# Electric Potential

Lecture 8.

# **Electric Potential**: Factor out the q!

- Analogy: F:E :: U:V

$$F_{\mathrm{Net},0} = q_0 \underbrace{\sum_{i\neq 0} \frac{q_i\,\vec{r}_{i,0}}{4\pi\epsilon_0\, r_{i,0}^3}}_{\vec{E}} = q_0\vec{E}$$

$$U_{\mathrm{Net},0} = q_0 \underbrace{\sum_{i\neq 0} \frac{q_i}{4\pi\epsilon_0\, r_{0,i}}}_{V} = q_0 V$$

- V is the **Electric Potential** (V = Volt = J/C)
- U is the **Electric Potential Energy** (J)

# What is **electric potential (V)**?

Energy in batteries:

$$dU = V\,dQ$$

Why is V so important?

- Voltage supplies
- Scalar
- …

- Common usage:
  What is the voltage (electric potential)?

# Relation between E and V

- The E field is the negative **gradient** of the potential:

$$\vec{E} = -\vec{\nabla}V = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V$$

- Geometric meaning: **Directional derivative**
  Given a **vector** displacement, the **Gradient** tells you the derivative in that direction.

- Potential is the negative **line integral** of the E field:

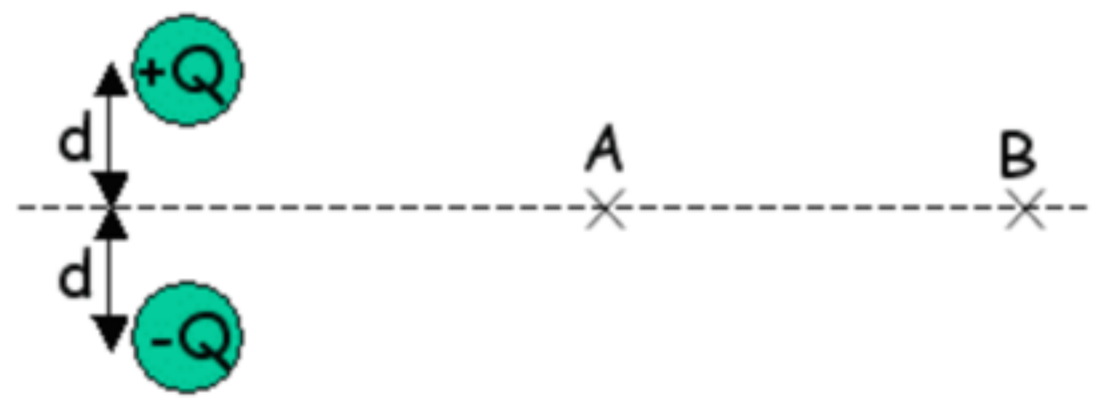
$$\Delta V = -\int_{\Gamma} d\vec{\ell} \cdot \vec{E}$$

# Calculating V:

- Use superposition!
- V for a single point charge: $V(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}$
- … charge isn't at origin: $V(\vec{r}) = \frac{q_{\vec{r}\,'}}{4\pi\epsilon_0|\vec{r} - \vec{r}\,'|}$
- Multiple charges: $V(\vec{r}) = \sum_i \frac{q_i}{4\pi\epsilon_0|\vec{r} - \vec{r}_i|}$
- Continuous distribution of charge: $V(\vec{r}) = \int \frac{dq_{\vec{r}\,'}}{4\pi\epsilon_0|\vec{r} - \vec{r}\,'|}$

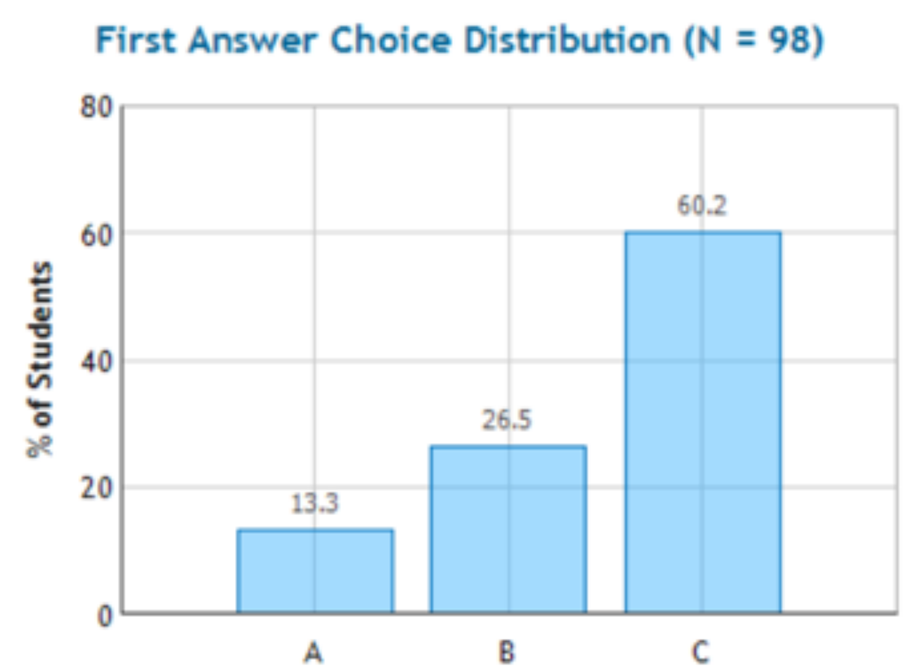
# Calculating V:

An electric dipole with charge magnitude $Q$ and separation $2d$ is oriented as shown below. Compare $V_A$, the electric potential at point A, with $V_B$, the electric potential at point B.



