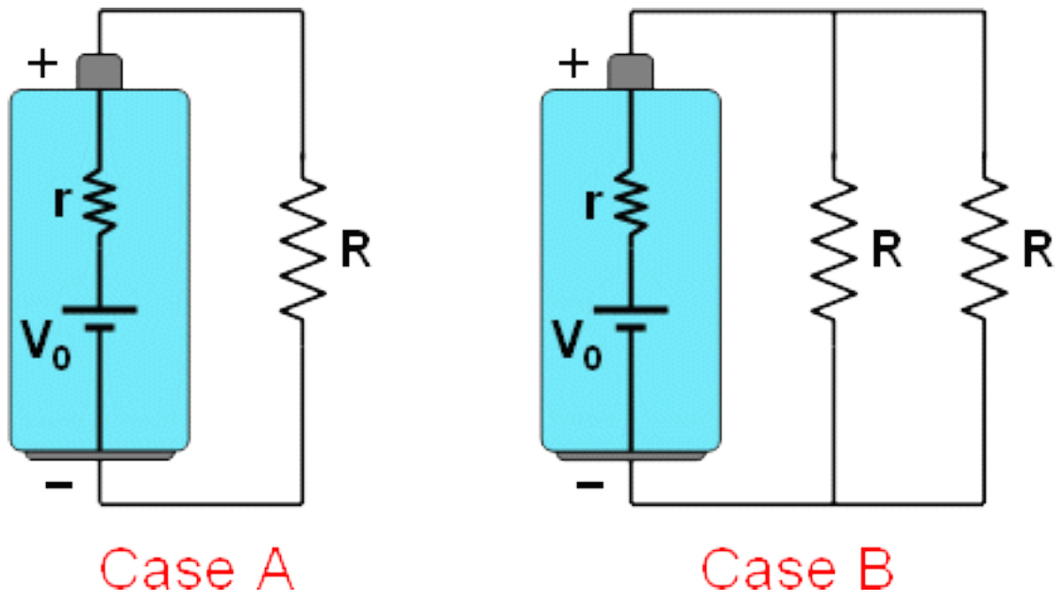
$$P = IV$$

- Power dissipated by a ohmic resistor (to heat):

$$P = IV = RI^2 = V^2/R$$

- Units: [Power] = J/s = Watts = W

Pre-lecture



- The 'real' batteries and the resistors in both cases illustrated above are identical. In which case is the voltage across the terminals of the battery closest to the ideal battery voltage V0?

Kirchhoff Laws:

- **Current Rule:** $\sum I_{\text{in}} = \sum I_{\text{out}}$

  DH: “What goes in must come out.”

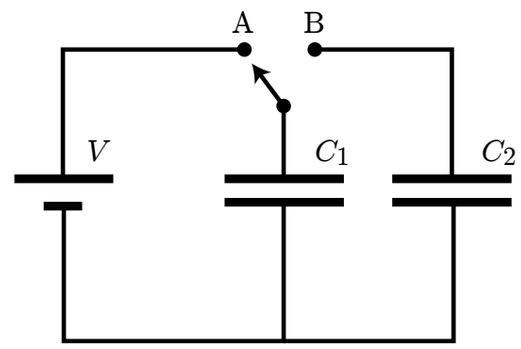
- **Voltage Rule:** $\sum \Delta V = 0$ around any loop

  DH: “What goes up must come down.”

- (Can be tricky to implement.)

Name _____ **Student ID** _____ **Score**_____
last *first*

For the next **three questions**, consider the following experiment: All capacitors start uncharged. Capacitor 1 with capacitance C_1 is charged to V using a battery. Once capacitor 1 is charged, it is disconnected from the battery and then used to charge capacitor 2 with capacitance C_2. (I.e. the switch is connected first to terminal A and then to terminal B.)



11. [2 points] When the switch is initially in the A position, what is the charge on capacitor 1?

