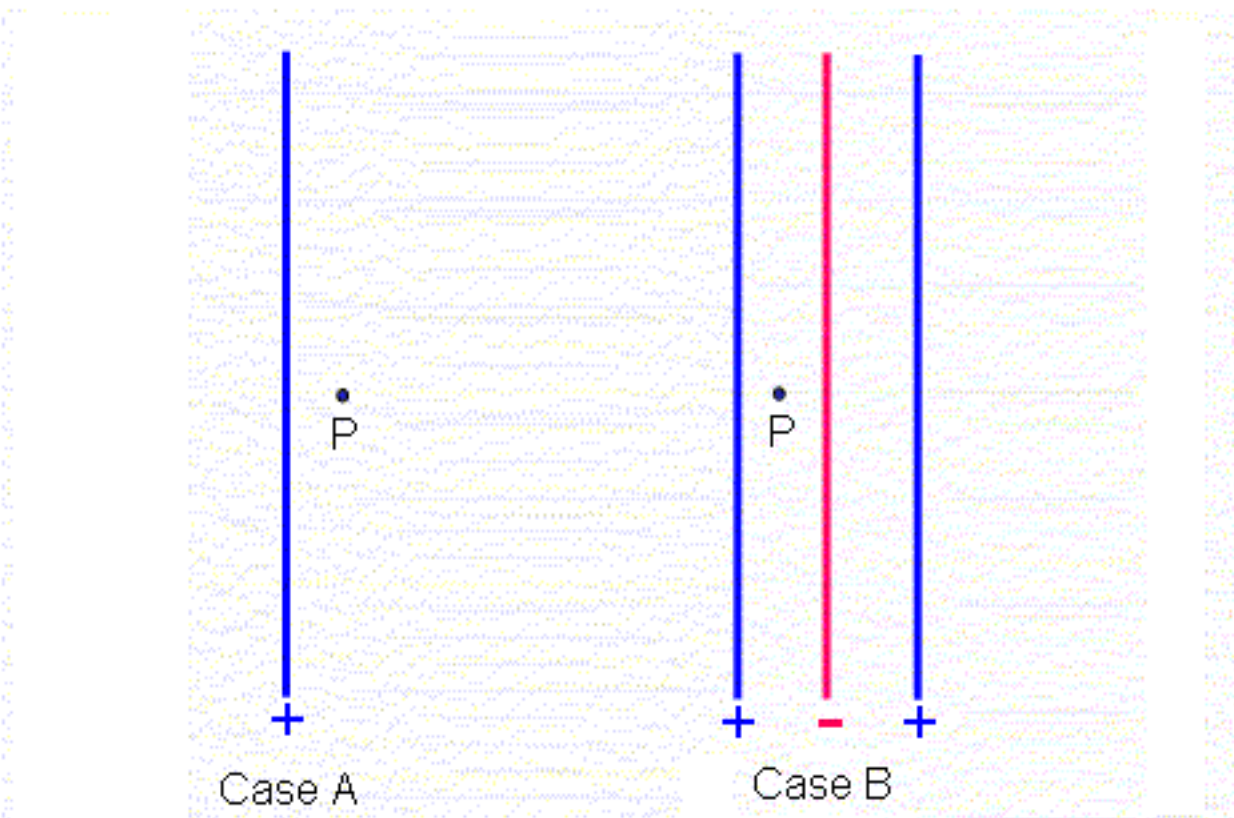
# Gauss Law & Electric Potential Energy

Lecture 6.