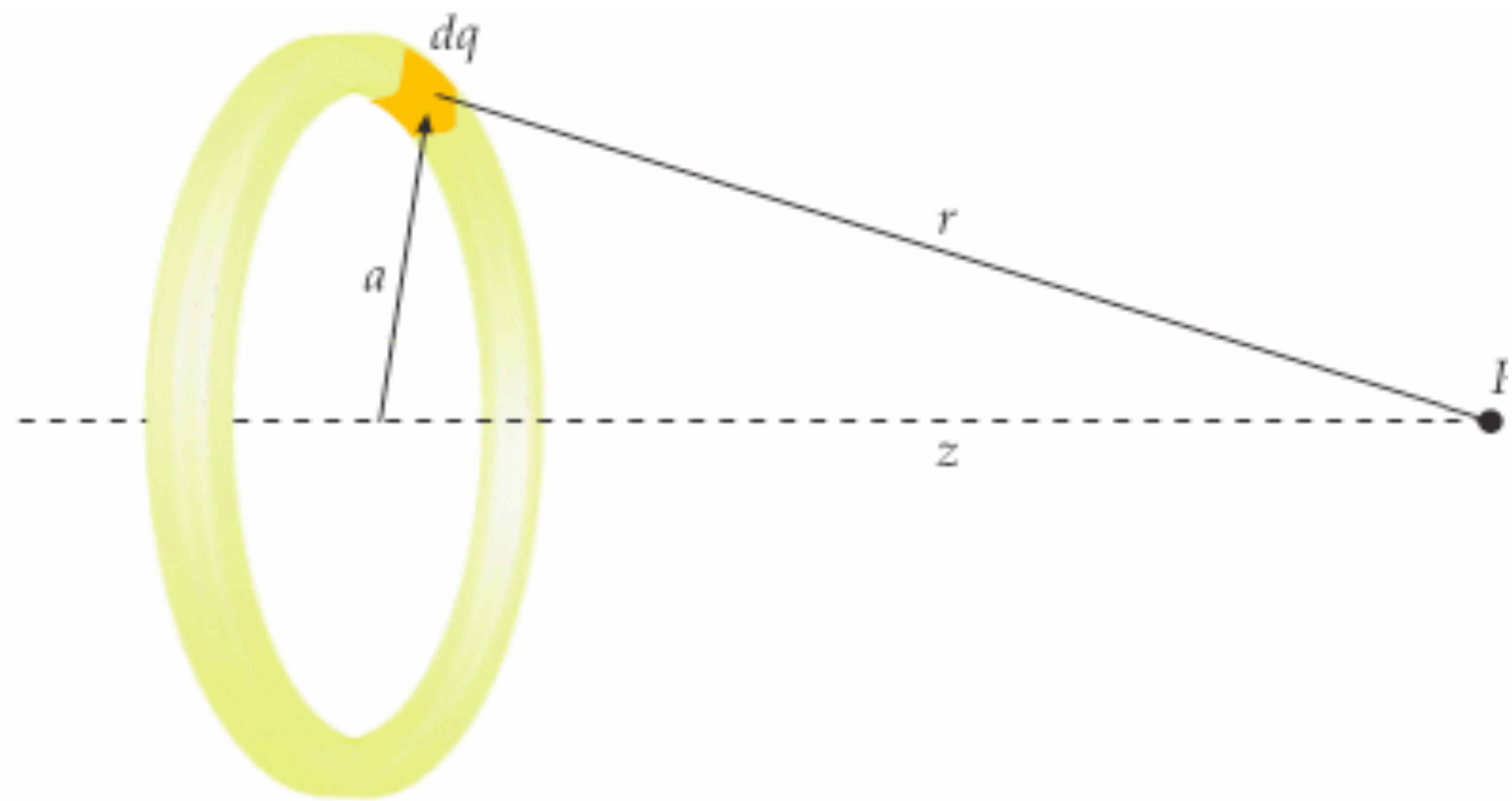

- $V_A < V_B$
- $V_A = V_B$
- $V_A > V_B$

Submit

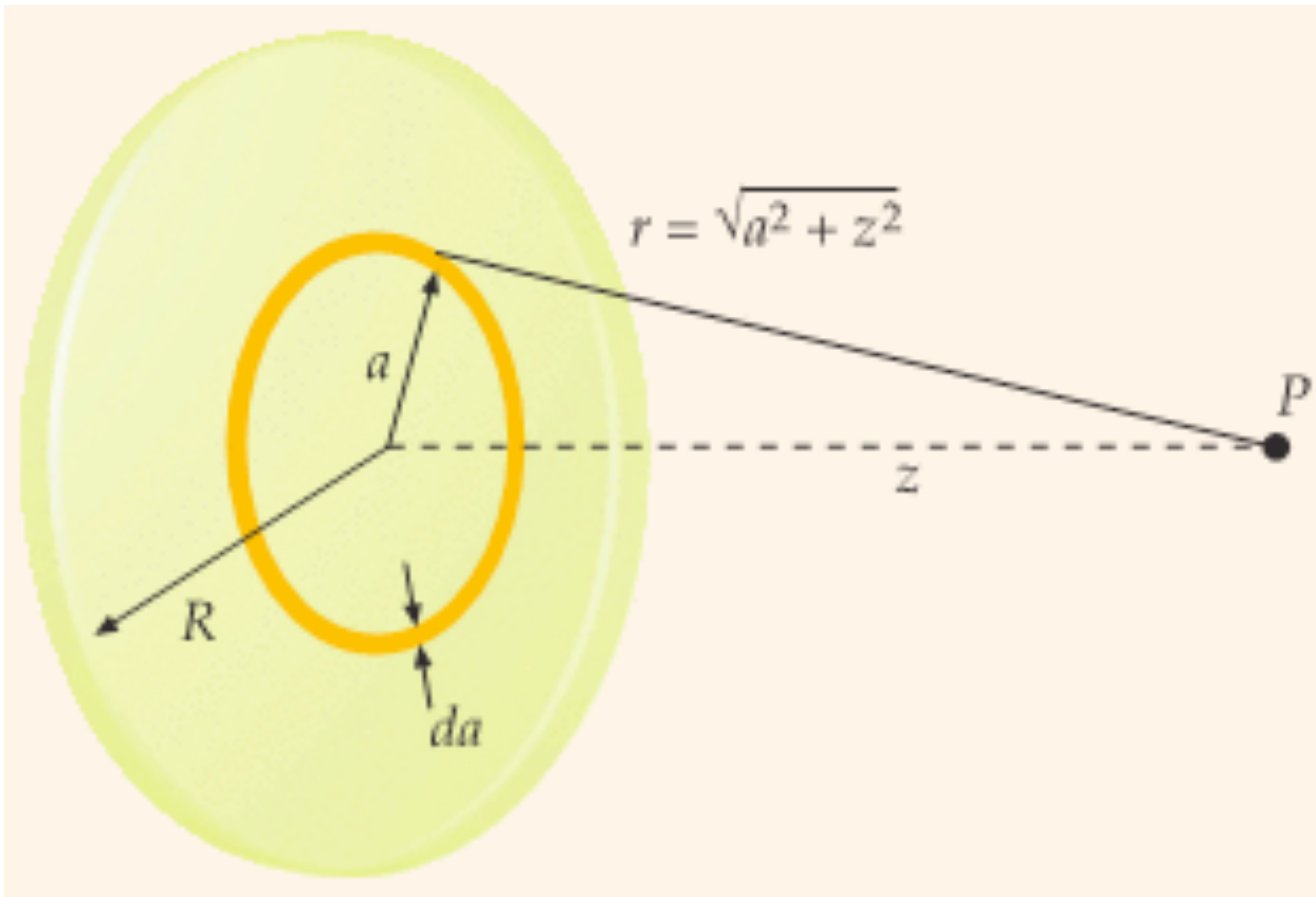



$$V(\vec{r}) = \sum_i \frac{q_i}{4\pi\epsilon_0|\vec{r} - \vec{r}_i|}$$

