Electric Potential Energy

Lecture 8.

Demo…

- Dielectric break down.

Electric Potential Energy

- Electro**static** forces is **conservative**:
- The **work** performed by the electric field on a charge is **path independent.**
- Proof:
    - Show for a point charge.
    - Use superposition of point charges to represent arbitrary charge distribution.

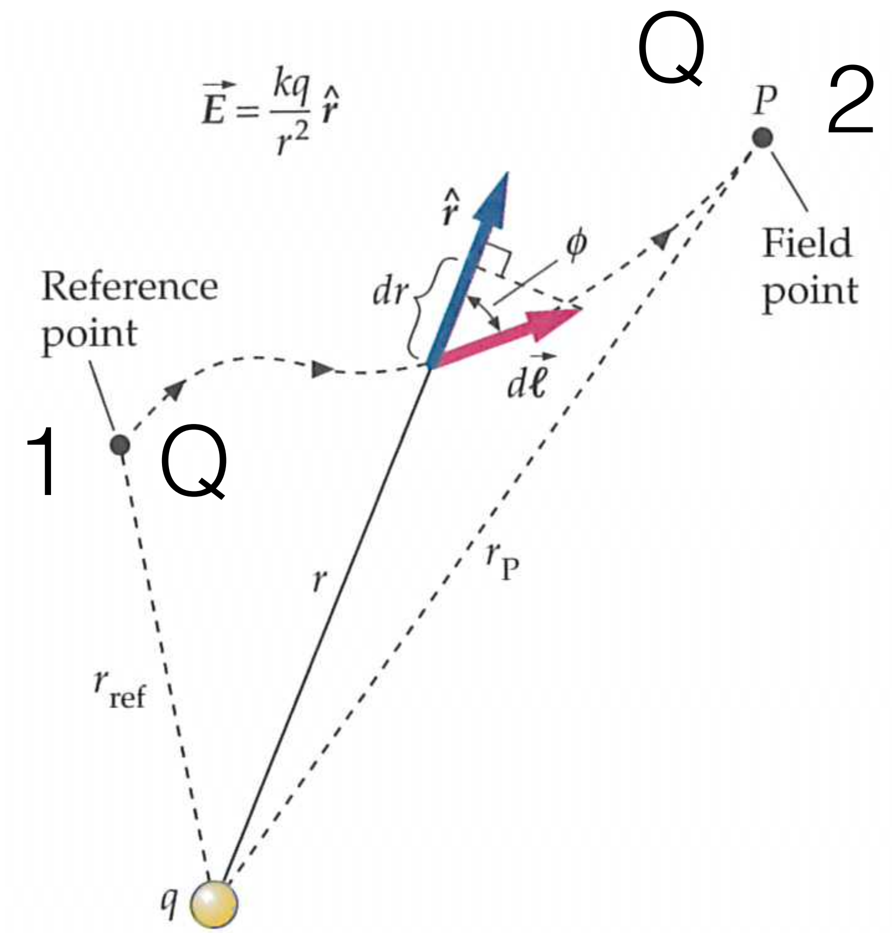
Electric Potential Energy

- Show for a point charge…
- Work done by field on charge:

$$dW = \vec{F} \cdot d\vec{\ell}$$

- (… on board …)
- Result:

$$\Delta W = W_{1\to 2} \equiv W_2 - W_1 = -\frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$



Electric Potential Energy

- Define the change is Electric Potential Energy as

$$\Delta U = U_{1\to 2} \equiv U_2 - U_1 = -W_{1\to 2}$$

- Electric Potential Energy difference:

$$\Delta U = U_{1\to 2} \equiv U_2 - U_1 = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Electric Potential Energy

- Define Electric Potential Energy as follows:

$$U_1 \equiv U_1 - U_\infty = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{\infty}\right] = \frac{qQ}{4\pi\epsilon_0 r_1}$$

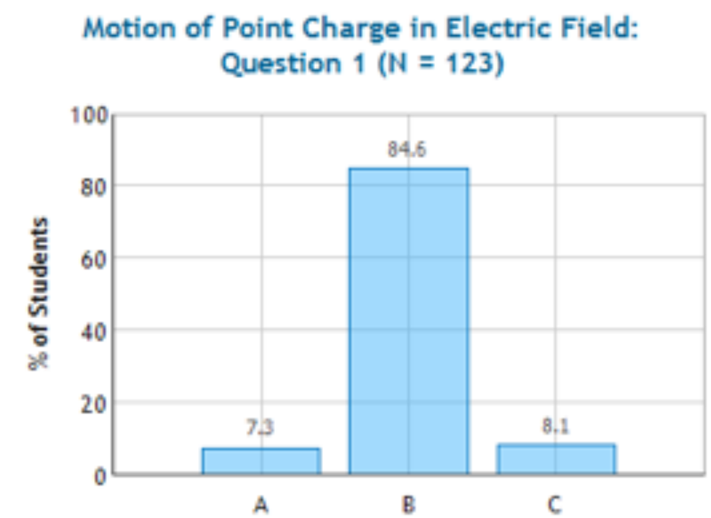
relative to a point at ∞. (Finite charge dist.)