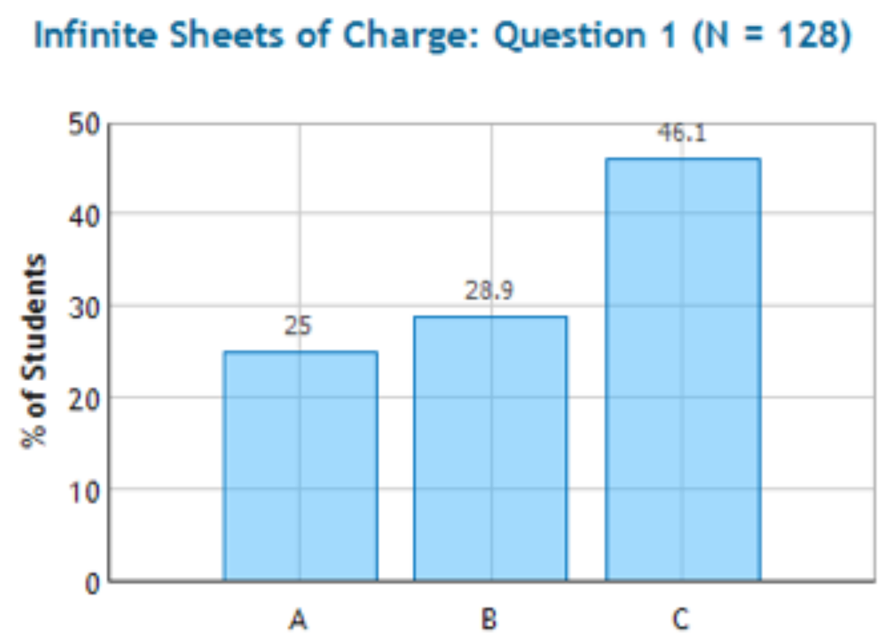
# Announcements

- Monday: No class (MLK Day)
- Exam next week on Thursday
  - Exam is **NOT** easy.
  - **Equation sheet** and **practice exam** have been posted!
- Reading for Wednesday: (None)
- Wednesday: Mostly review…

# From the checkpoint…



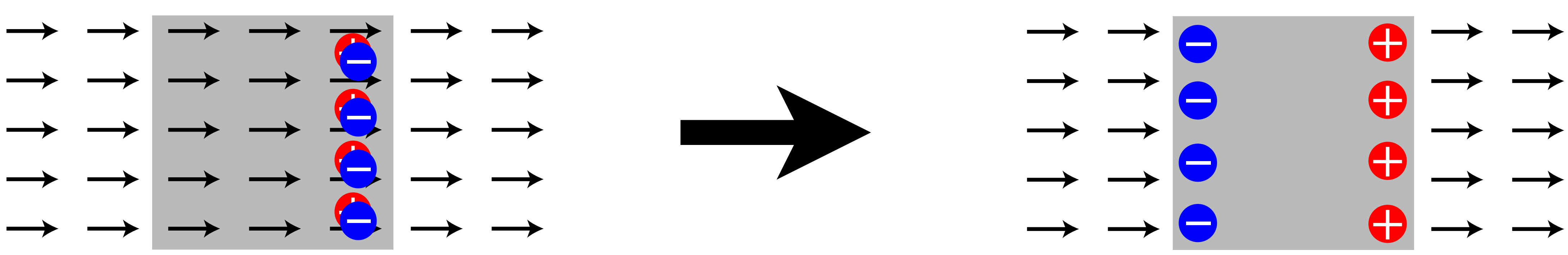

In which case is the magnitude of the electric field at point P bigger?




Use superposition!

