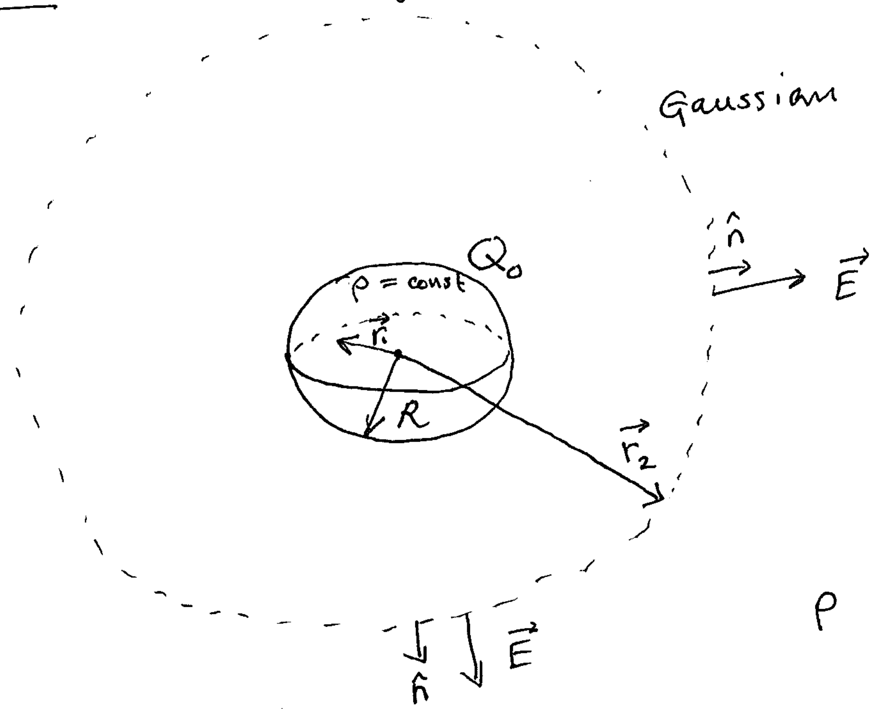
Gauss Law applied

Lecture 6.

Gauss Law for charged insulating Sphere

Gaussian Surface.



$\hat{n} \cdot \vec{E} = E$

$\hat{n}$ + $\vec{E}$ are $\parallel$

$$\rho \equiv \frac{Q_0}{\frac{4\pi}{3} R^3}$$

Total Charge (Q_0)

Volume ($\frac{4\pi}{3} R^3$)

$|\vec{r}| < R$:

Flux:

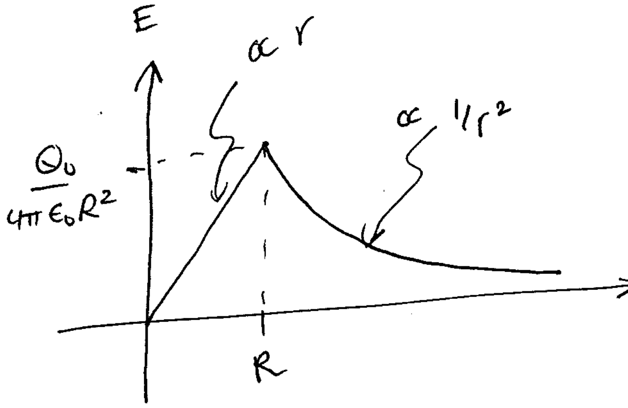
$$\Phi_{\ominus} \equiv \oint_{\ominus} d^2A \; \hat{n} \cdot \vec{E} = 4\pi r^2 E$$

Gauss Law:

$$\Phi_{\ominus} = \frac{Q_{inside}}{\epsilon_0} = \frac{1}{\epsilon_0} \underbrace{\frac{4}{3}\pi r^3}_{V_{\ominus}} \cdot \rho = \left(\frac{r}{R}\right)^3 \frac{Q_0}{\epsilon_0}$$

$$E = \frac{Q_0}{4\pi\epsilon_0} \frac{r}{R^3}$$

$$\vec{E} = \frac{Q_0}{4\pi\epsilon_0} \frac{\vec{r}}{R^3}$$

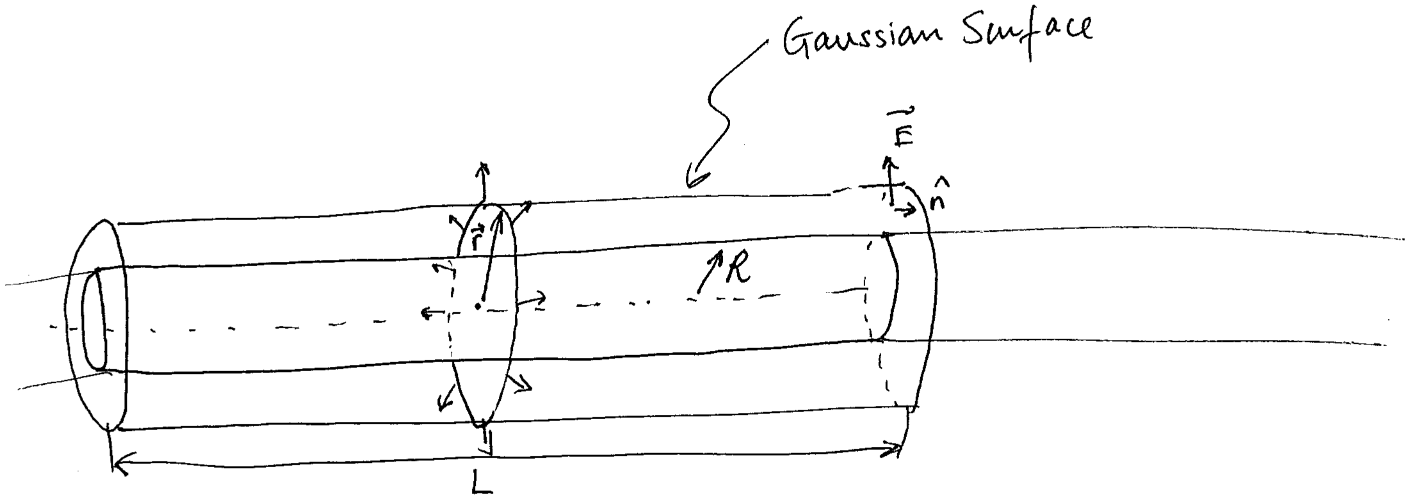


$|\vec{r}| > R$:

$$Q_{inside} = Q_0$$

$$\vec{E} = \frac{Q_0}{4\pi\epsilon_0} \frac{\vec{r}}{r^3}$$

Gauss Law for cylinder w/ surface charge σ



$$\Phi_M = \Phi_{\substack{\text{end}\\\text{caps}}} + \Phi_{\text{cylinder}} = \frac{Q_{\text{inside}}}{\epsilon_0} = \begin{cases} \frac{\sigma L \cdot 2\pi R}{\epsilon_0}, & r > R \\ 0, & r < R \end{cases}$$