- Superposition applies: $U_{\mathrm{Net},0} = q_0 \sum_{i \neq 0} \frac{q_i}{4\pi\epsilon_0\, r_{0,i}}$

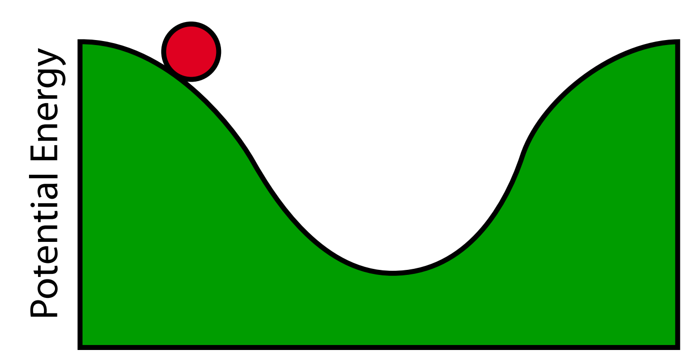
- Potential energy of a charge distribution = energy required to assemble from ∞.

Electric Potential Energy

- Plot potential $U(\vec{r}) = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$
- (… on board …)



- Heuristic for remembering signs:
  "Physics is always going down hill."



Electric Potential: Factor out the q!

- Analogy: F:E :: U:V

$$F_{\mathrm{Net},0} = q_0 \underbrace{\sum_{i\neq 0} \frac{q_i\, \vec{r}_{i,0}}{4\pi\epsilon_0\, r_{i,0}^3}}_{\vec{E}} = q_0 \vec{E}$$

$$U_{\mathrm{Net},0} = q_0 \underbrace{\sum_{i\neq 0} \frac{q_i}{4\pi\epsilon_0\, r_{0,i}}}_{V} = q_0 V$$

- V is the **Electric Potential** (V = Volt = J/C)

- U is the **Electric Potential Energy** (J)

What is **electric potential (V)**?

Energy in batteries:

$$dU = V\,dQ$$

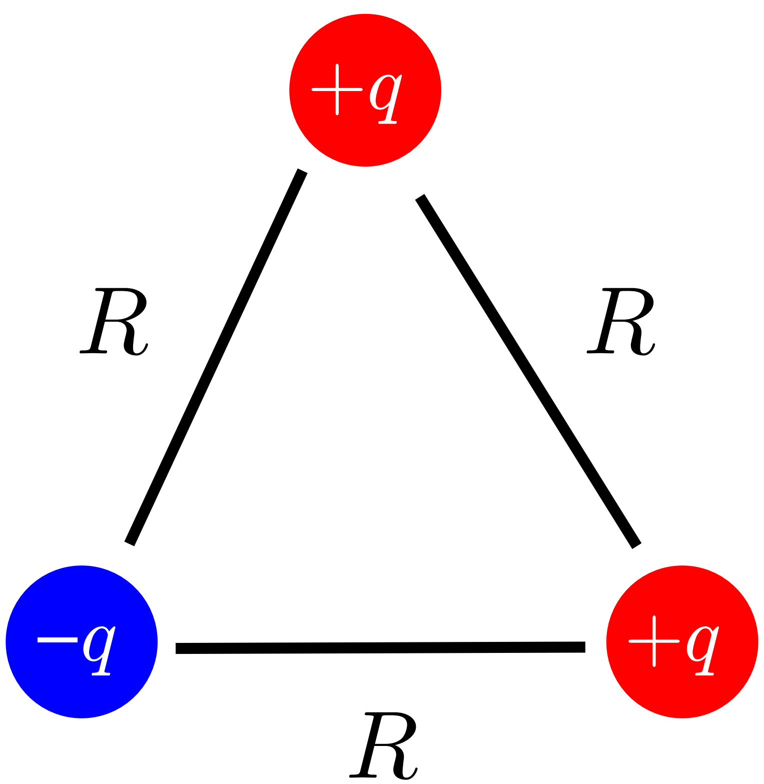
Why is V so important?

