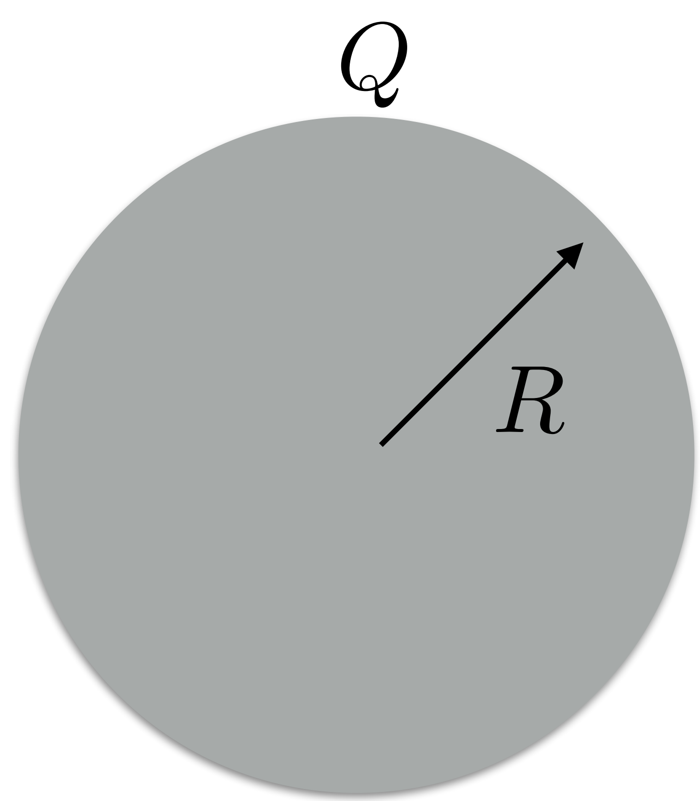
E from V & V from E.

Lecture 9.

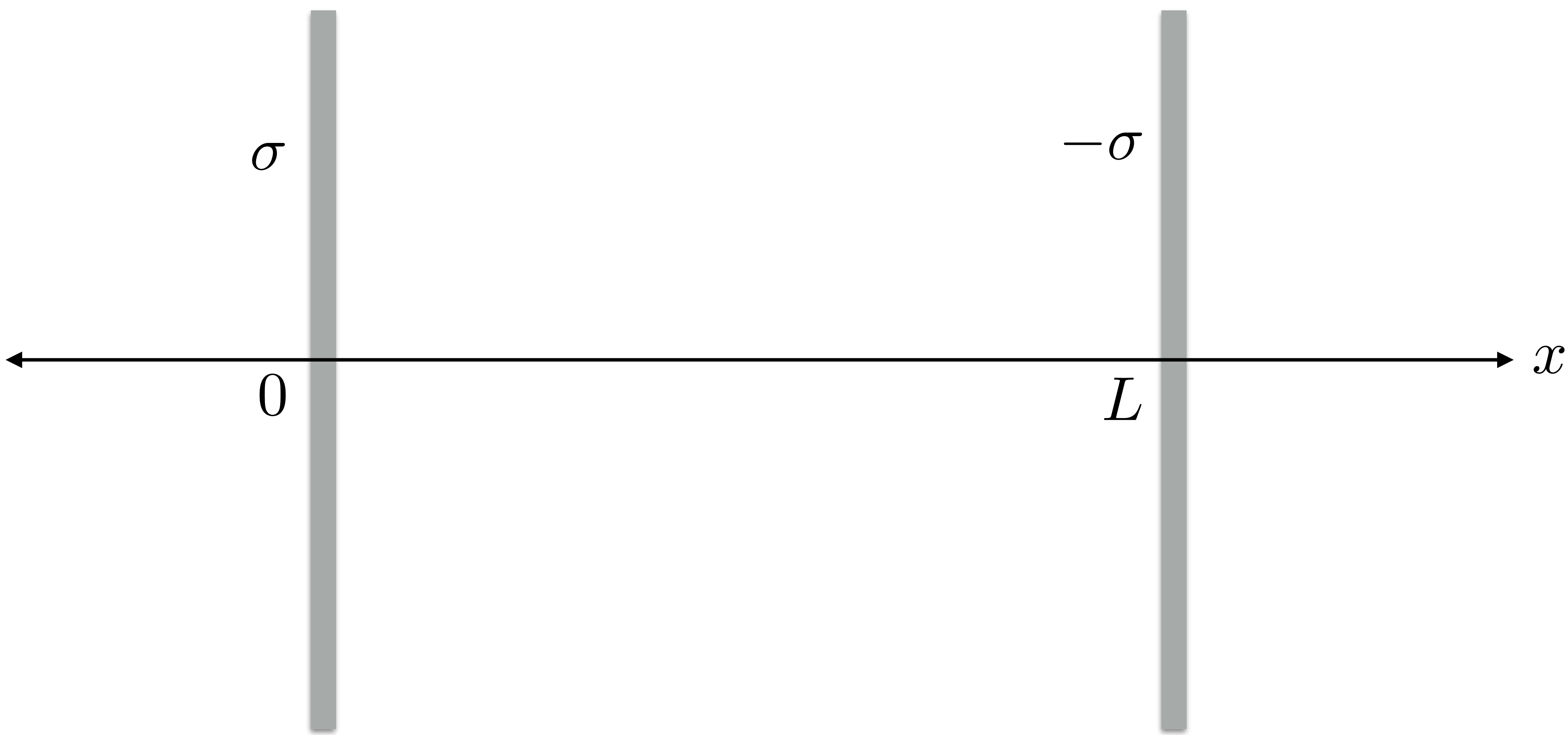
Calculate V from E:

- Calculate the potential from a charged conducting sphere



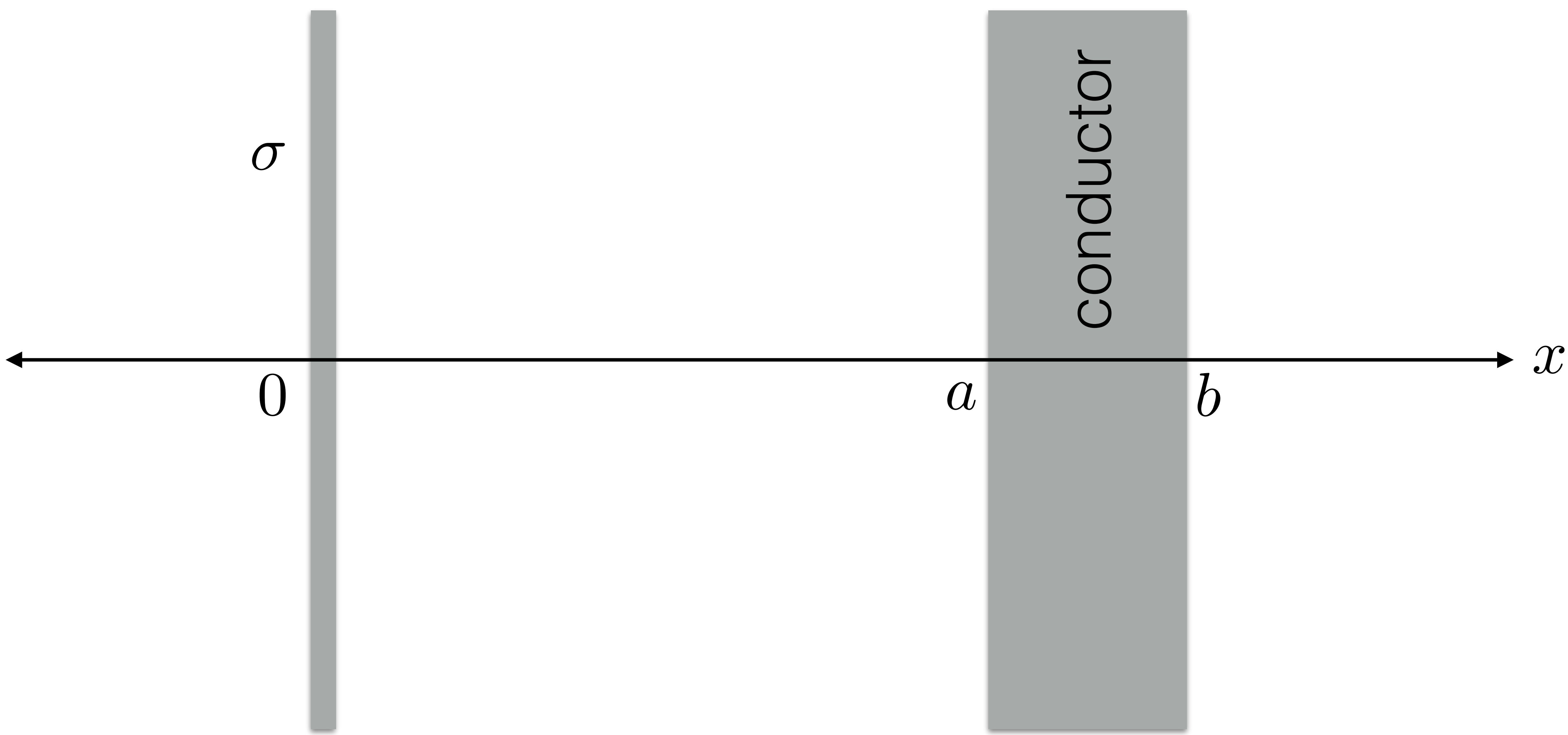
Potential from Planes of charge

$\sigma > 0$

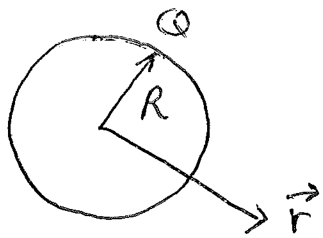


Potential from Planes of charge

$\sigma > 0$



(1)



$$\vec{E}(\vec{r}) = \begin{cases} \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r}, & r > R \\ 0, & r < R \end{cases}$$

$r > R$

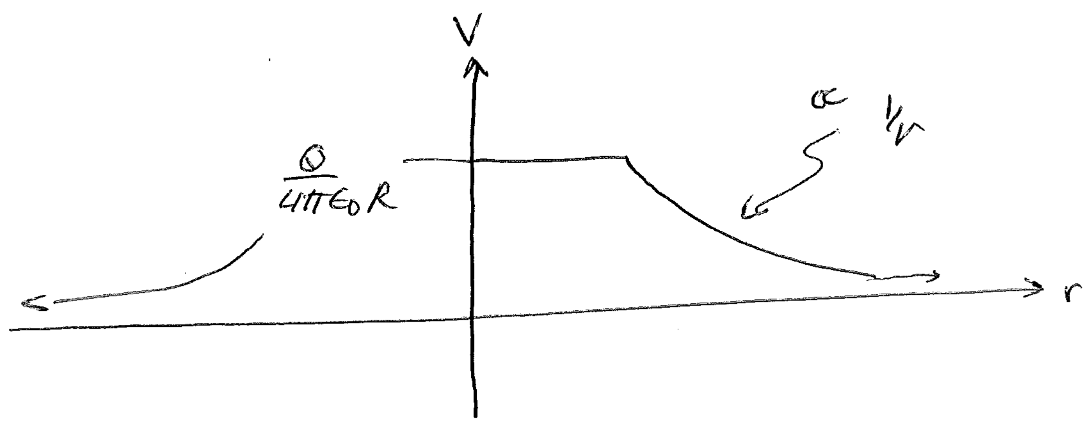
$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} d\vec{l} \cdot \vec{E}(\vec{r})$$

$$V(\vec{r}) = -\int_{\infty}^{r} dr \frac{Q}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r}$$