# Compute the potential from a ring of charge

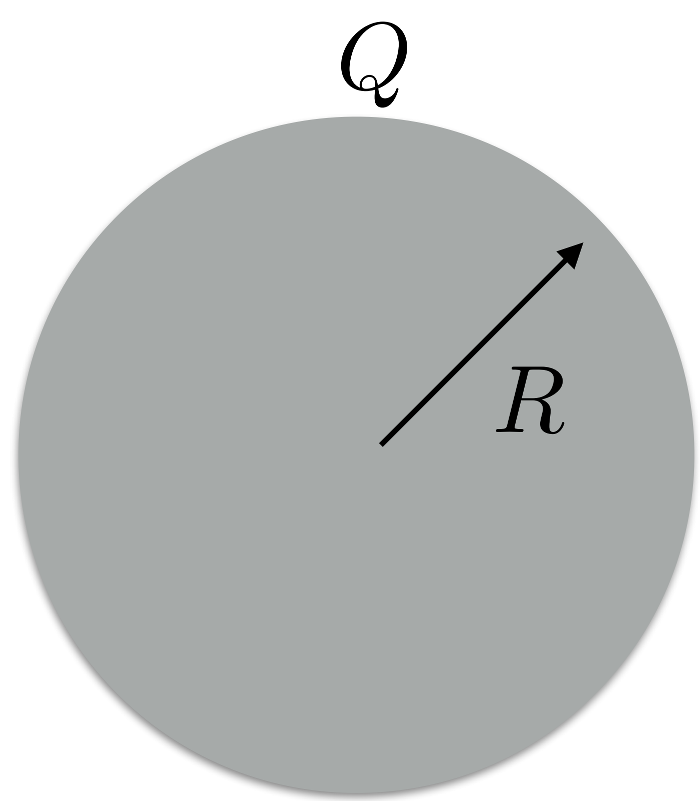


# Compute the potential from a charged disk



# Calculate V from E:

- Calculate the potential from a charged conducting sphere