- Voltage supplies
- Scalar
- …

- Common usage:
  What is the voltage (electric potential)?

Example: **Electric Potential Energy** of a charge distribution

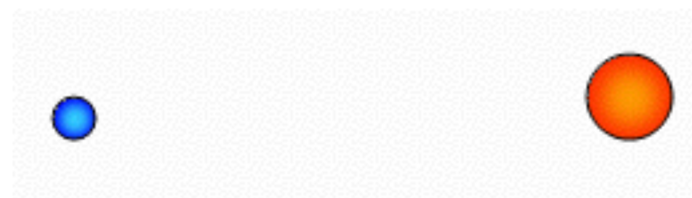
Meaning: Energy required to assemble charge configuration from infinity



$$U_{\text{Tot}} = \sum_{\text{pairs}} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{1}{2}\sum_{i\neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

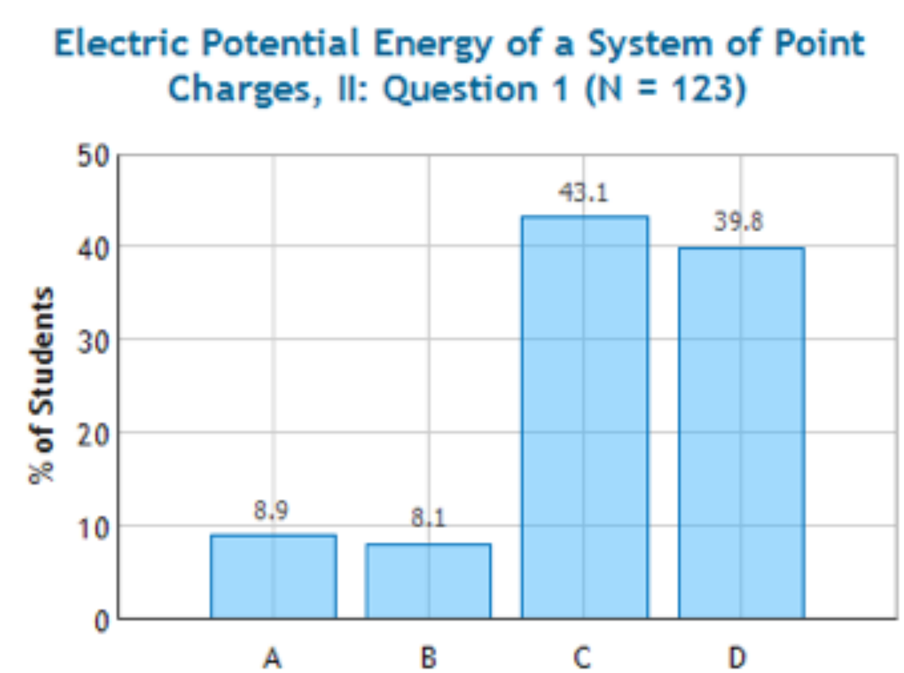
From the checkpoint…

Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first.



Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

(A) YES, as long as the third charge is positive
(B) YES, as long as the third charge is negative
(C) YES, no matter what the sign of the third charge
(D) NO



Relation between E and V

- The E field is the negative **gradient** of the potential:

$$\vec{E} = -\vec{\nabla} V = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] V$$

- Geometric meaning: **Directional derivative**
  Given a **vector** displacement, the **Gradient** tells you the derivative in that direction.

- Potential is the negative **line integral** of the E field:

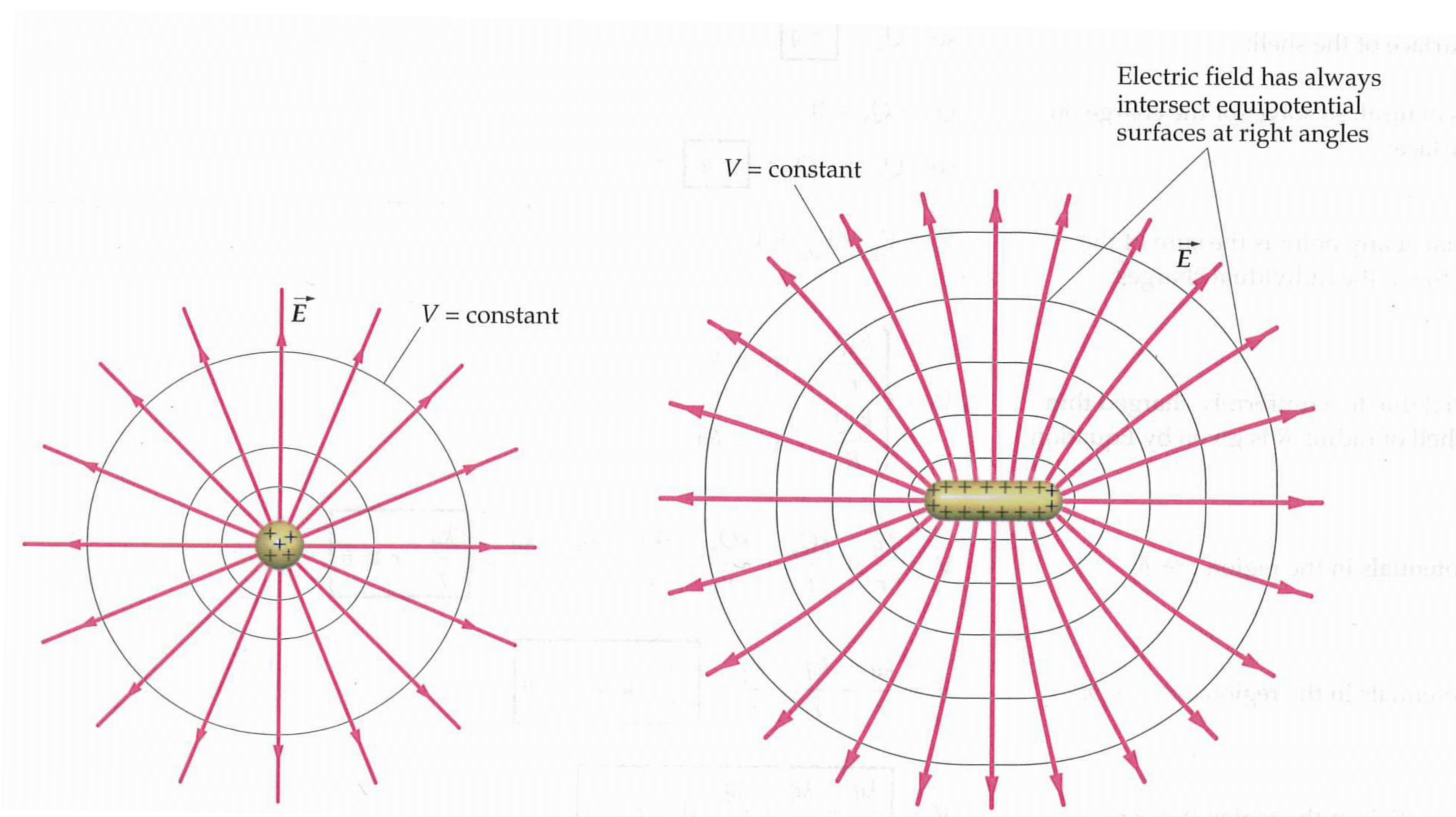
$$\Delta V = -\int_{\Gamma} d\vec{\ell} \cdot \vec{E}$$

Question from smartPhysics:

Is there a connection between **field lines** and **electric potential (energy)**?