# Conductors vs insulators

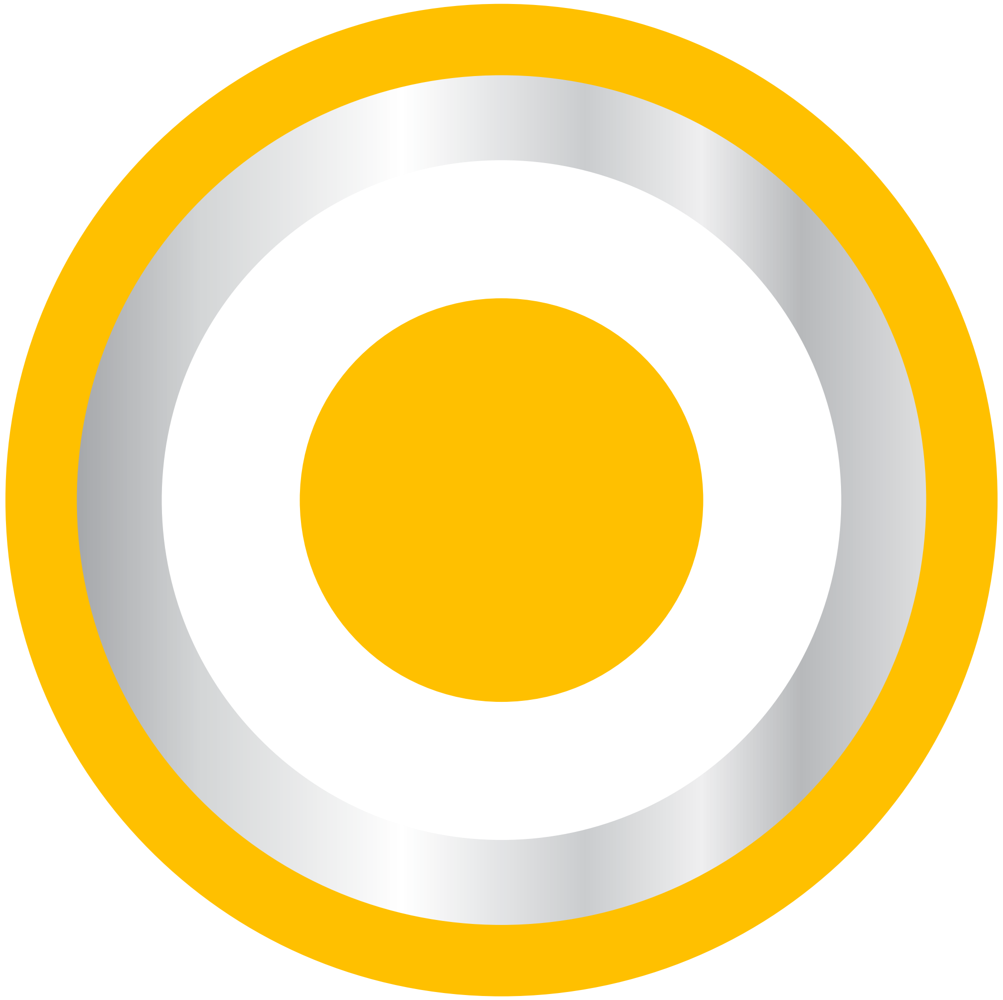
- Conductor (Insulator): Charge is free (not free) to move to establish electrostatic equilibrium
- **Induction:** A sufficient and equal number of + and - charges can be **induced** to cancel any E field in the conductor



# Demo…

- Trash can of science (a.k.a. Faraday Pail)

# E field from a multi-layer Sphere/Coax cable

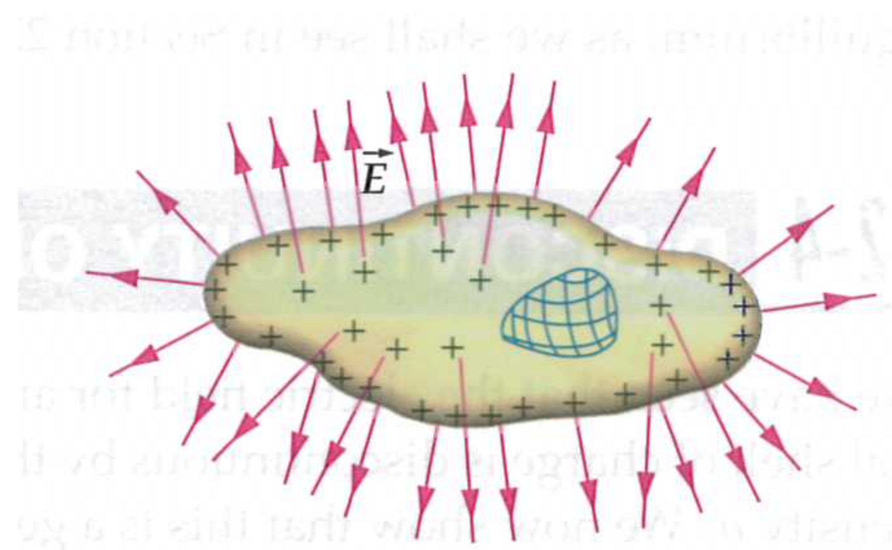


…approximations…

# Clicker questions

# All free charge is at the surface of conductors

- Use Gauss Law…




- Charge at the surface:

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

$$\vec{E}_{\|} = 0$$ (Prove this soon!)