$\Phi_{\text{end caps}} \to 0$ since $\hat{n} \cdot \vec{E} = 0$

$\Phi_{\text{cylinder}} \to AE = 2\pi r \cdot L$ since $\hat{n} \cdot \vec{E} = E$

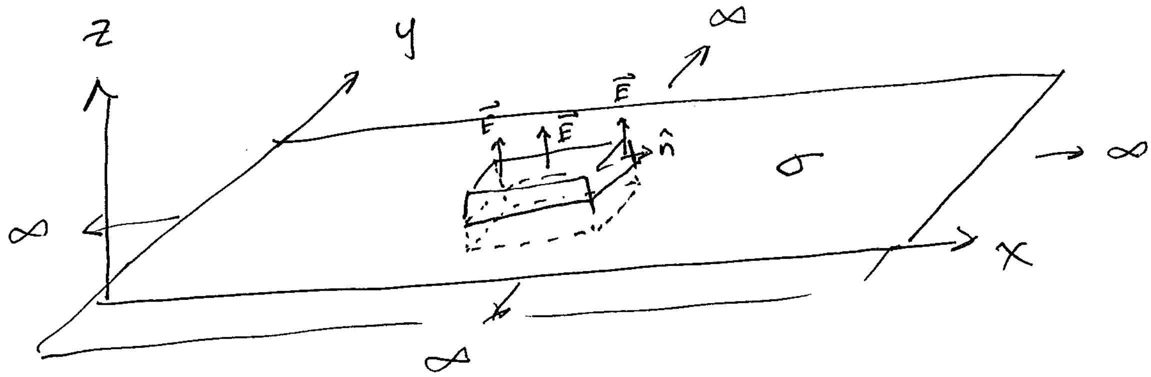
$$\vec{E} = \hat{r} \cdot \frac{\cancel{2\pi} \sigma R \cdot \cancel{L}}{\cancel{2\pi} \epsilon_0 r \cdot \cancel{L}} \Rightarrow \vec{E} = \hat{r} \frac{\sigma}{\epsilon_0}\left(\frac{R}{r}\right) \cdot \begin{cases} 1, \\ 0, \end{cases}$$

$\hat{r}$: cylindrical radial unit vector

For a line charge, $R \to 0$, $\sigma \to \frac{\lambda}{2\pi R}$

$$\vec{E} = \hat{r} \frac{\lambda}{2\pi \epsilon_0 r}$$

Gauss Law for charge plane



$$\Phi_M = \underbrace{\cancel{\Phi_{\text{vertical sides}}}}_{0} + \Phi_{TOP} + \Phi_{Bottom}$$

since: $\hat{n} \cdot \vec{E} = 0$

These are =

$$= 2\,\Phi_{top} = 2A \cdot E$$

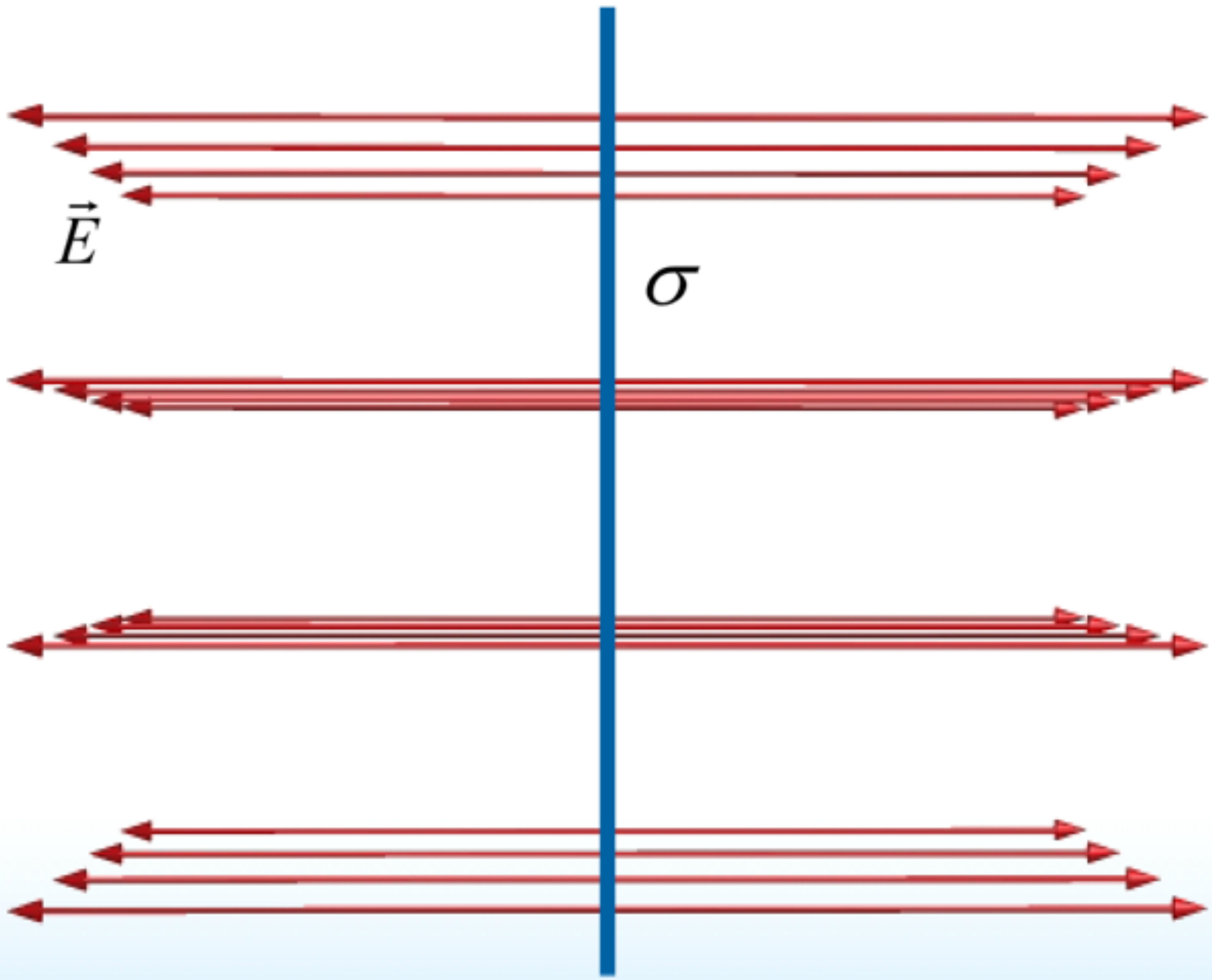
$$\frac{Q_{inside}}{\epsilon_0} = \frac{A\sigma}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\,\hat{z}\begin{cases} 1, & z>0 \\ -1, & z<0 \end{cases}$$