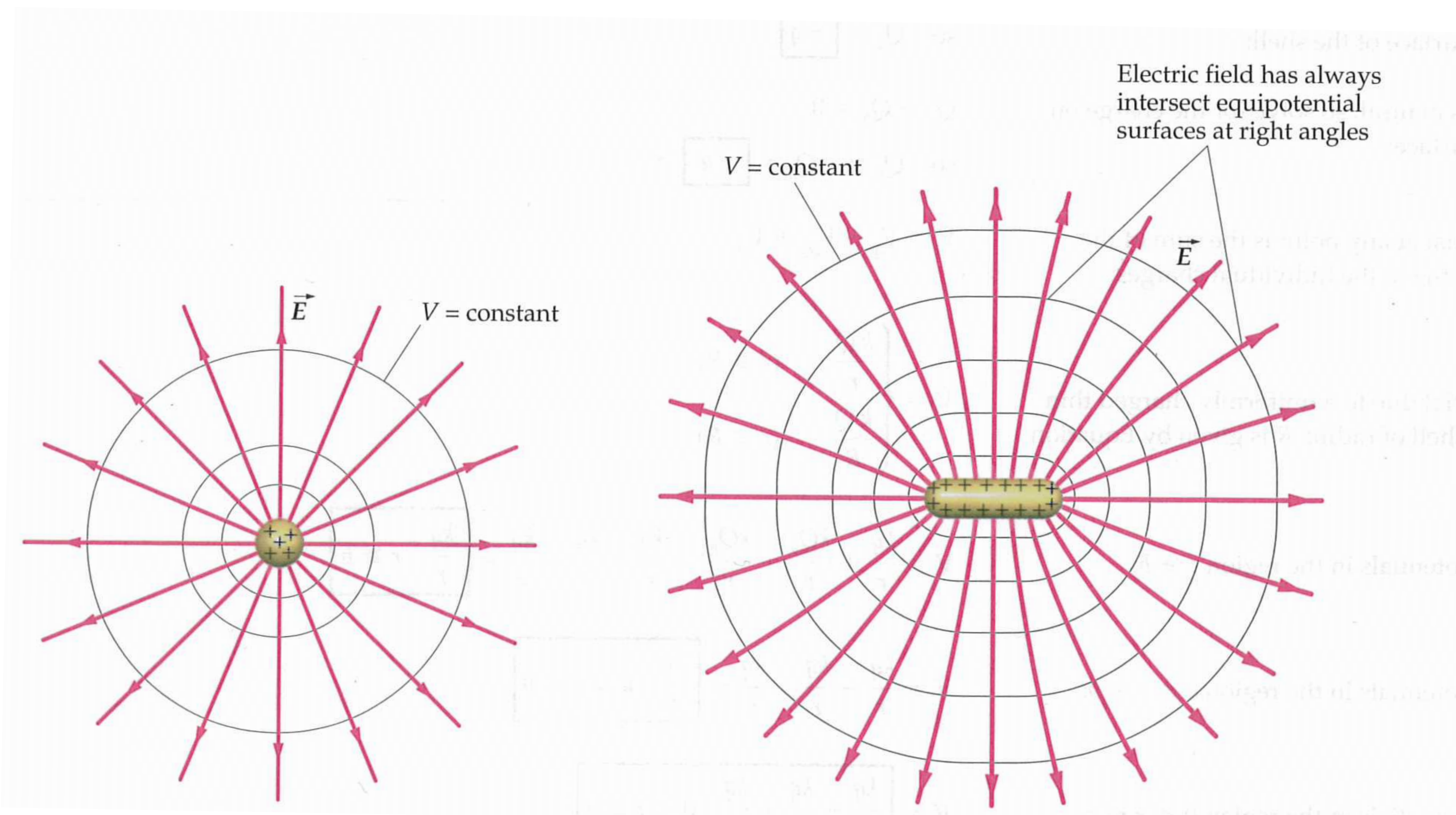


Question from FlipIt Physics:

Is there a connection between **field lines** and **electric potential (energy)**?

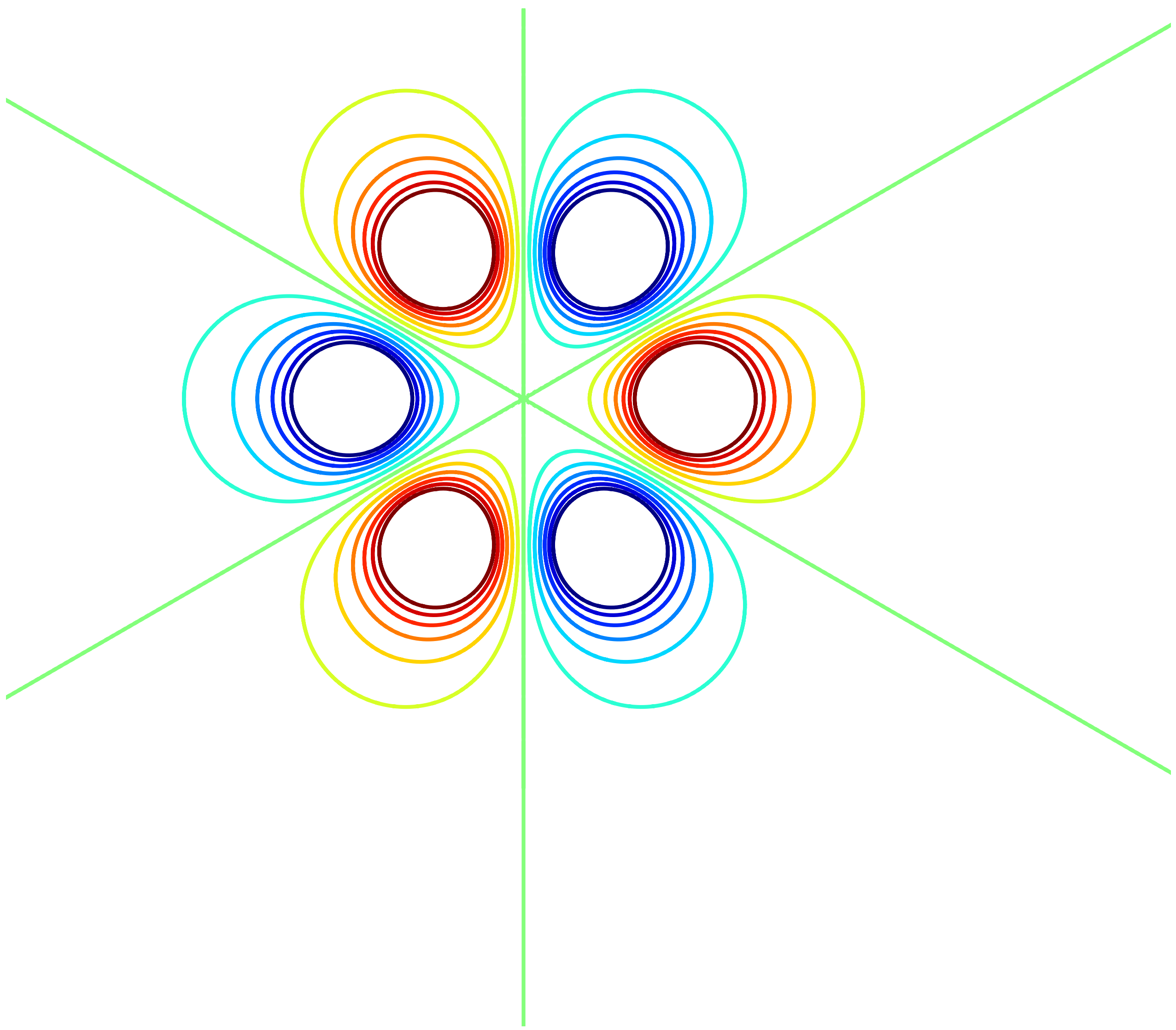
Yes… let's talk about that now!