# Induced surface charge density is typically non-uniform in conductors.


Draw field lines…

# Clicker questions

# Electric Potential Energy

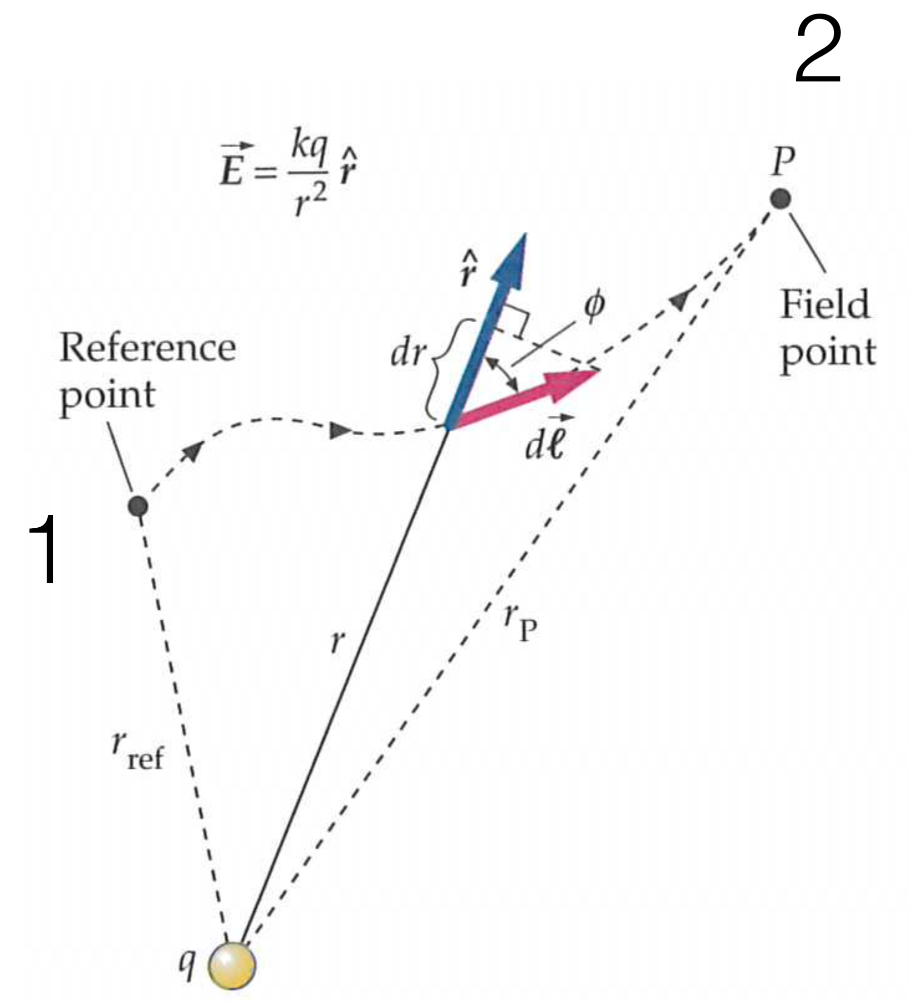
- Electric forces is **conservative**:
- The **work** performed by the electric field on a charge is **path independent.**
- Proof:
  - Show for a point charge.
  - Use superposition of point charges to represent arbitrary charge distribution.

# Electric Potential Energy

- Show for a point charge…
- Work done by field on charge:

$$dW = \vec{F} \cdot d\vec{\ell}$$

- (… on board …)
- Result:




$$\Delta W = W_{1\rightarrow 2} \equiv W_2 - W_1 = -\frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

# Electric Potential Energy

- Define the change is Electric Potential Energy as

$$\Delta U = U_{1\to 2} \equiv U_2 - U_1 = -W_{1\to 2}$$

- Electric Potential Energy difference:

$$U_{1\to 2} \equiv U_2 - U_1 = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

# Electric Potential Energy

- Define Electric Potential as follows:

$$U_1 \equiv U_1 - U_\infty = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{\infty}\right] = \frac{q_1 q_2}{4\pi\epsilon_0 r_1}$$

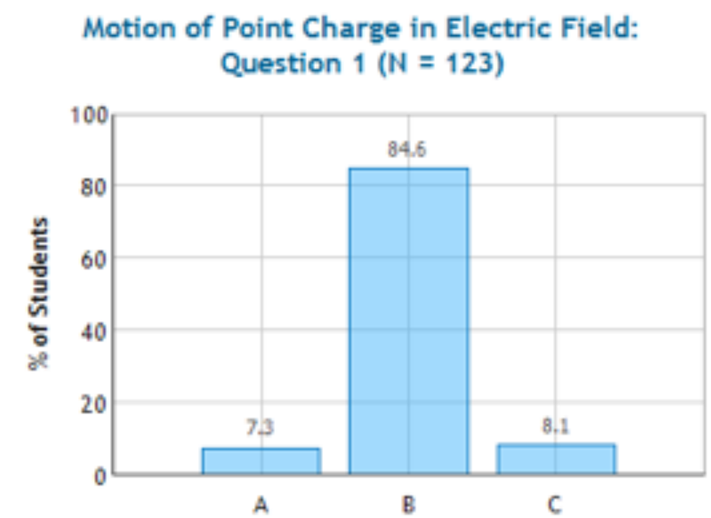
  relative to a point at $\infty$.