(1) Infinite plane/plate

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

Side View

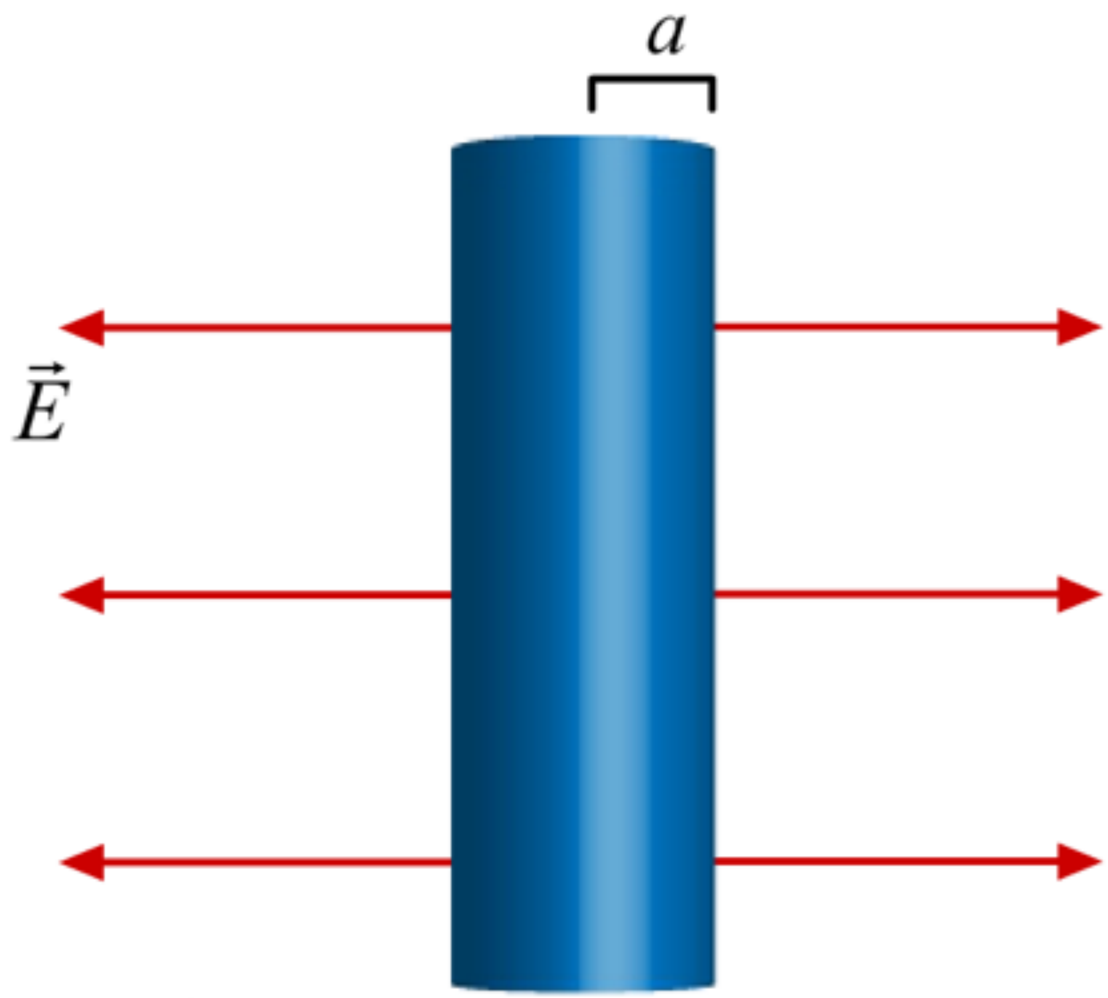


$$E = \frac{\sigma}{2\varepsilon_o}$$

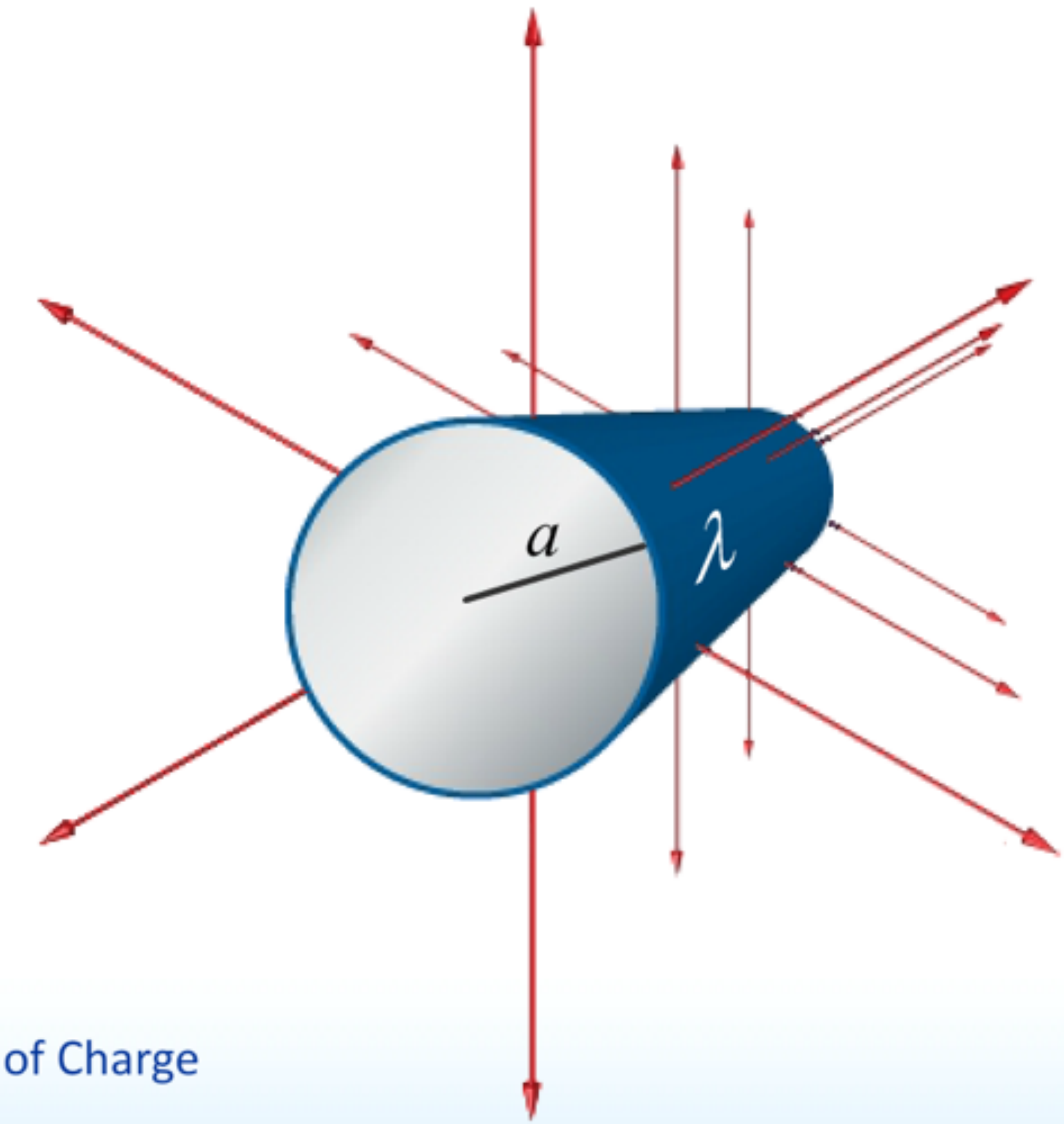
(2) Infinite line