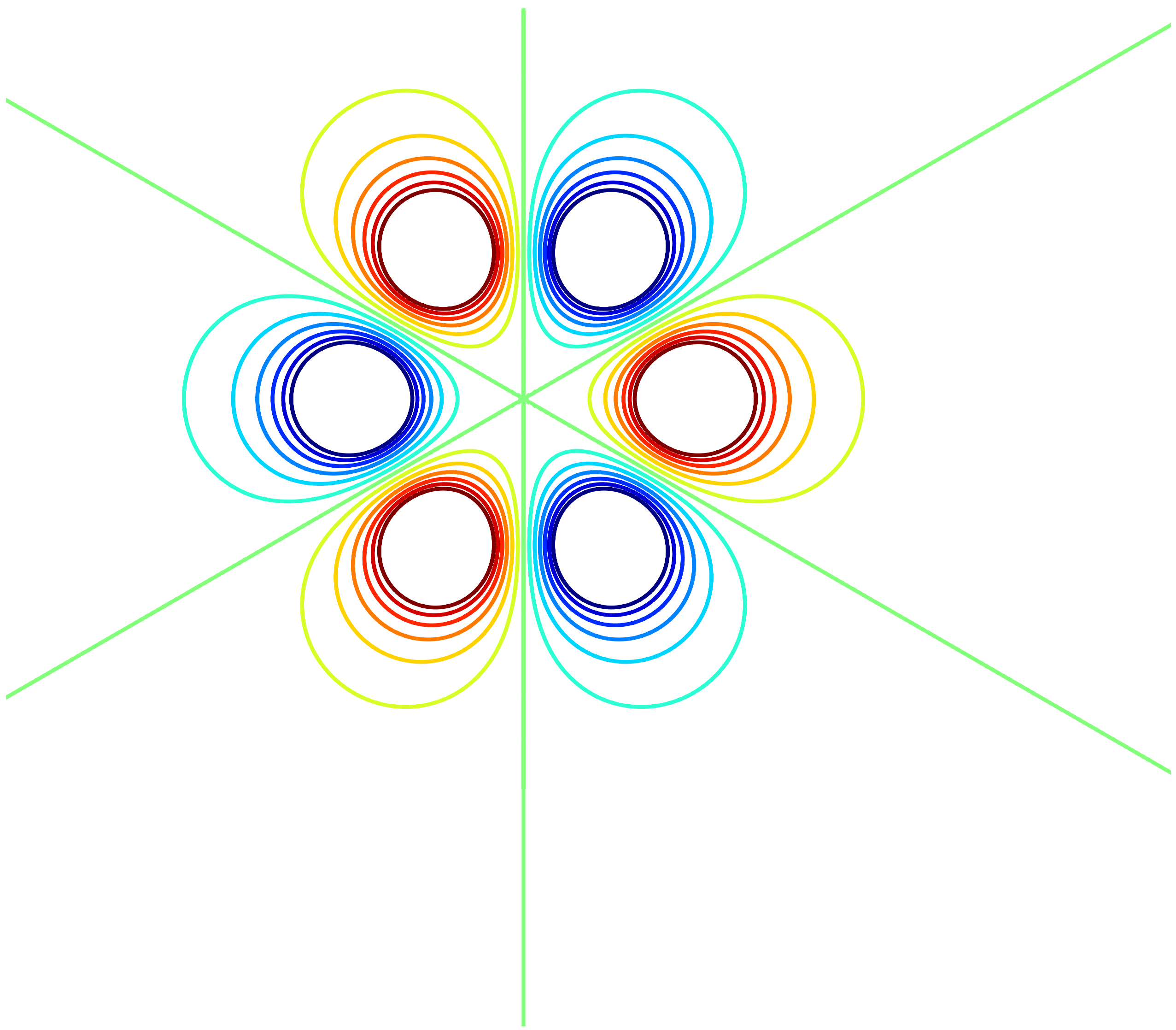
Yes… let's talk about that now!