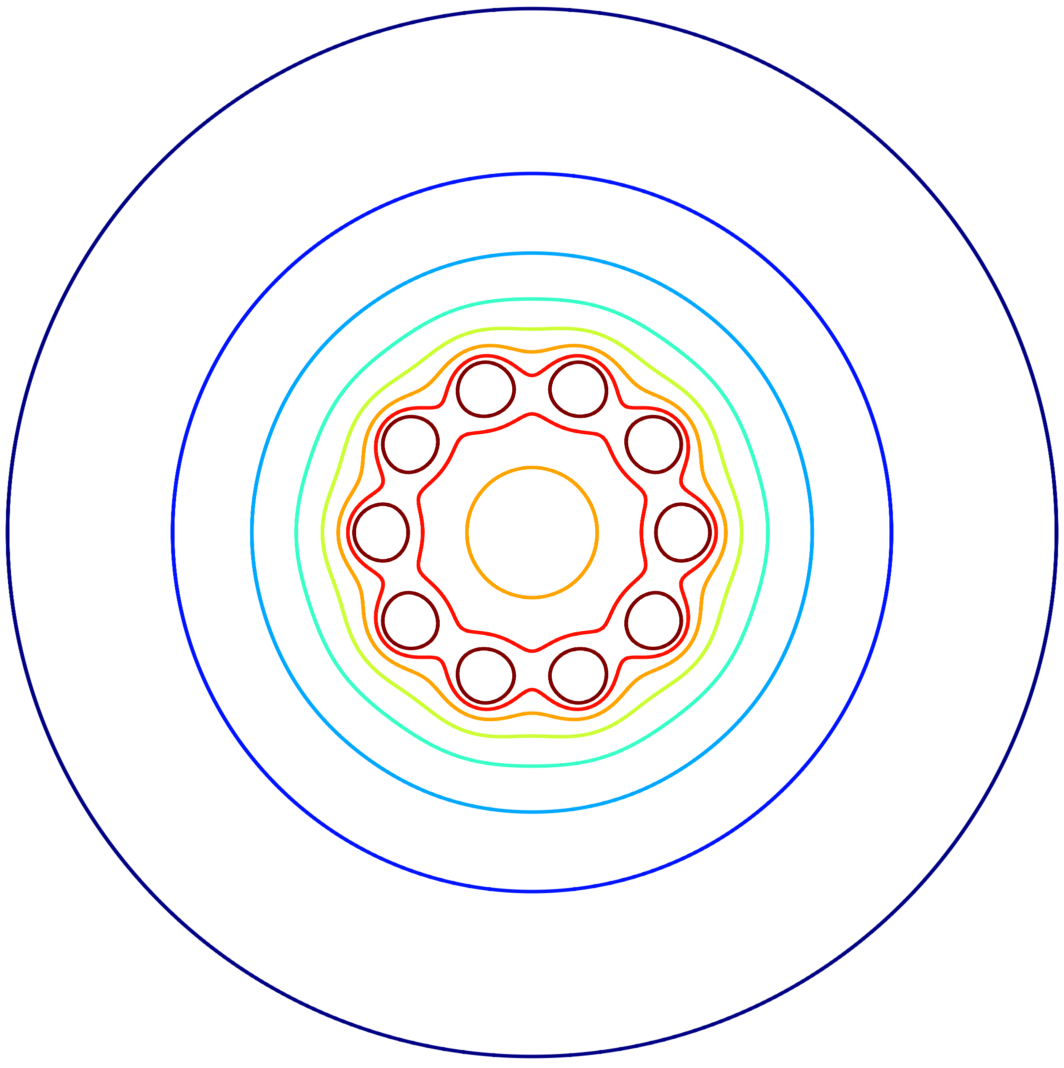
# Visualizing the potential: **Equipotential Surfaces**