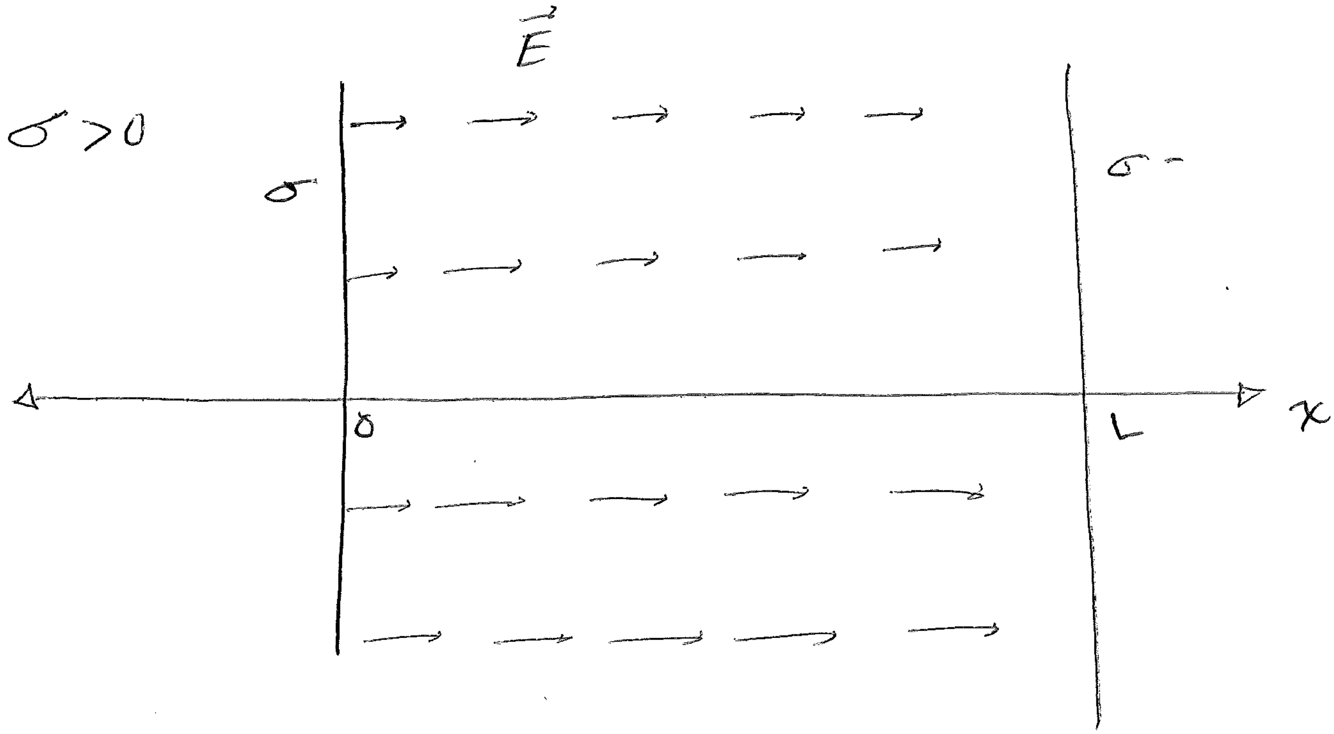
$r < R$

$$V(\vec{r}) = -\int_{\infty}^{R} dr \frac{Q}{4\pi\epsilon_0 r^2} + -\int_{R}^{r} dr \cdot 0 = \frac{Q}{4\pi\epsilon_0 R}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \begin{cases} r, & r > R \\ R, & r < R \end{cases}$$

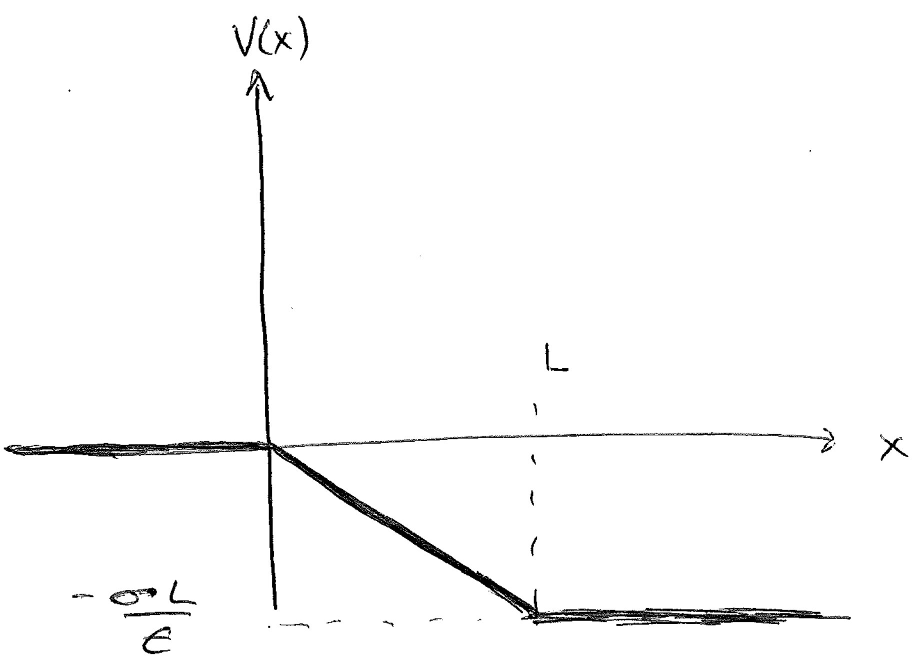


2



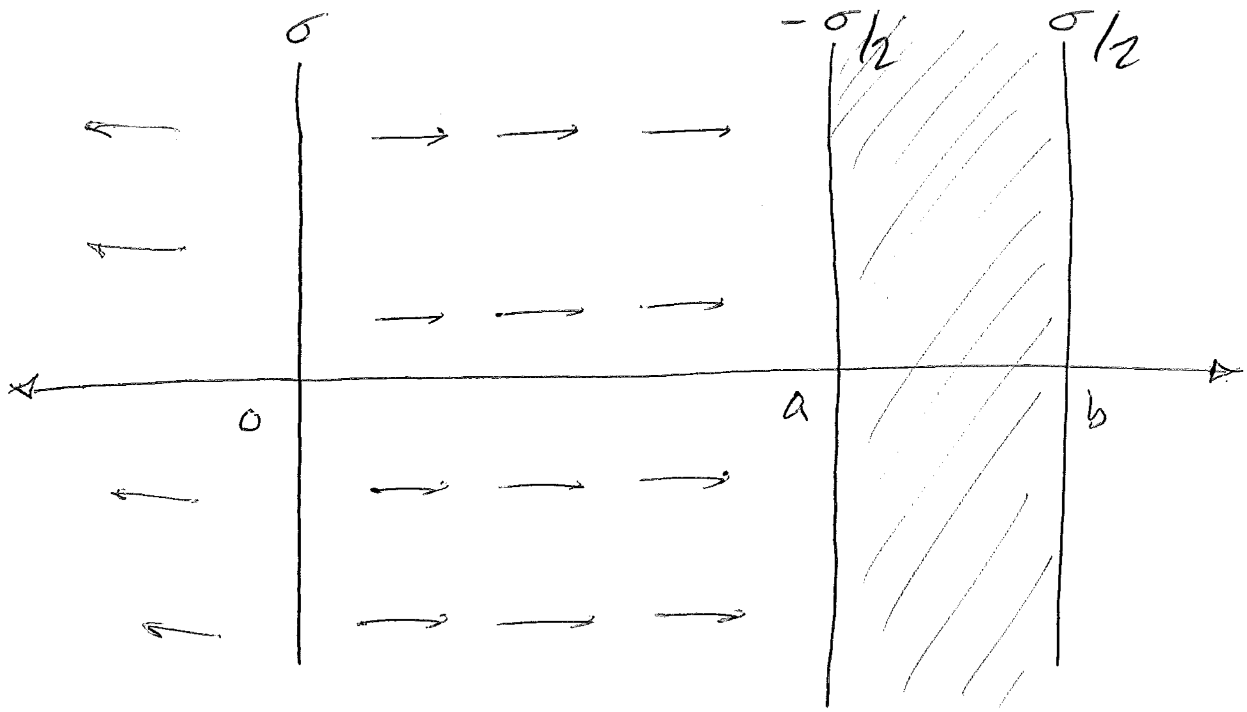
$$\vec{E} = \begin{cases} \dfrac{\sigma}{\epsilon_0}\hat{x}, & 0 < x < L \\ 0, & x < 0 + x > L \end{cases}$$

$$V(\vec{r}) = -\int_{-\infty}^{x} dx\, E(x) = \begin{cases} -\dfrac{\sigma x}{\epsilon_0}, & 0 < x < L \\ -\dfrac{\sigma L}{\epsilon}, & x > L \\ 0, & x < 0 \end{cases}$$