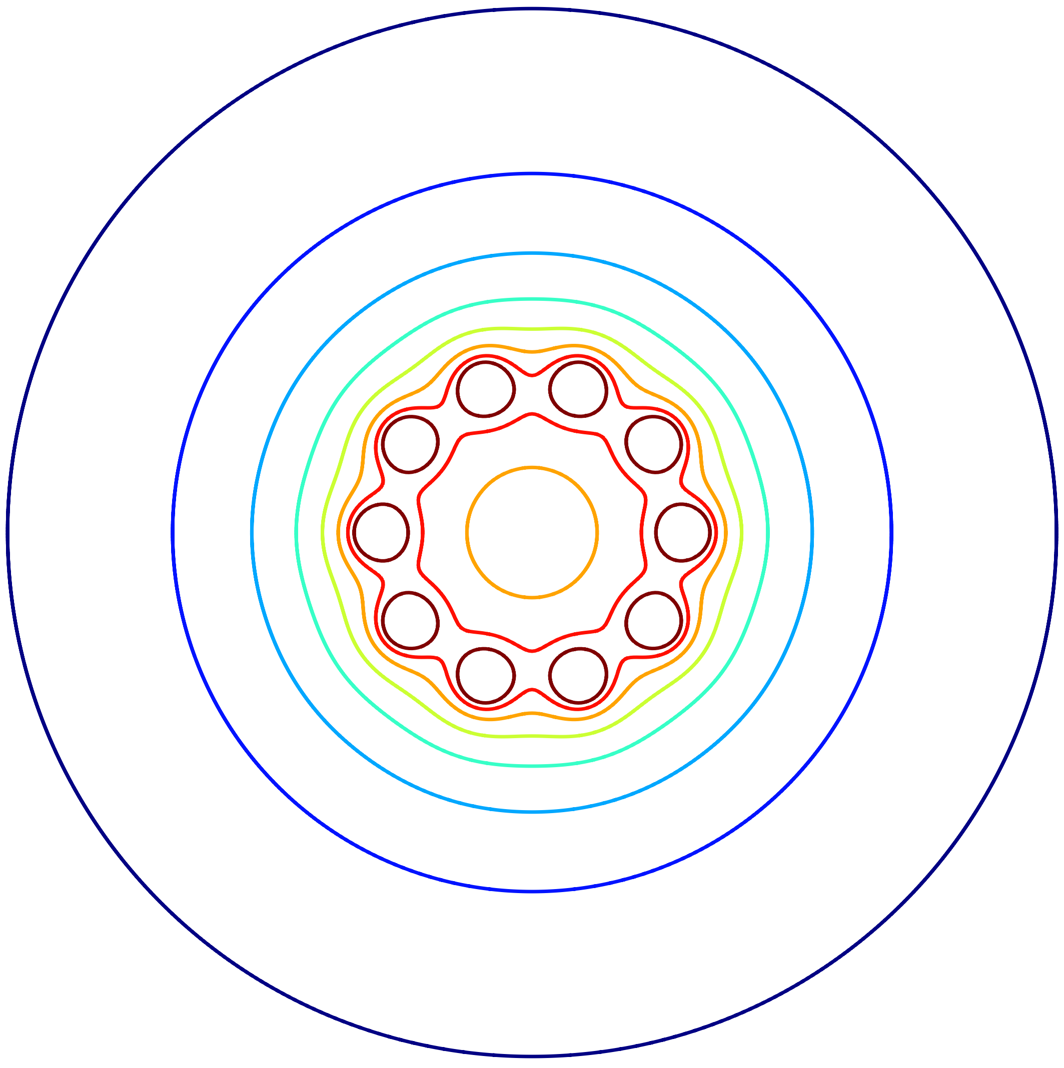
Visualizing the potential: **Equipotential Surfaces**



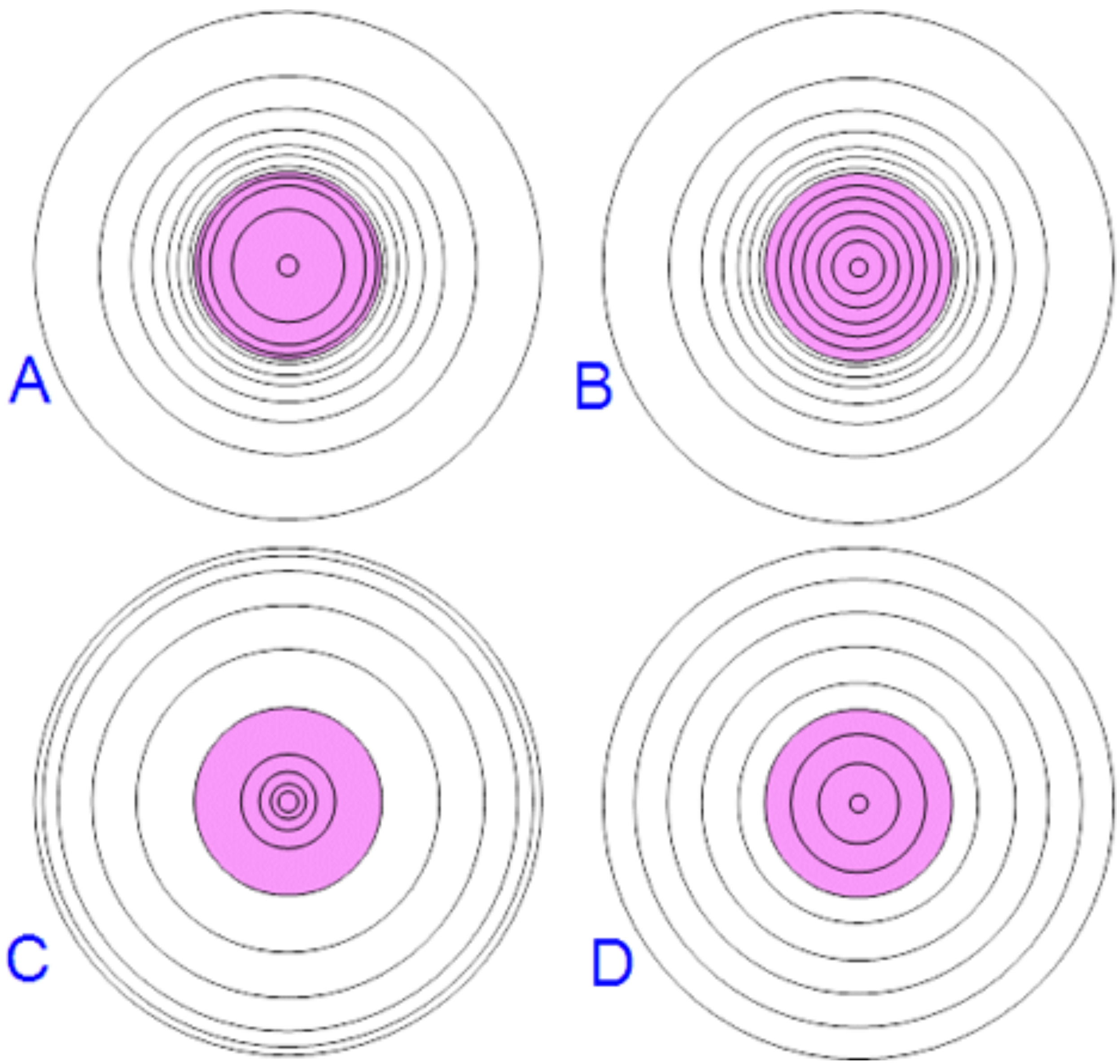
- Normal to E field/lines
- Spacing ∝ E magnitude