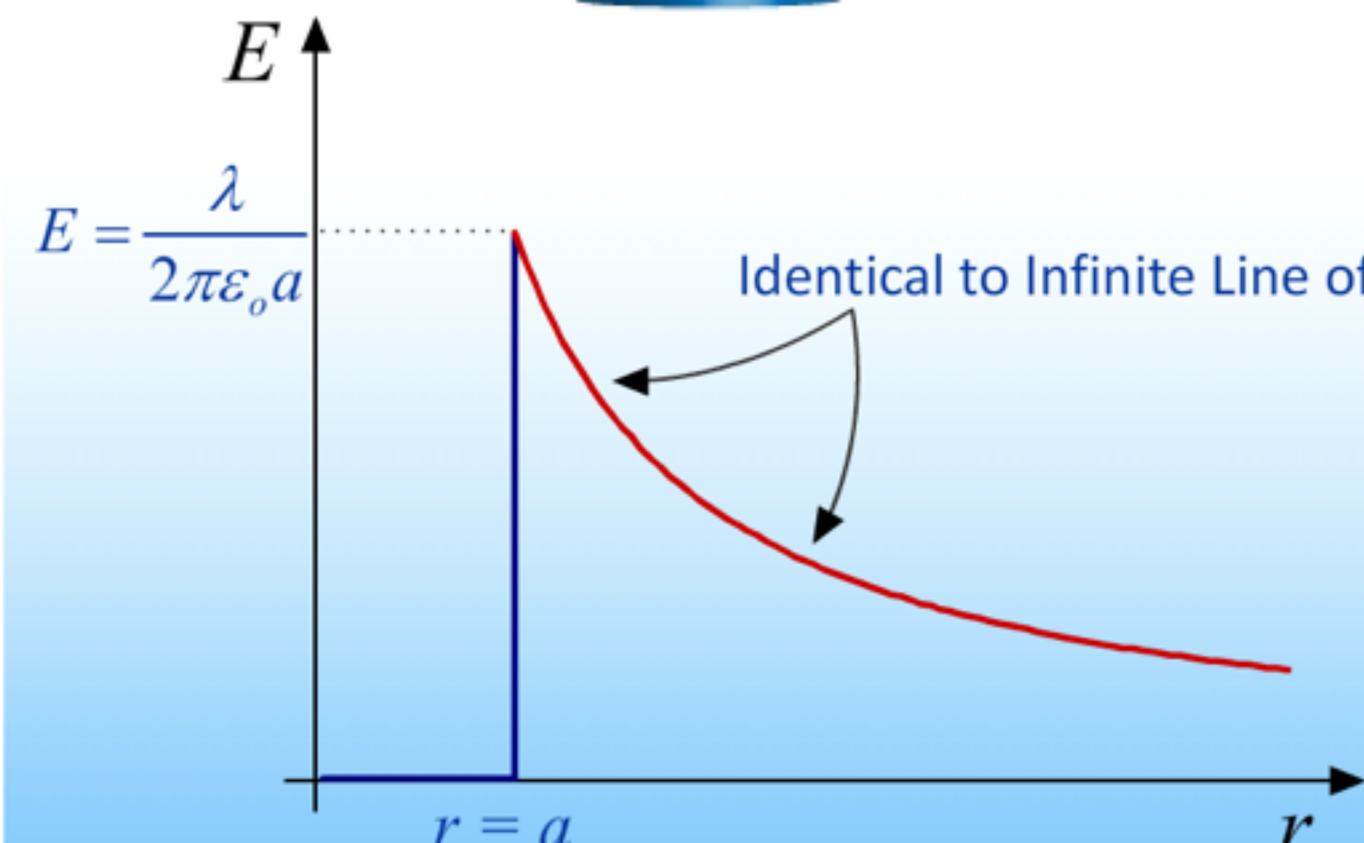
1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

Side View



View with Perspective





$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}} \quad (r > a)$$

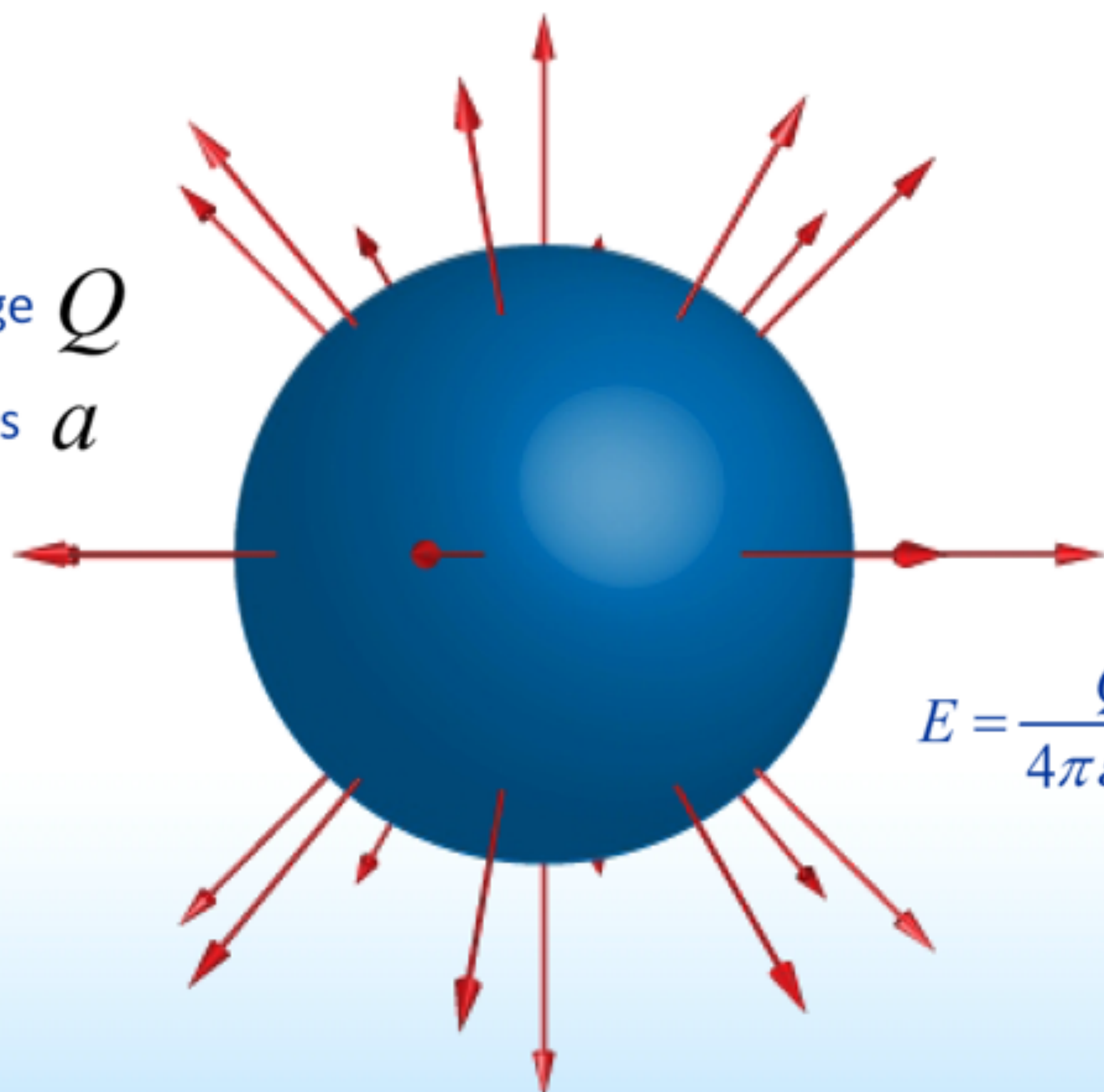
(3) Point (or sphere)