- Superposition applies

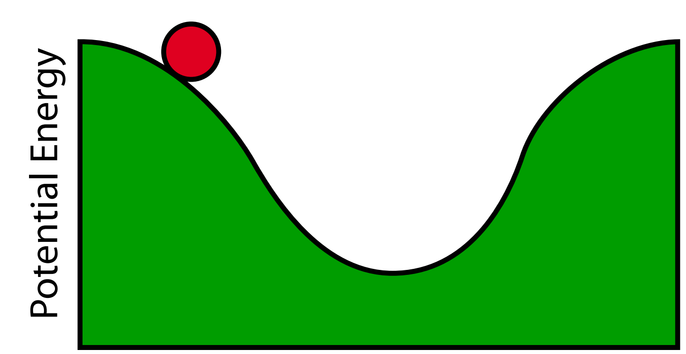
- Potential energy of a charge distribution
  = energy required to assemble from $\infty$.

# Electric Potential Energy

- Plot potential $$U(\vec{r}) = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$
- (… on board …)




- Heuristic for remembering signs: "Physics is always going down hill."

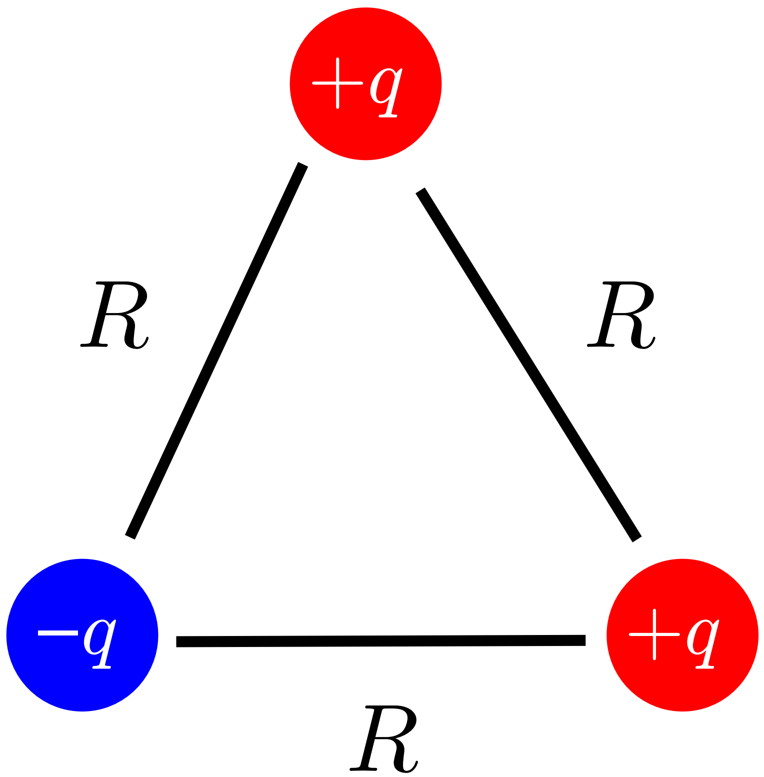


# Question from smartPhysics:

Is there a connection between
**field lines** and **electric potential**?

(Soon!)

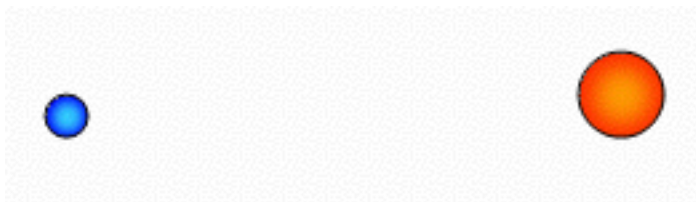
# Electric Potential Energy of a charge distribution




$$U_{\text{Tot}} = \sum_{\text{pairs}} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

# From the checkpoint…

Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first.



Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

(A) YES, as long as the third charge is positive
(B) YES, as long as the third charge is negative
(C) YES, no matter what the sign of the third charge
(D) NO