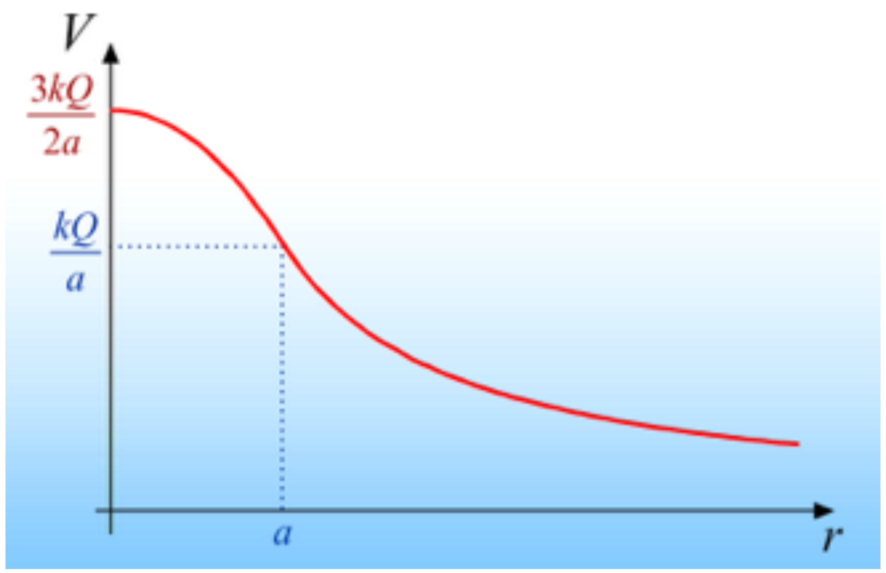
Examples of Equipotential Surfaces: What is the charge distribution?