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

(3B) Charged sphere

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

Solid Insulator



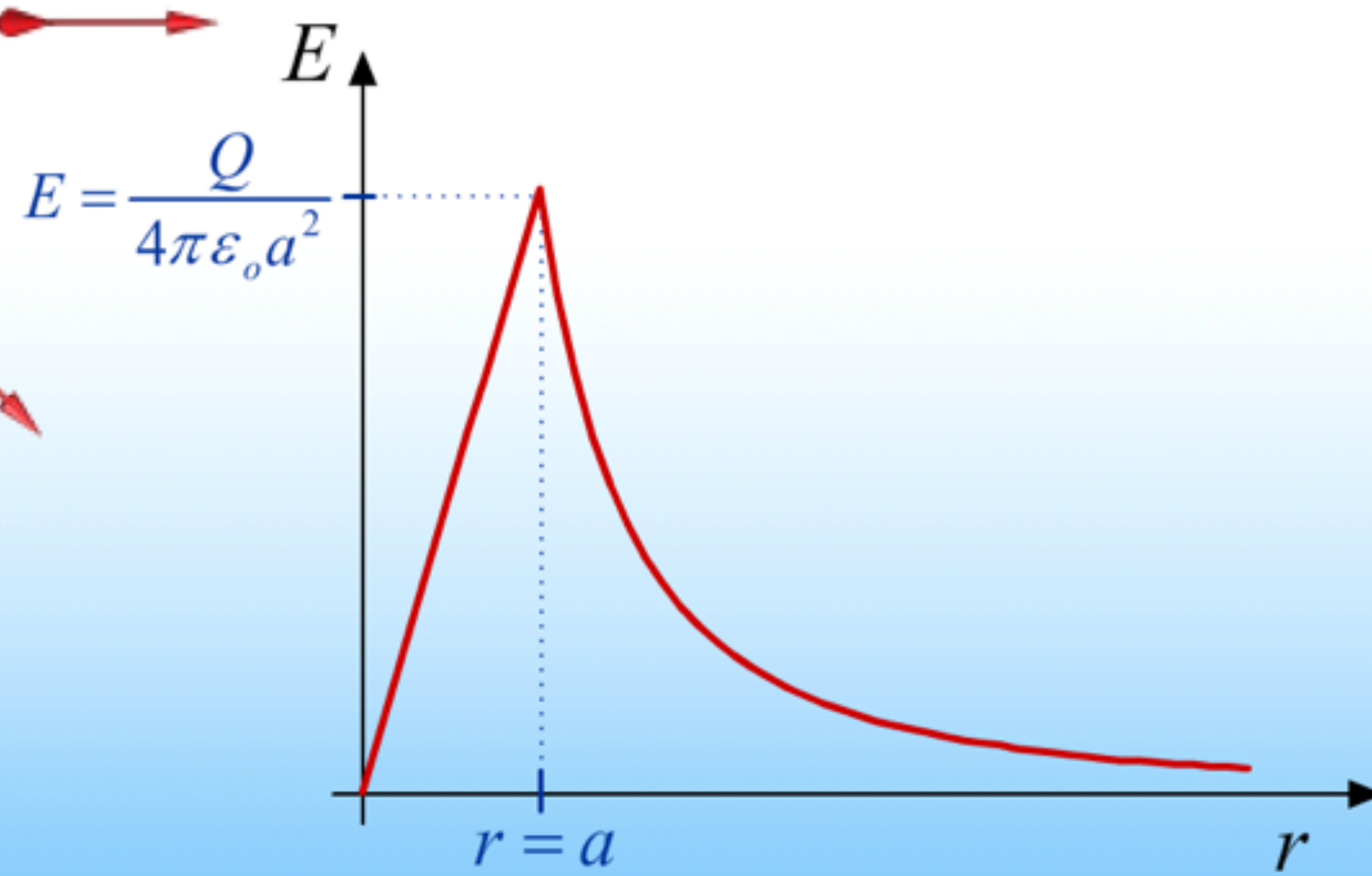
charge Q

radius a

