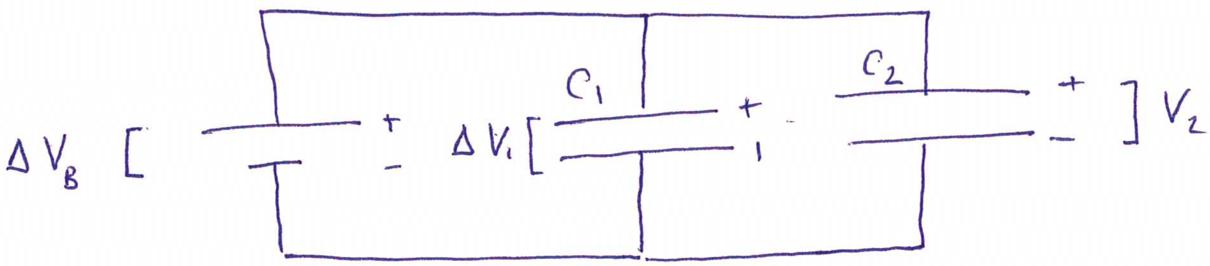
Capacitors in Parallel



$$\Delta V_B = \Delta V_1 = \Delta V_2 = \Delta V$$

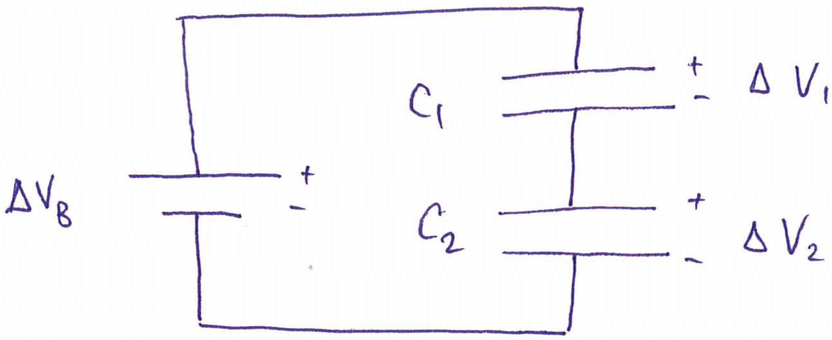
$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

$$Q_{TOT} = Q_1 + Q_2 = (C_1 + C_2)\Delta V$$

$$= C_{Equiv} \Delta V$$

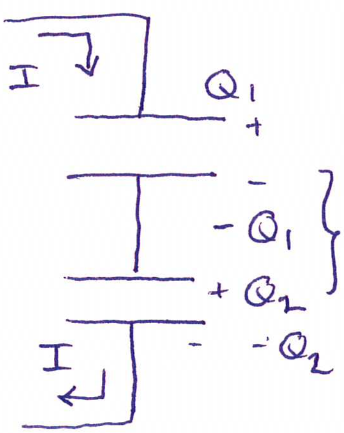
$$\boxed{C_{Equiv} = C_1 + C_2}$$

Capacitors in Series



$\Delta V_B = \Delta V_1 + \Delta V_2$

But we assume Q_1 & $Q_2 = 0$ before circuit is charged. $\Rightarrow$ $Q_1 = Q_2 = Q$ after charging



these must sum to 0!

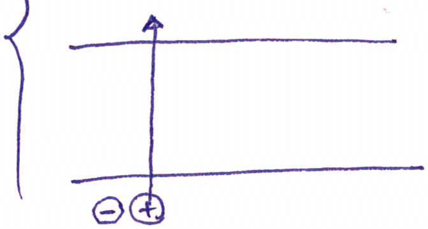
$$\Delta V_B = \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q = \frac{Q}{C_{equiv}}$$

$$C_{Equiv}^{-1} = C_1^{-1} + C_2^{-1}$$

Compute energy require to charge a capacitor

why isn't the answer $U = Q\overset{\swarrow\ \text{depends on } Q}{V}$?

easier

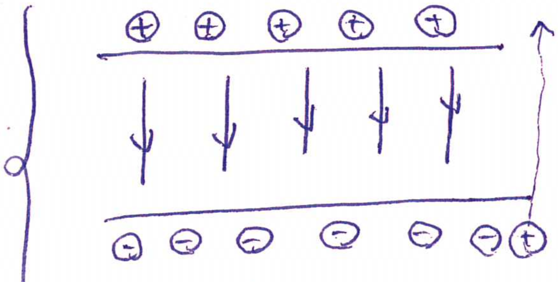


$$dU = V\,dQ$$

$$\uparrow$$

$$V = V(Q)\ !$$

$$V = \frac{Q}{C}$$



harder!

$$U_0 = \int_0^{U_0} dU = \int_0^{Q_0} dQ\,\frac{Q}{C} = \frac{1}{2}\frac{Q_0^2}{C}$$

$$= \frac{1}{2} C V_0^2 = \frac{\varepsilon_0 A}{2d}(dE)^2$$

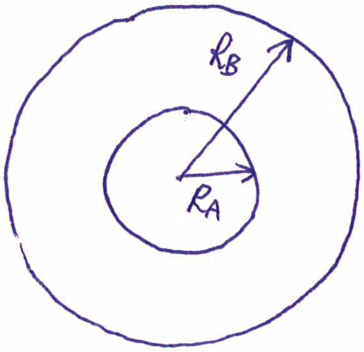
$$= \frac{1}{2}\varepsilon_0 \underbrace{(Ad)}_{\text{Volume between plates.}} E^2$$

$$\Rightarrow \quad U = \int d^3x \underbrace{\tfrac{1}{2}\varepsilon_0 E^2}_{\text{energy density.}}$$

Energy Density

$$u \equiv \frac{1}{2}\varepsilon_0 E^2$$

Capacitance of a Cylindrical Capacitor



$$\vec{E} = (-\hat{r}) \frac{Q}{L} \frac{1}{2\pi \epsilon_0 r}$$

$$\Delta V = -\int_{R_A}^{R_B} dr \; \hat{r} \cdot (-\hat{r}) \frac{Q}{L} \frac{1}{2\pi \epsilon_0 r}$$

$$= \frac{Q R_A}{2\pi R_A L \epsilon_0} \log \frac{R_B}{R_A}$$

$$\Delta V = \frac{Q}{2\pi \epsilon_0 L} \log \frac{R_B}{R_A}$$

$$C = \frac{2\pi L \epsilon_0}{\log \frac{R_B}{R_A}}$$

limit as $R_A \to R_B^- \Rightarrow R = \frac{1}{2}(R_B + R_A) \quad \delta R = R_B - R_A$

$$C = \frac{\overbrace{2\pi L R}^{A} \; \epsilon_0}{R \log \frac{1 + \frac{\delta R}{2R}}{1 - \frac{\delta R}{2R}}} = \frac{A \epsilon_0}{R\left(\frac{\delta R}{R}\right)} = \frac{A \epsilon_0}{\delta R}$$