(3)

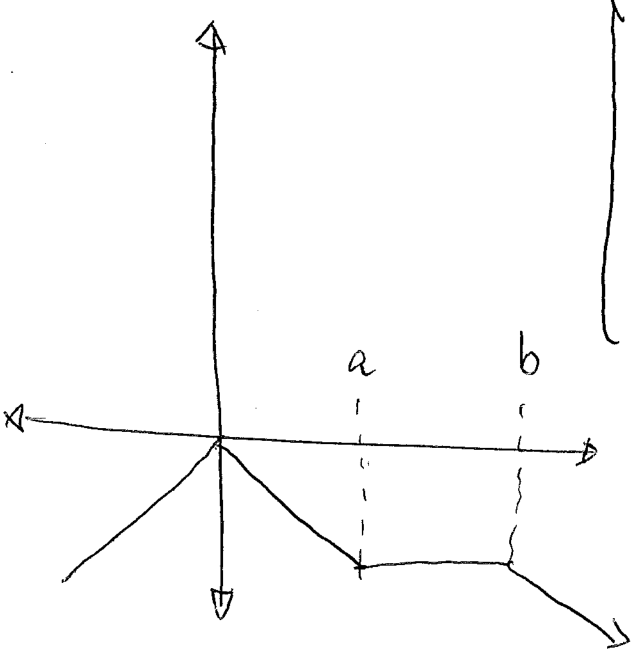
$\sigma > 0$



$E = -\frac{\sigma}{2\epsilon_0}$ $\quad E = \frac{\sigma}{2\epsilon_0}$ $\quad E = 0$ $\quad E = \frac{\sigma}{2\epsilon_0}$

$$V(\vec{r}) - V(x=0) = \begin{cases} \frac{\sigma x}{2\epsilon_0}, & x < 0 \\ -\frac{\sigma x}{2\epsilon_0}, & 0 < x < a \\ -\frac{\sigma a}{2\epsilon_0}, & a < x < b \\ -\frac{\sigma(x-b+a)}{2\epsilon_0}, & b < x \end{cases}$$

$V(x)$



(4)

$$V(\vec{r}) = Ax + By + Cz$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}) = -A\cdot\hat{x} - B\hat{y} - C\hat{z}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0 (x^2+y^2+z^2)^{1/2}}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}) = +\frac{1}{\cancel{2}}\frac{Q}{4\pi\epsilon_0}\frac{\cancel{2}x\cdot\hat{x} + \cancel{2}y\hat{y} + \cancel{2}z\hat{z}}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{\vec{r}}{r^3}$$

Spherical Coords:

$$\vec{\nabla} = \underline{\underline{\hat{r}\partial_r}} + \hat{\theta}\frac{1}{r}\partial_\theta + \hat{\phi}\frac{1}{r\sin\theta}\partial_\phi$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

Same answer!

Practice with the gradient: V to E

- Compute the E field:

$$V(\vec{r}) = A\,x + B\,y + C\,z$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r}$$

Gradient (in different coordinates)

- Cartesian:

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

- Spherical:

$$\vec{\nabla} \equiv \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

- Cylindrical:

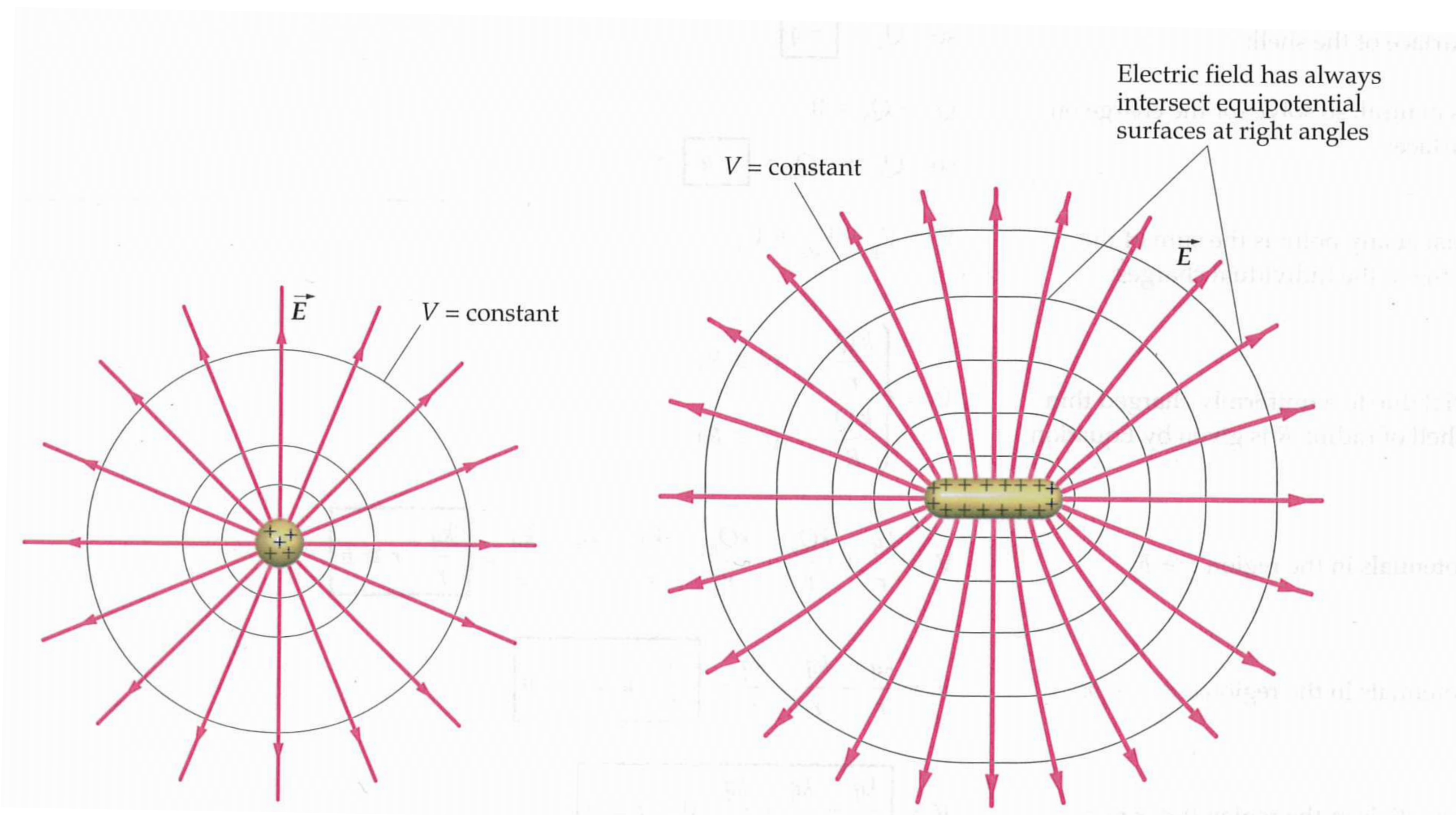
$$\vec{\nabla} \equiv \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$$

Question from FlipIt Physics:

Is there a connection between **field lines** and **electric potential (energy)**?