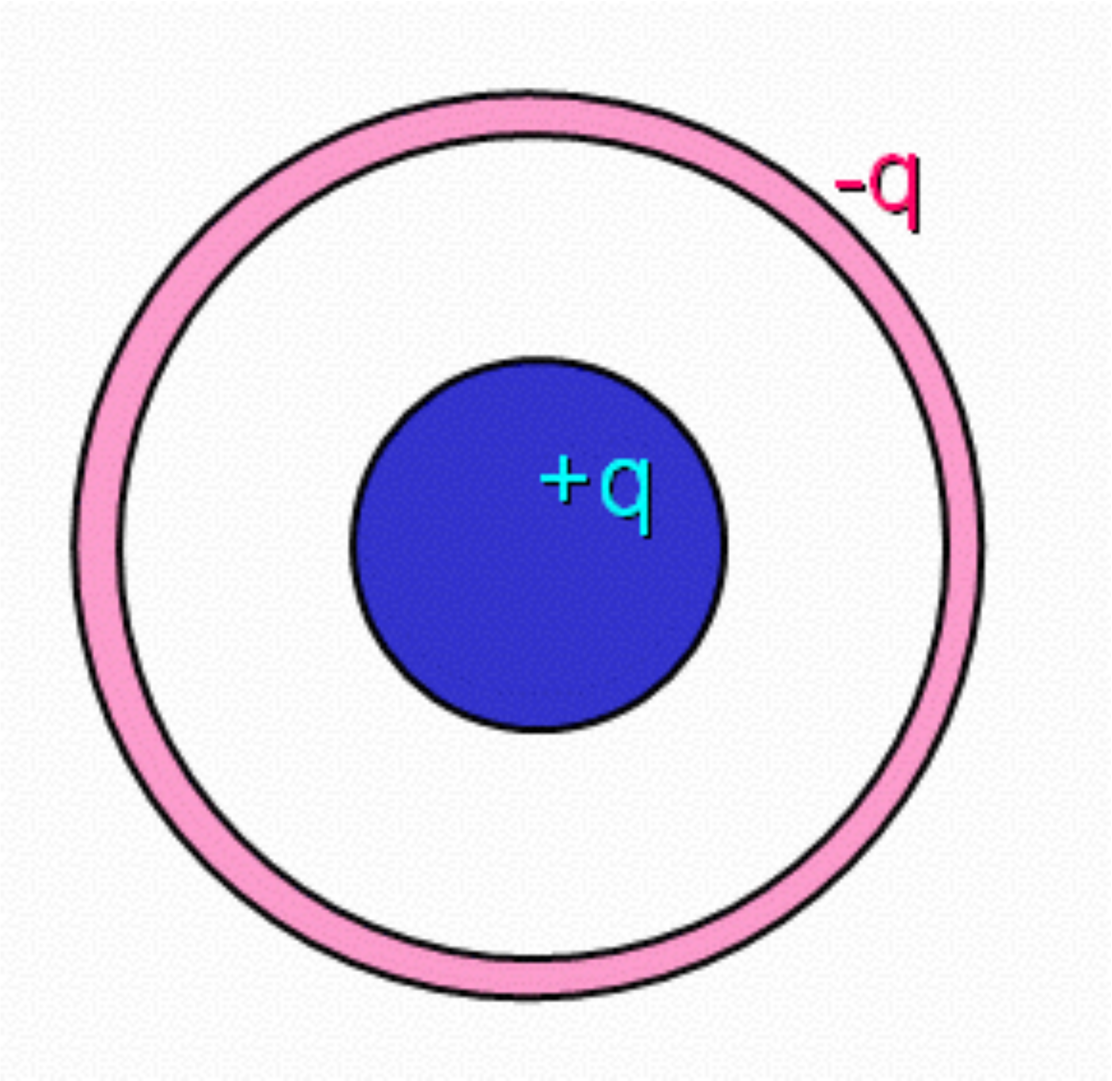
Charge Density $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

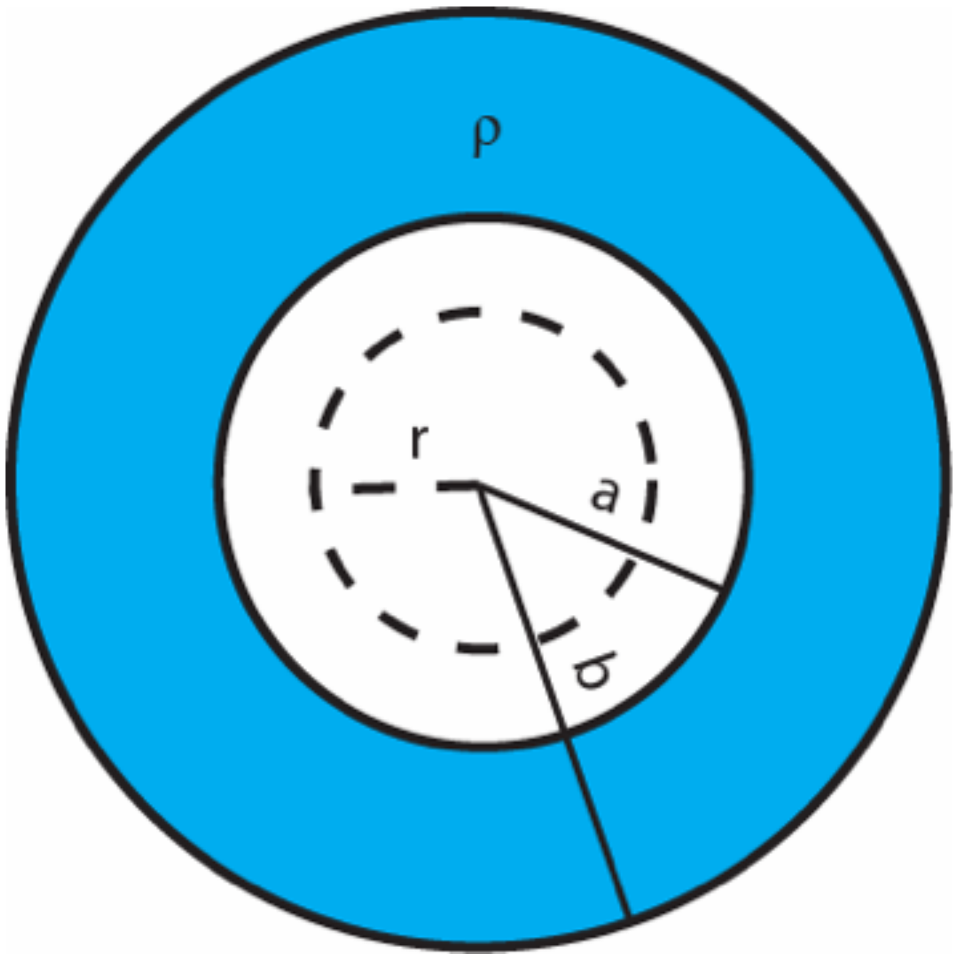
$$\boxed{E = \frac{Q}{4\pi \varepsilon_o r^2}} \quad (r > a)$$