A. $Q_1 = C_1V$, **B.** $Q_1 = C_2V$, **C.** $Q_1 = 0$, **D.** $Q_1 = (C_1 + C_2)V$, **E.** $Q_1 = \frac{C_1C_2}{C_1+C_2}V$,

12. [2 points] When the switch is in the B position, what is the total charge on both capacitors Q?

A. $Q = C_1V$, **B.** $Q = C_2V$, **C.** $Q = 0$, **D.** $Q = (C_1 + C_2)V$, **E.** $Q = \frac{C_1C_2}{C_1+C_2}V$,

Solving Circuit Problems

1. Use equivalent resistance & re-order components

2. Define and draw current arrow in all wires

3. Define and draw loops for voltage rule (covering all wires)

4. Apply Kirchhoff Rules to generate equations (**tricky**)

5. Solve algebraic equations for currents (**time consuming**)

Kirchhoff's Current Rule

$$I_1 + I_2 = I_3$$

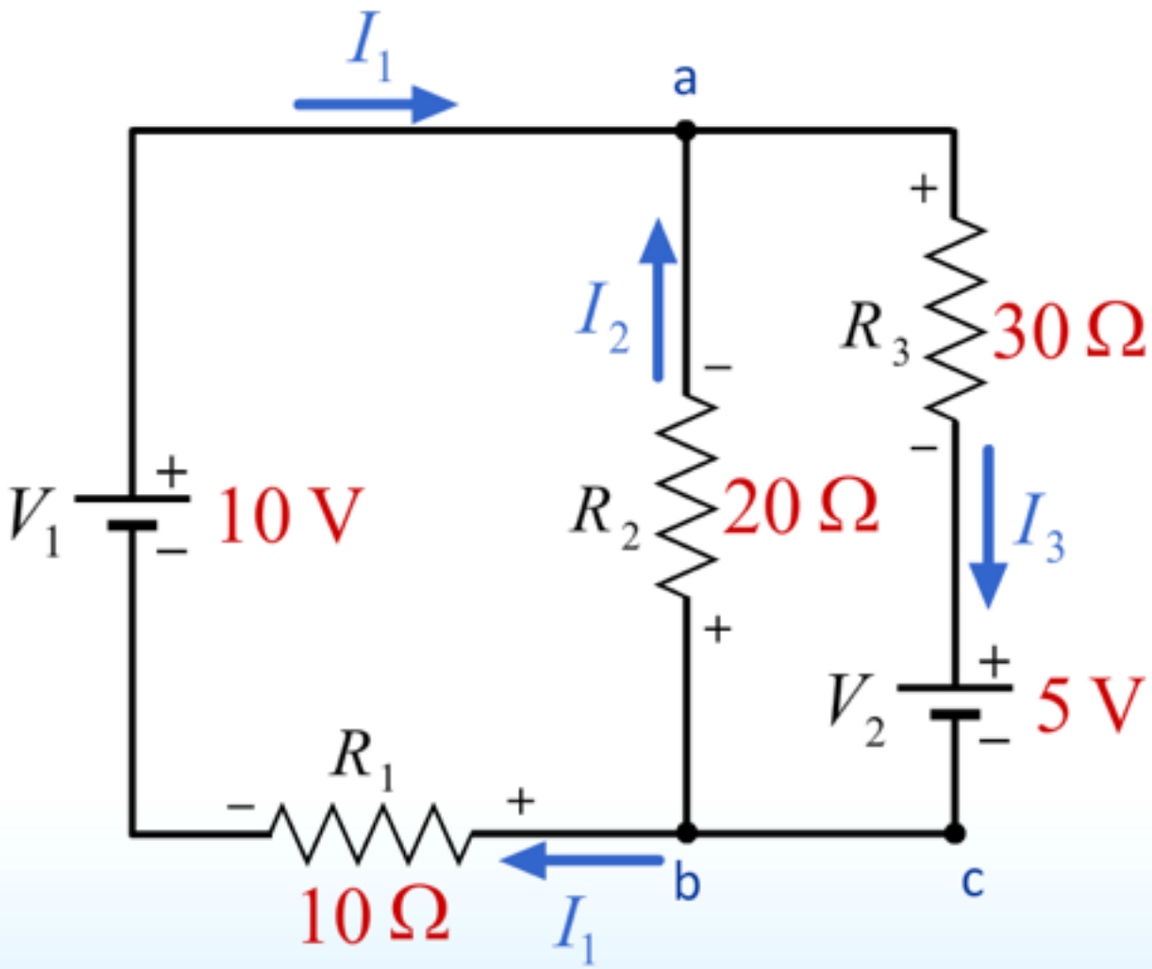
Kirchhoff's Voltage Rule

outer loop

$$I_3 R_3 + V_2 + I_1 R_1 - V_1 = 0$$

left loop

$$-I_2 R_2 + I_1 R_1 - V_1 = 0$$