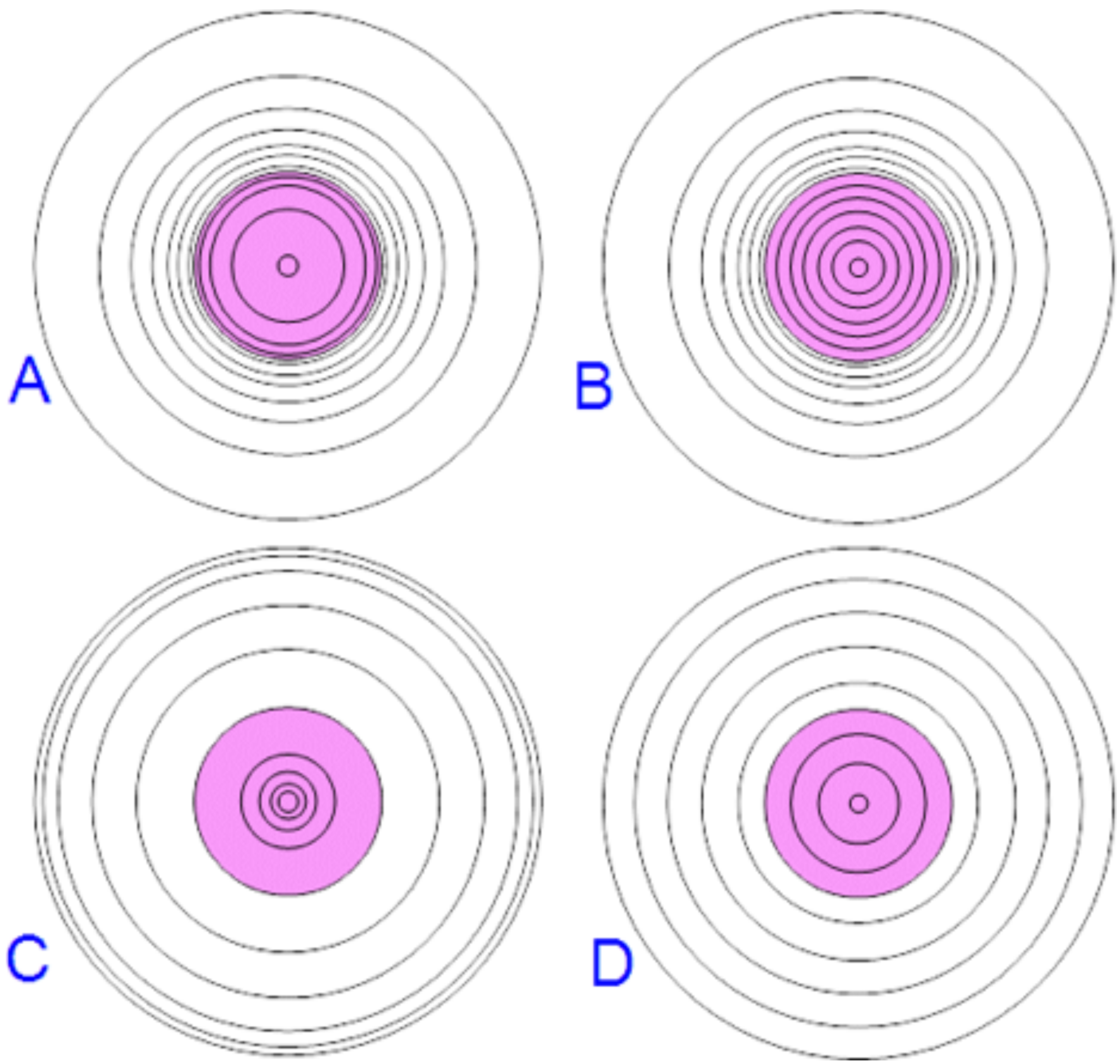

- Normal to E field/lines
- Spacing ∝ 1/E

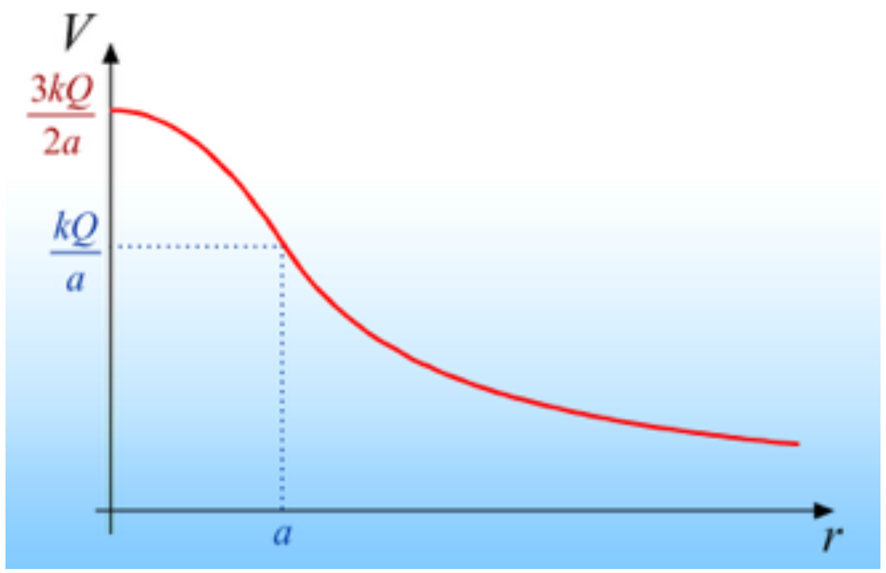
# Examples of Equipotential Surfaces: What is the charge distribution?