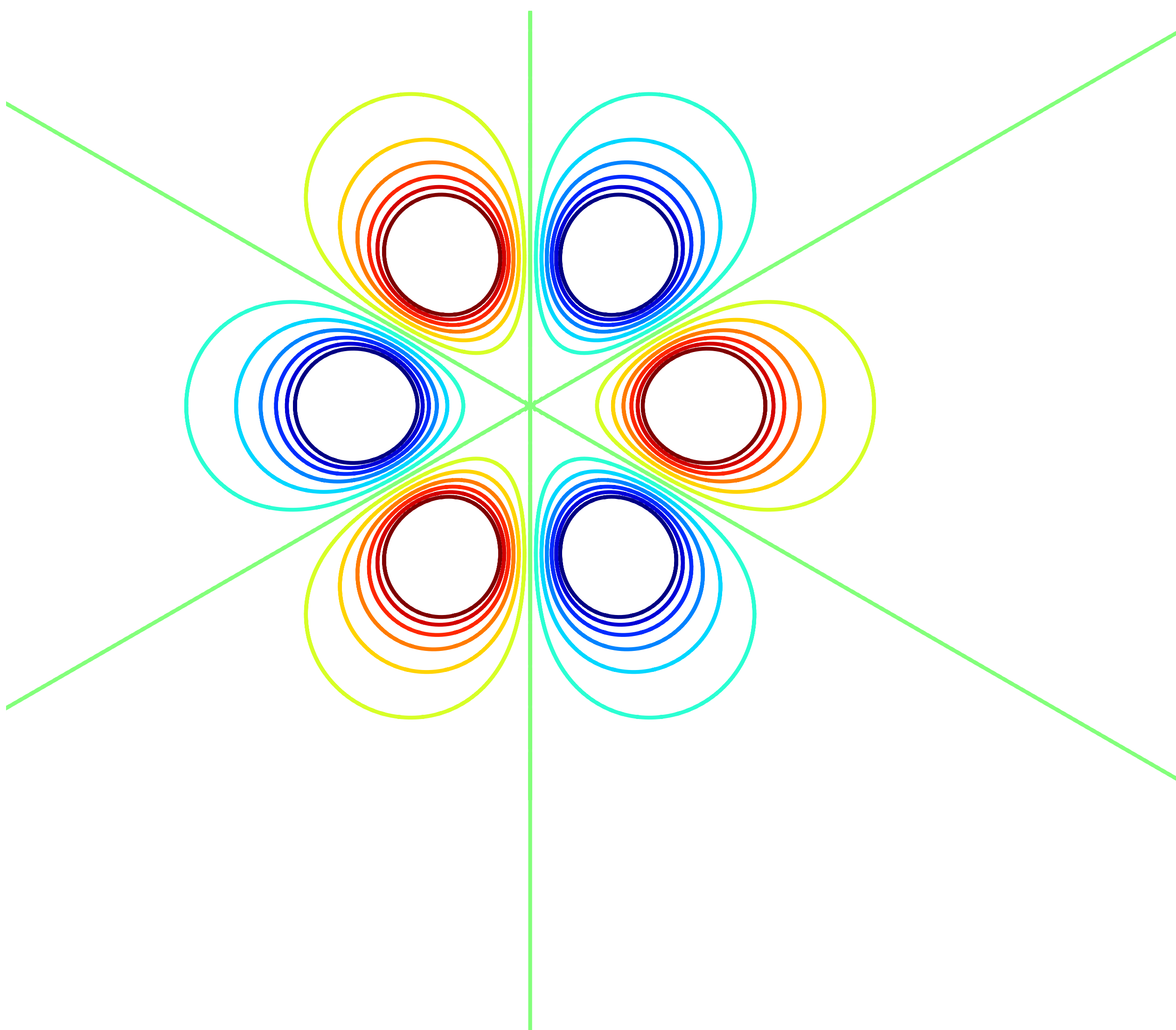
Yes… let's talk about that now!

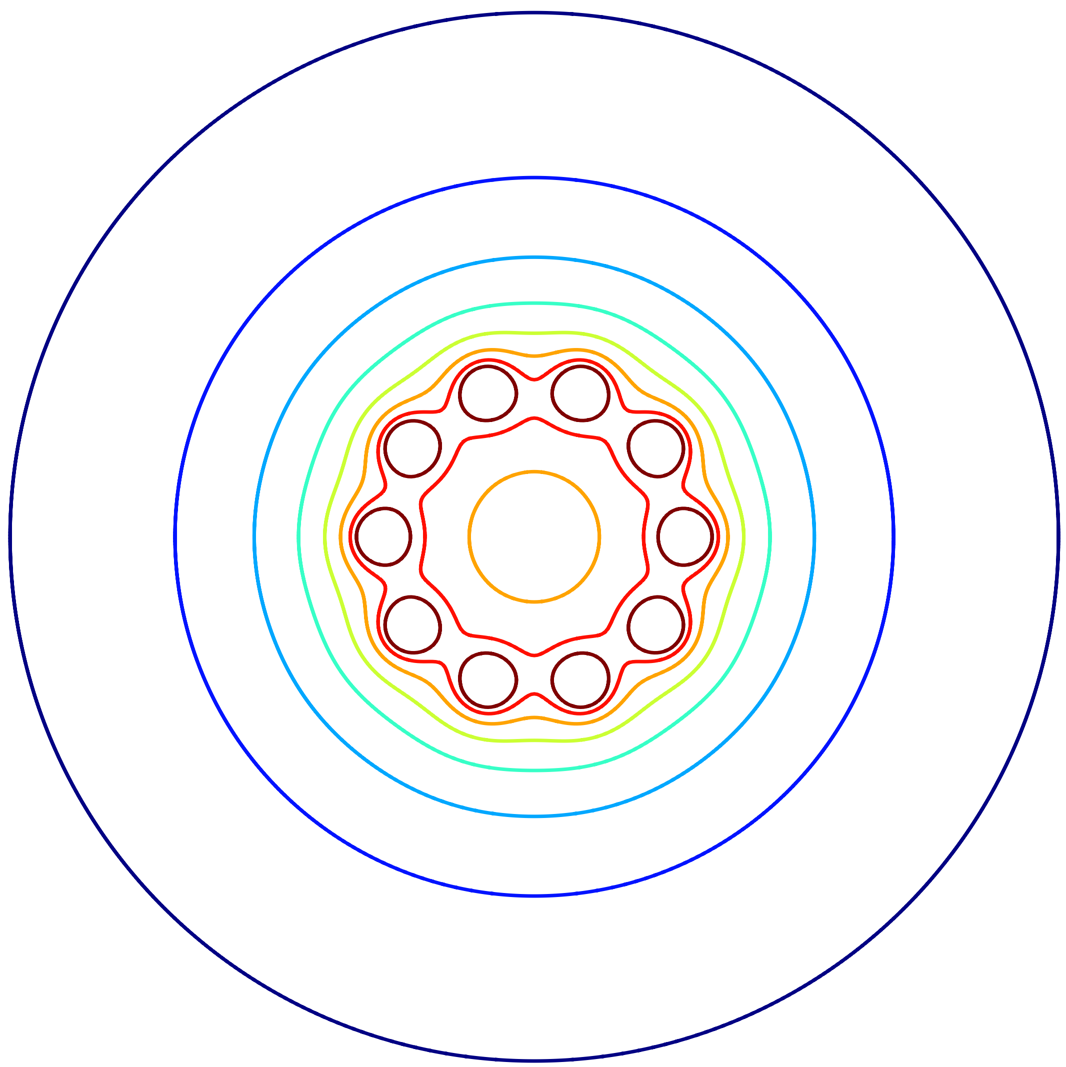
Visualizing the potential: **Equipotential Surfaces**