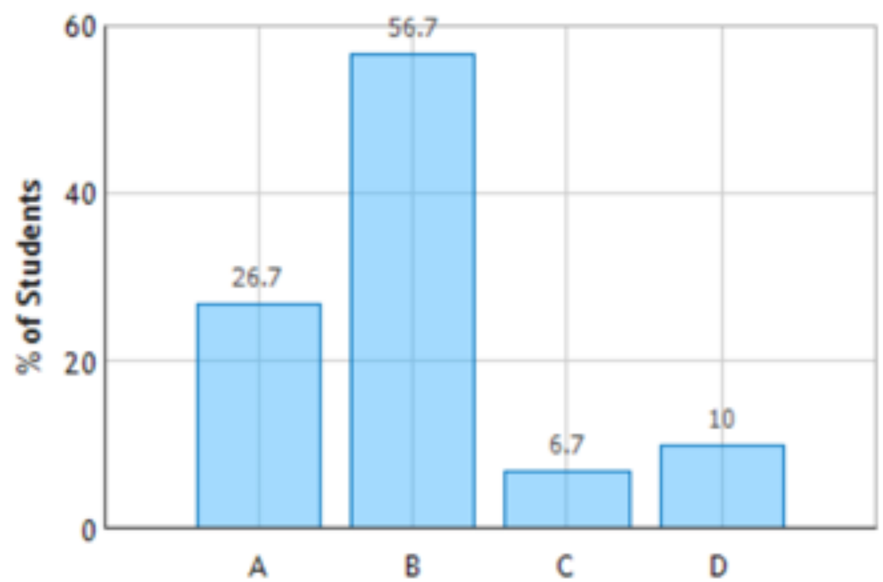




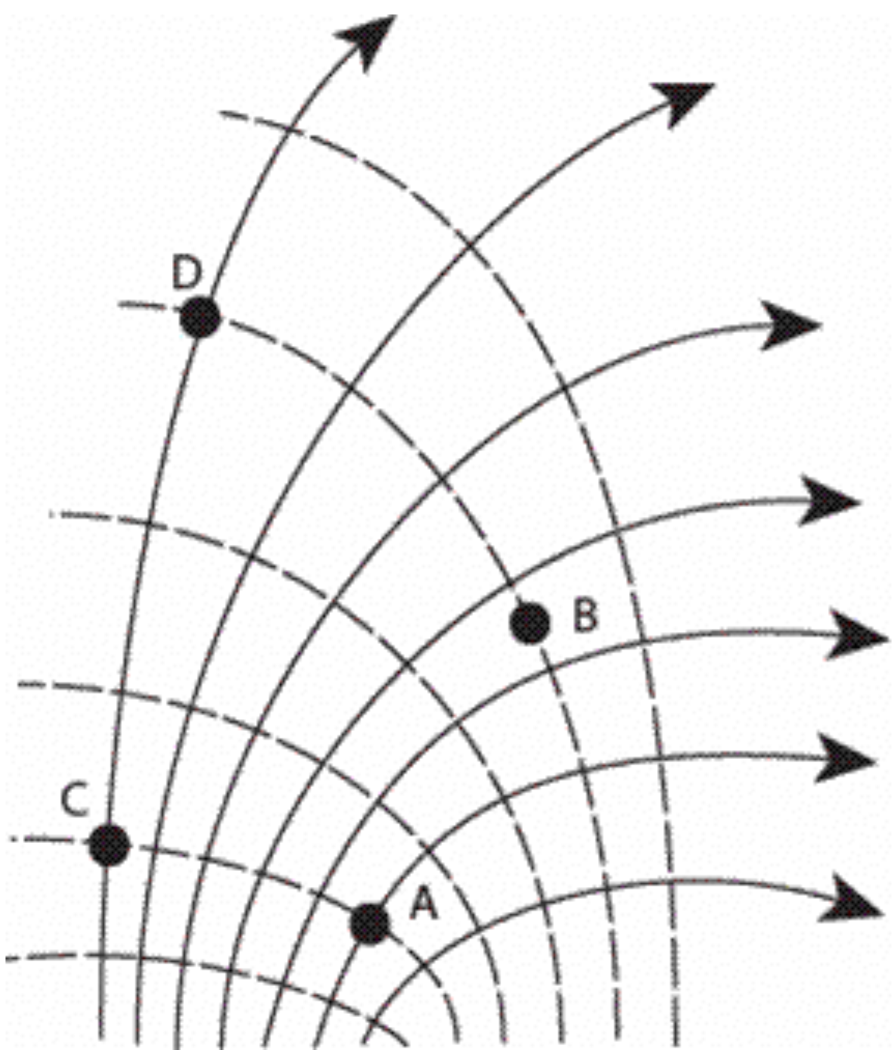
Example from the pre-lecture…



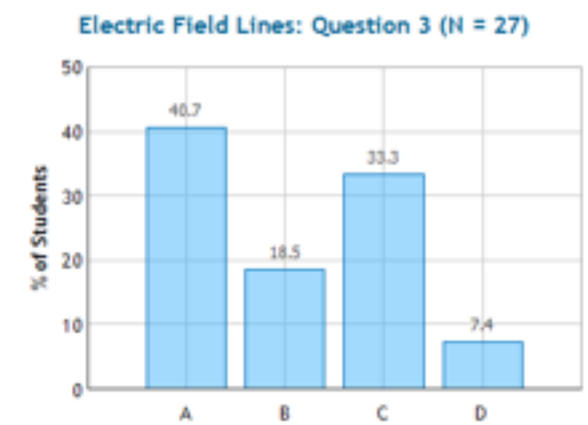




Questions from the checkpoint…



3) Compare the work done moving a negative charge from **A to B** and from **C to D.** Which one requires more work?



5) Compare the work done moving a negative charge from **A to B** and from **A to D.** Which one requires more work?