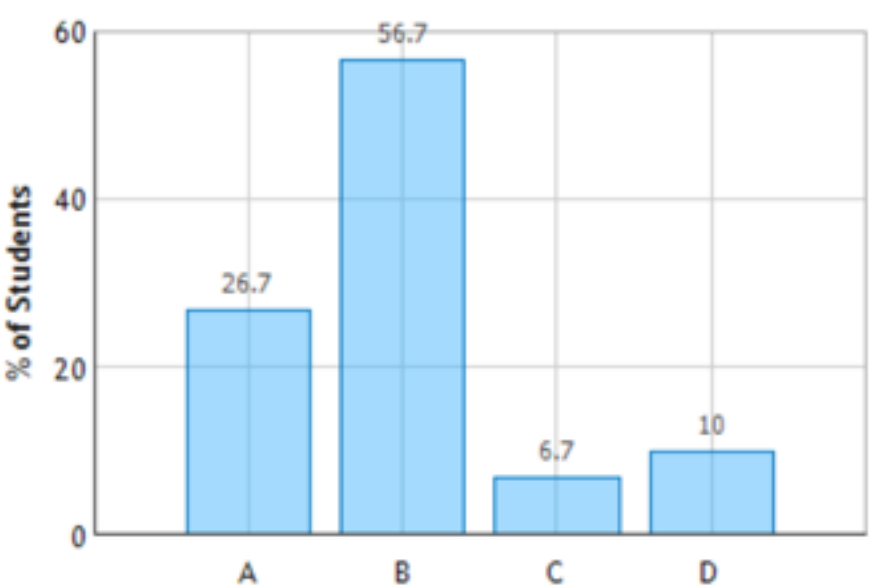




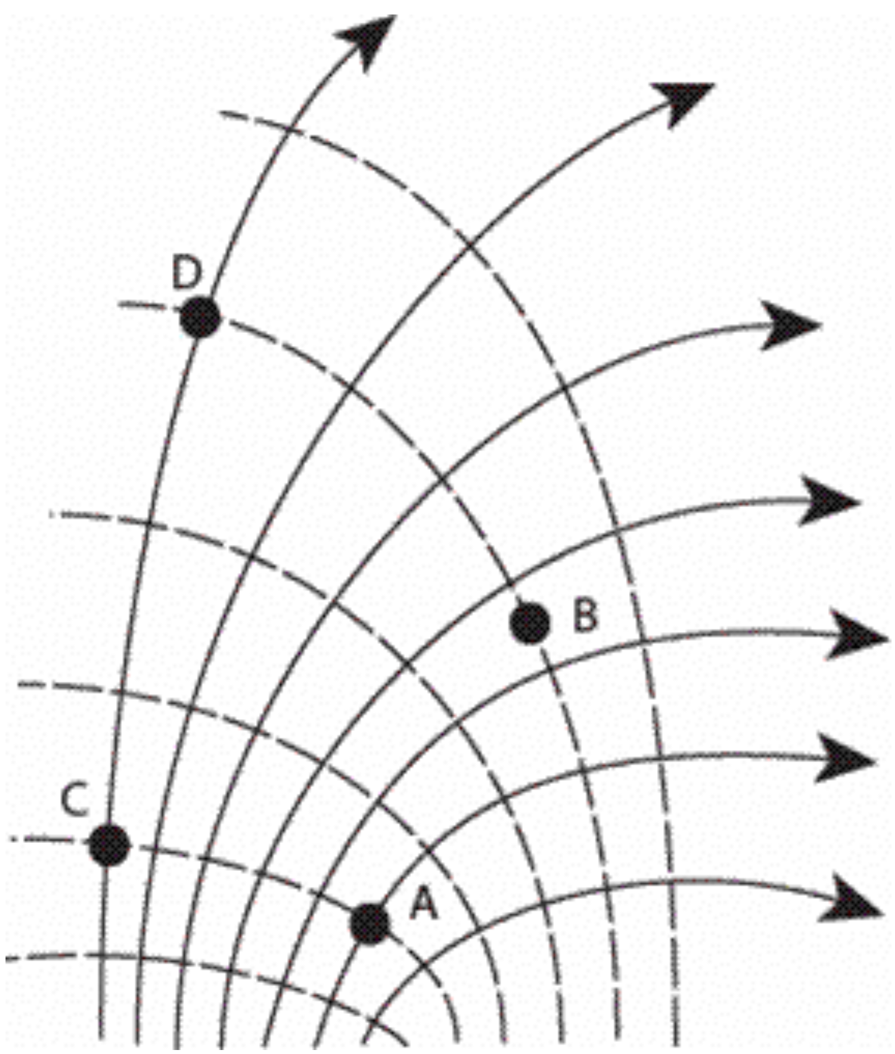
# Example from the pre-lecture…



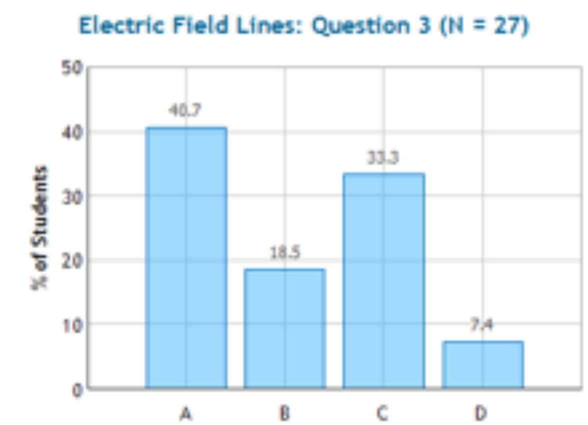



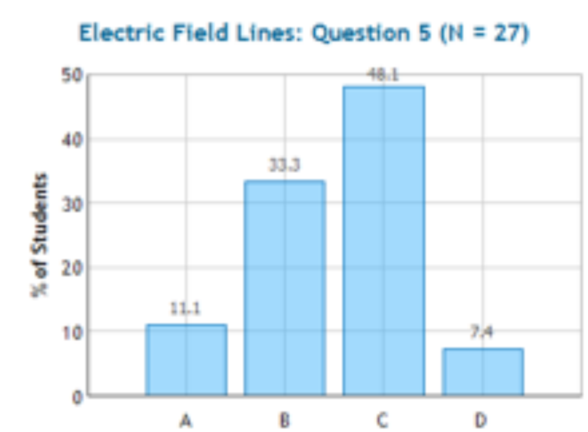
# Questions from the checkpoint…



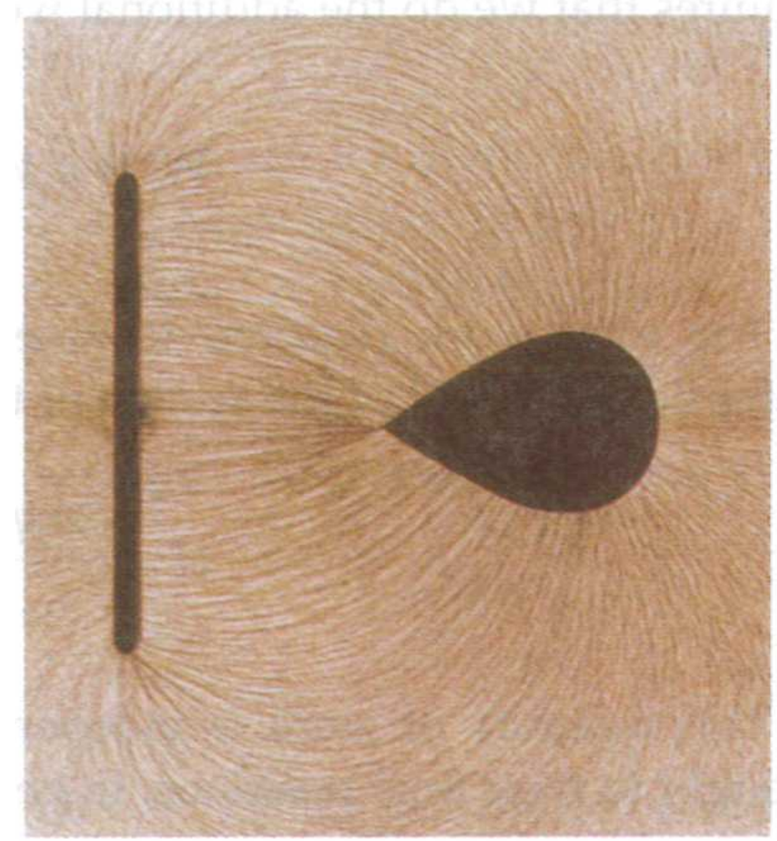

3) Compare the work done moving a negative charge from **A to B** and from **C to D.** Which one requires more work?