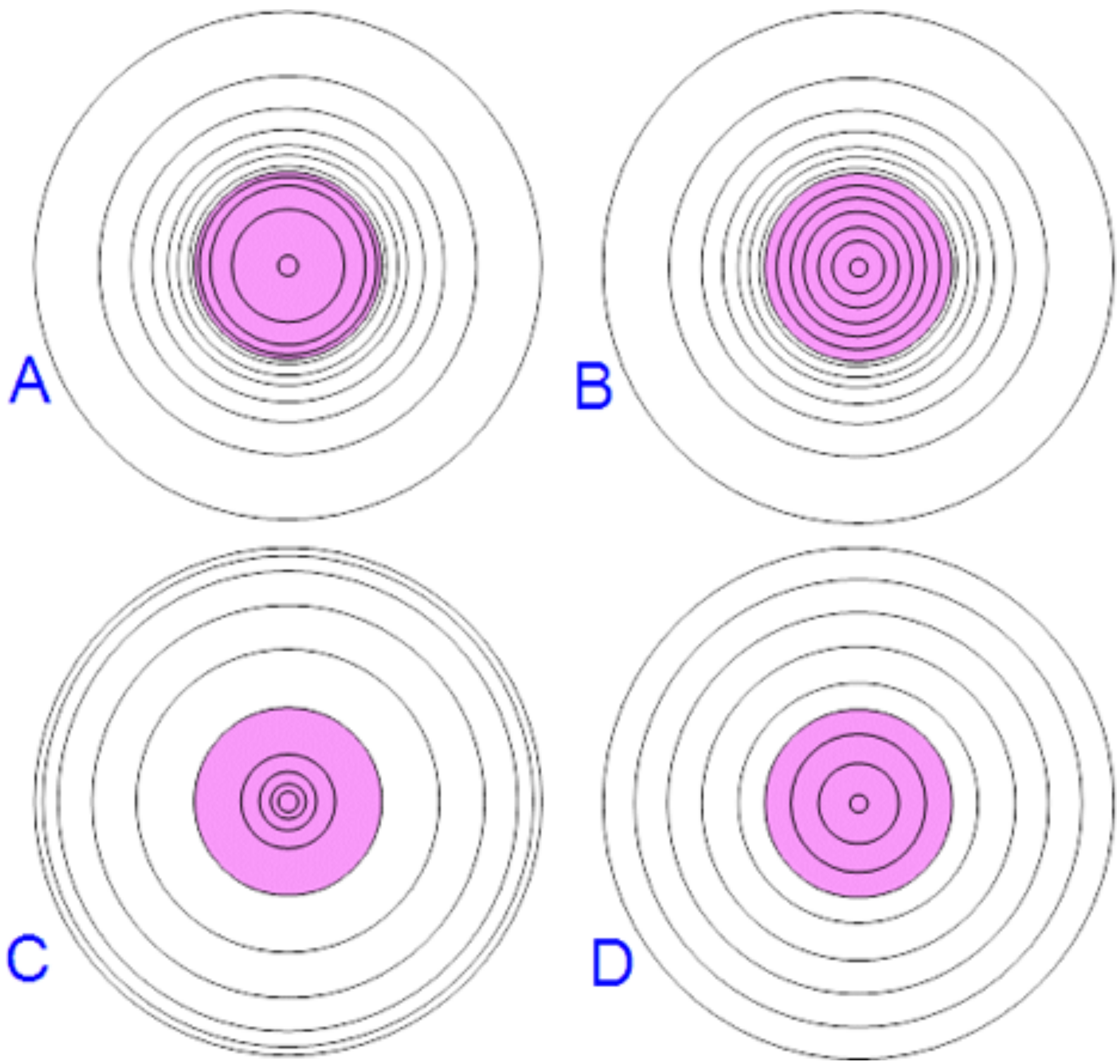
- Normal to E field/lines
- Spacing $\propto$ 1/E

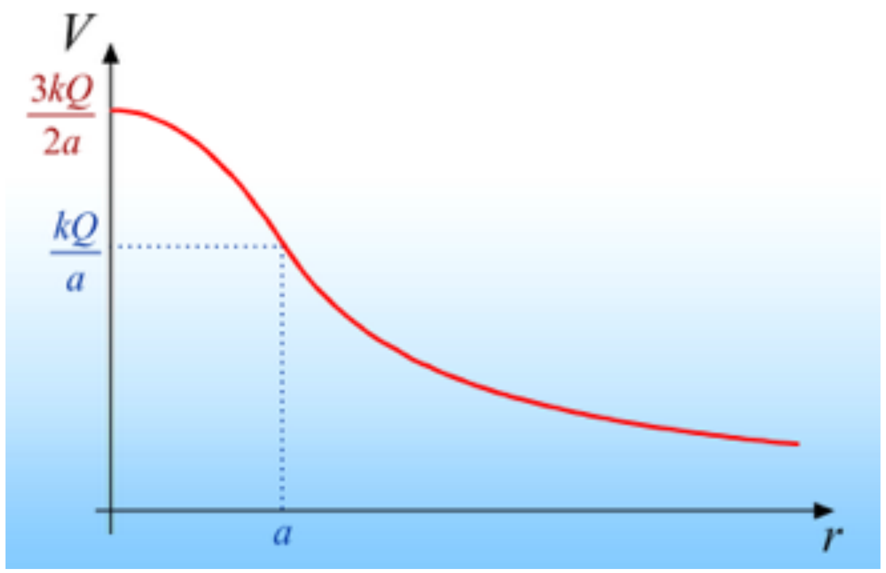
Examples of Equipotential Surfaces: What is the charge distribution?