$$\boxed{E = \frac{\rho}{3\varepsilon_o} r} \quad (r < a)$$

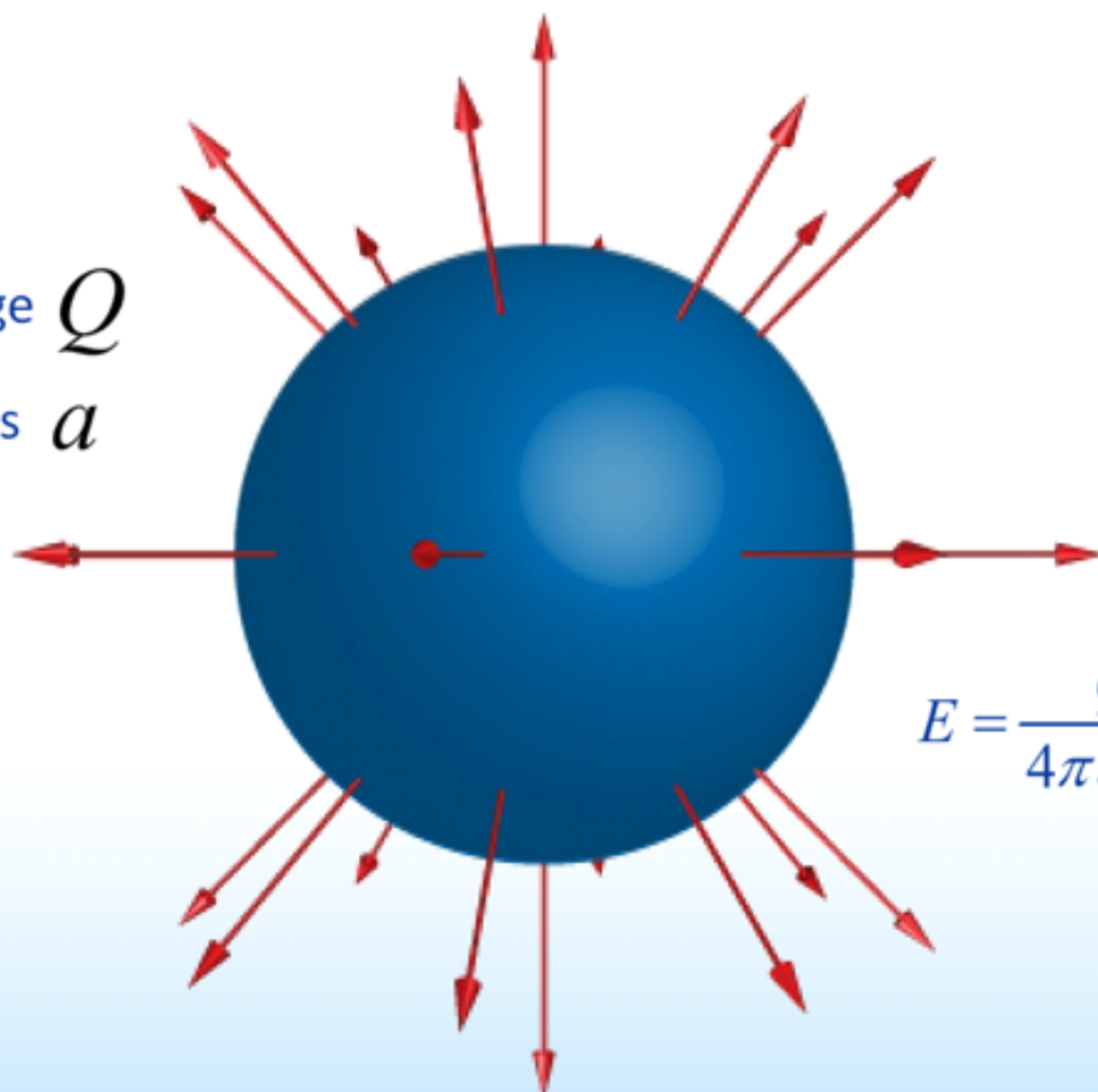


FlipIt Physics:
Where do I draw the surface?



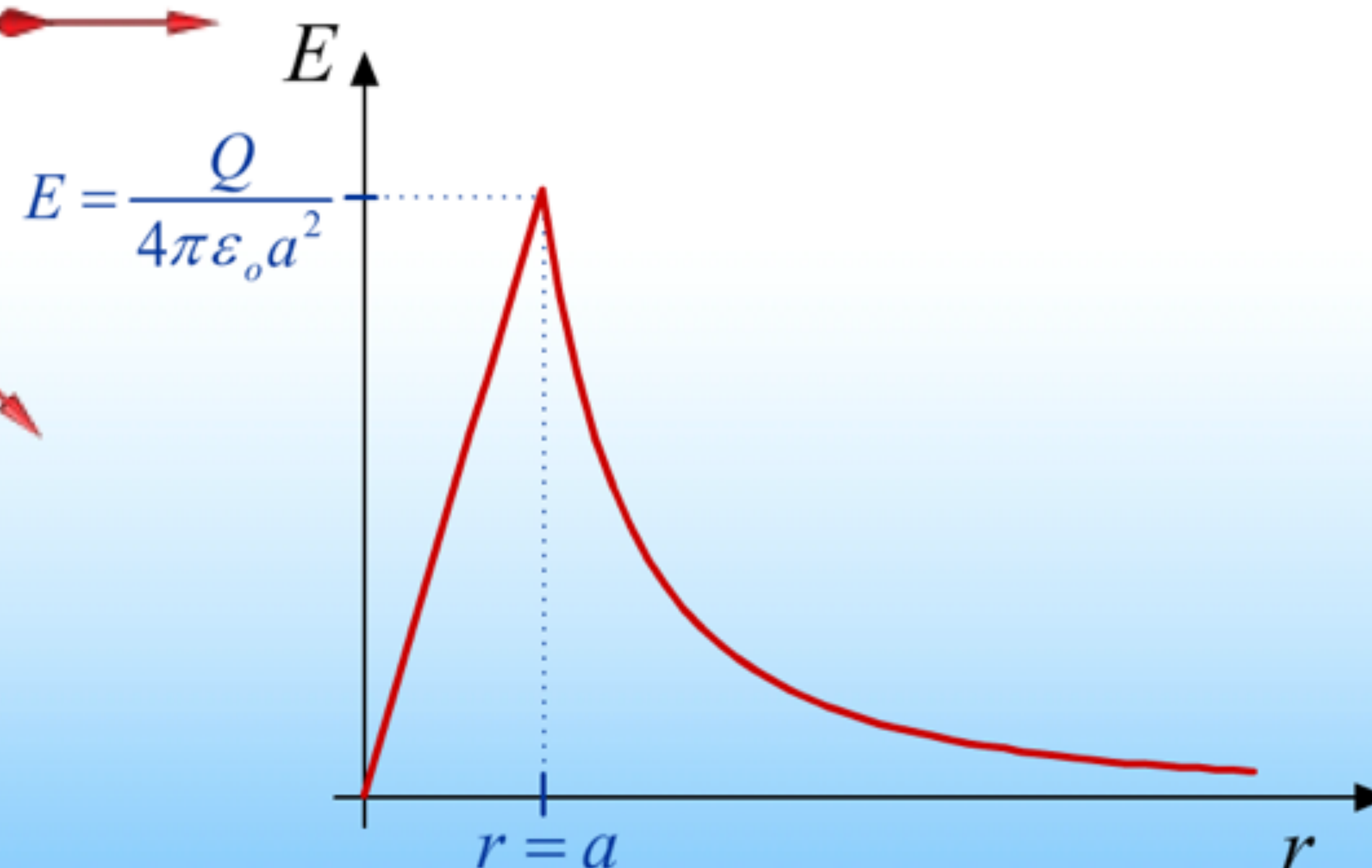


Conductor: Where do charges want to go?