$$I_1 = \frac{V_1(R_2 + R_3) - V_2 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_2 = \frac{-V_2 R_1 - V_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_3 = \frac{V_1 R_2 - V_2(R_1 + R_2)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_1 = +0.364\ \text{A}$$

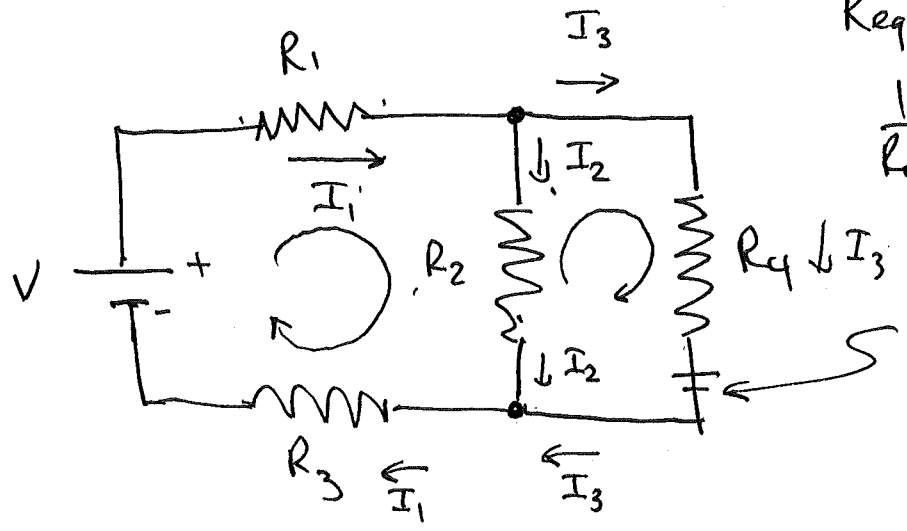
$$I_2 = -0.318\ \text{A}$$

$$I_3 = +0.045\ \text{A}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_4}$$

$$\frac{1}{R_{eq}} = \frac{R_2 + R_4}{R_2 R_4}$$

$$R_{eq} = \frac{R_2 R_4}{R_2 + R_4}$$



$I_1 = I_2 + I_3$ ✓

$$0 = \sum_i \Delta V = R_1 I_1 + I_2 R_2 + I_1 R_3 - V$$

$$0 = \sum_i \Delta V = R_4 I_3 - R_2 I_2 = 0 \checkmark$$

$$R_4 I_3 = R_2 I_2$$

$$I_3 = \frac{R_2}{R_4} I_2$$

$$I_1 = I_2 + \frac{R_2}{R_4} I_2 = \frac{R_4 + R_2}{R_4} I_2$$

$$I_2 = \frac{R_4}{R_4 + R_2} I_1$$

$$0 = R_1 I_1 + \frac{R_4 R_2}{R_4 + R_2} I_1 + R_3 I_1 - V$$

$$V = \left(R_1 + \frac{R_4 R_2}{R_4 + R_2} + R_3 \right) I_1$$

R_{eq}