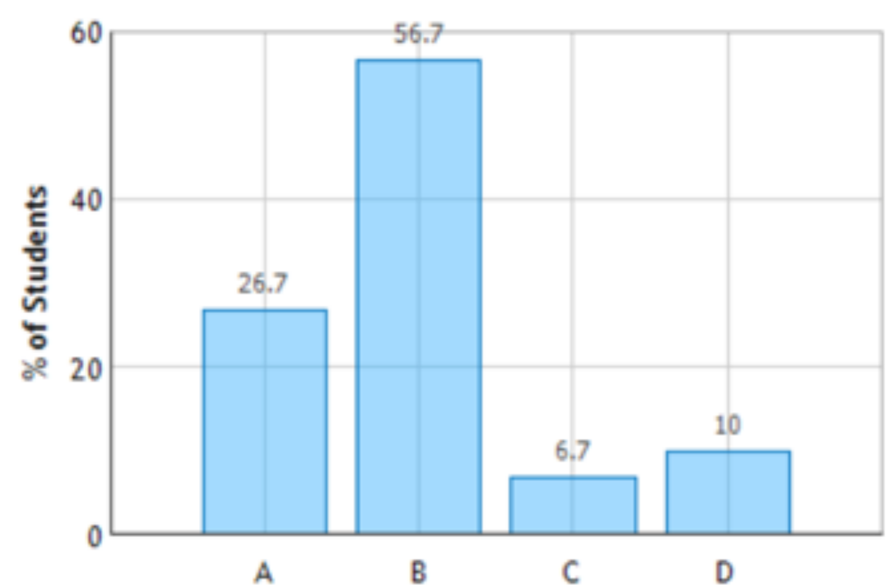




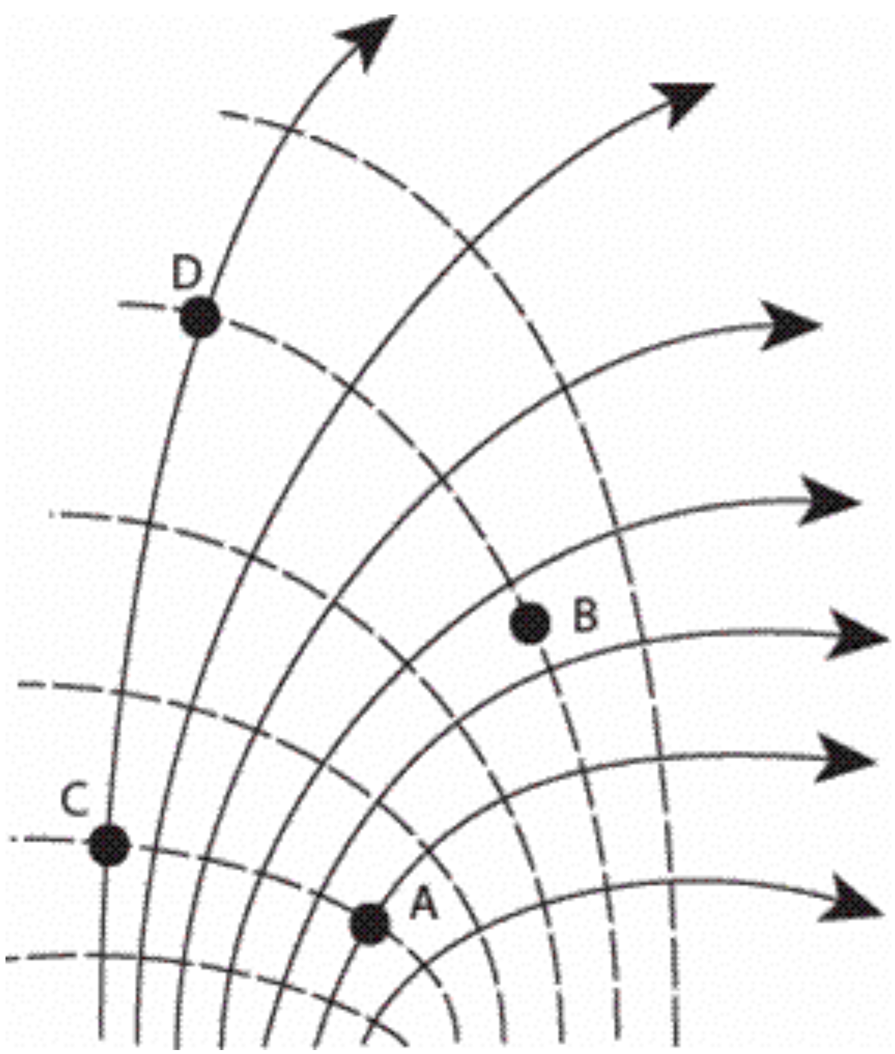
Example from the pre-lecture…



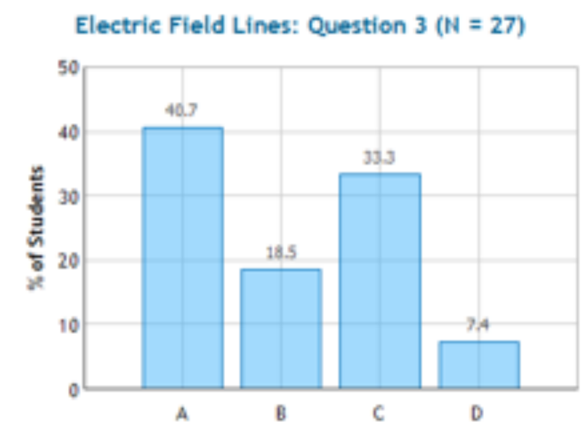




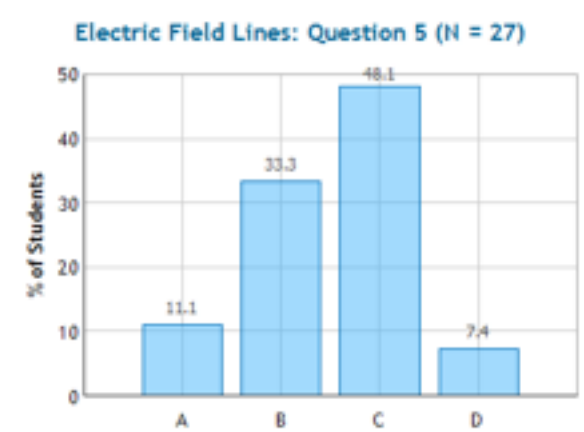
Questions from the checkpoint…



3) Compare the work done moving a negative charge from **A to B** and from **C to D.** Which one requires more work?