5) Compare the work done moving a negative charge from **A to B** and from **A to D.** Which one requires more work?

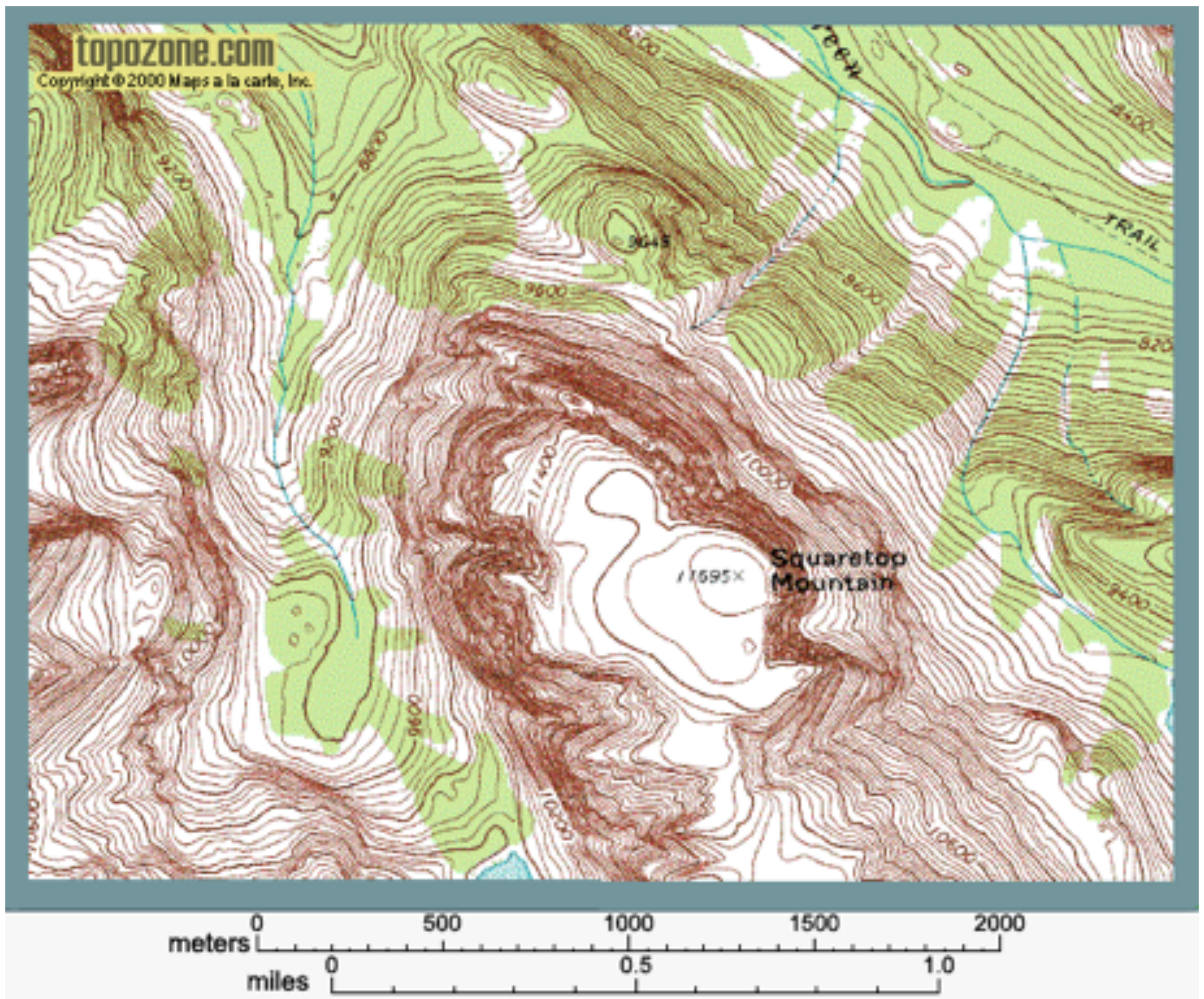


# Conductors review…



- Charges free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

# Understanding potential




- Only **relative** values of potential matter
- Changes (gradients) in potential generate force

# Potential is always relative to some reference (often ∞)


**… whereas E field can be defined locally.**