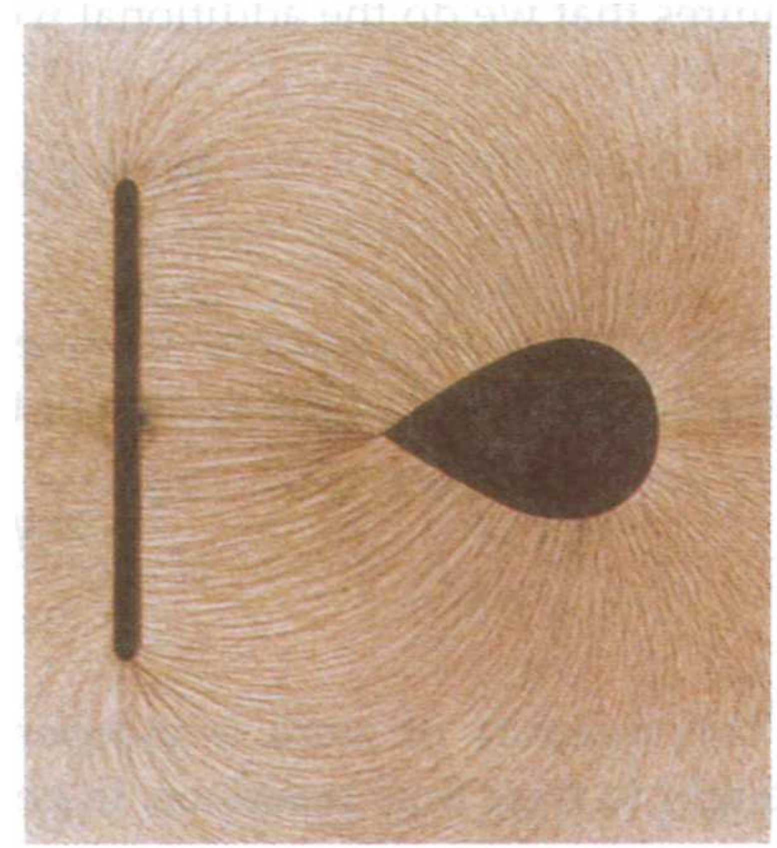


5) Compare the work done moving a negative charge from **A to B** and from **A to D.** Which one requires more work?

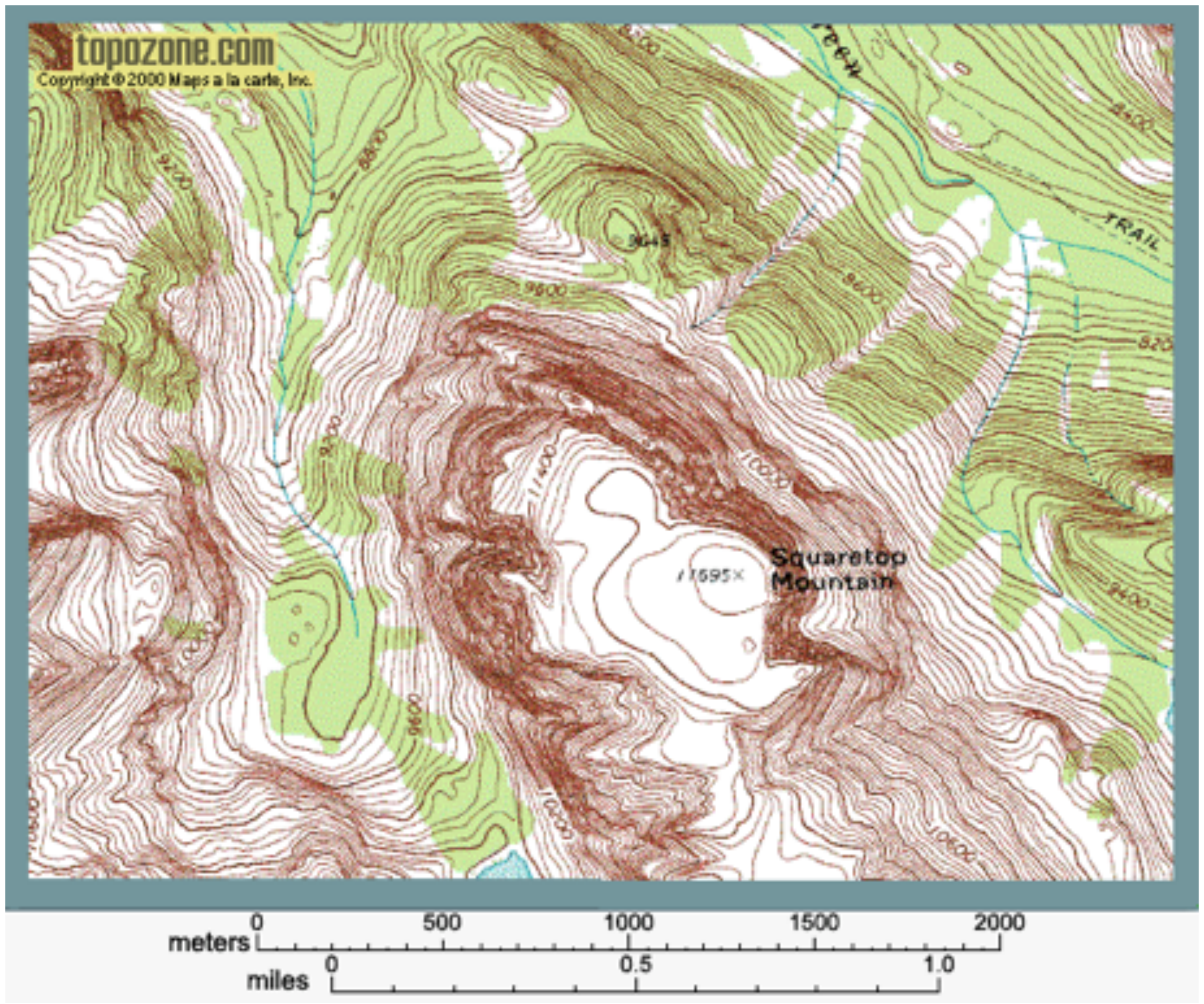


Conductors review…

- Charges free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface



Understanding potential



- Only **relative** values of potential matter
- Changes (gradients) in potential generate force

Potential is always relative to some reference (often ∞)

… whereas E field can be defined locally.