$$E = \frac{Q}{4\pi\varepsilon_o r^2} \quad (r > a)$$

$$E = \frac{\rho}{3\varepsilon_o} r \quad (r < a)$$

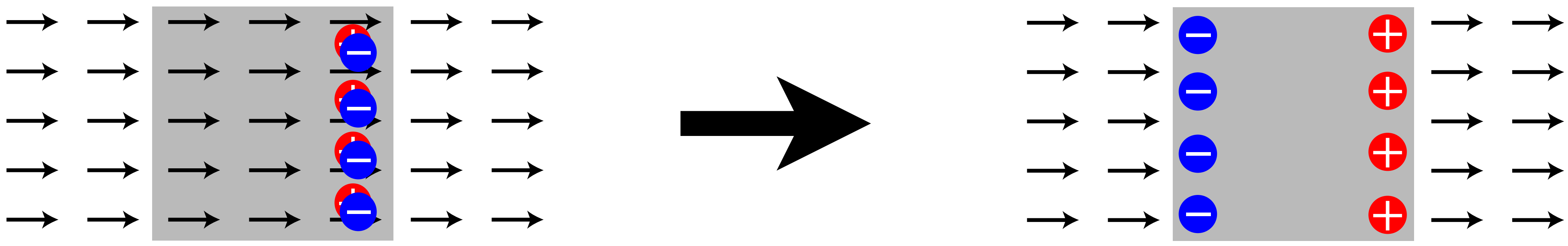


Charge Density $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

Conductors vs insulators

- Conductor (Insulator): Charge is free (not free) to move to establish electrostatic equilibrium

- Conductor have free charge: A sufficient and equal number of + and - charges can be "created" to cancel any E field in the conductor



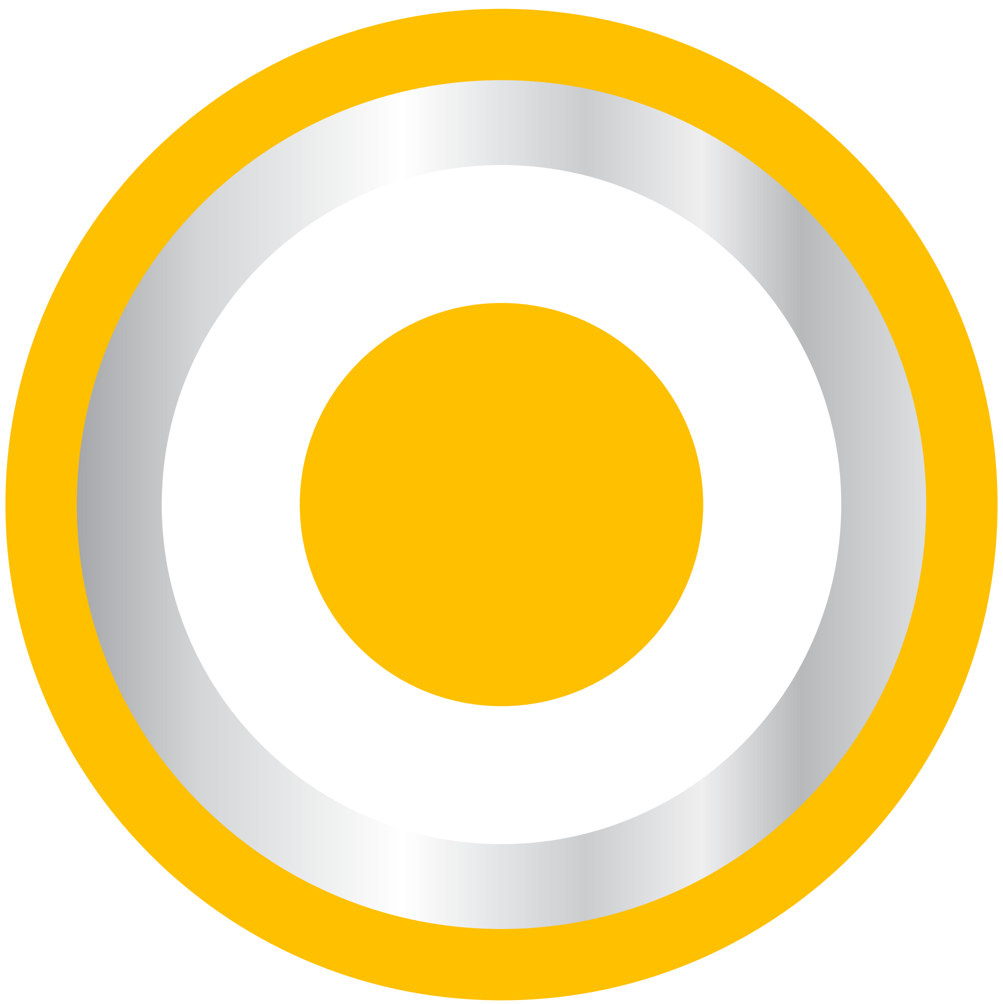
All free charge is at the surface of conductors

- Use Gauss Law…

Demo…

- Trash can of science (a.k.a. Faraday Pail)

E field from a multi-layer Sphere/Coax cable

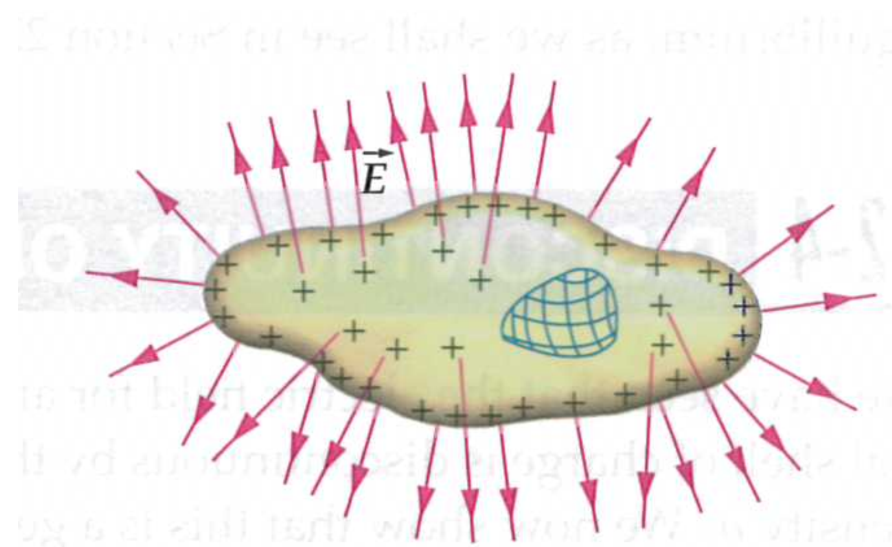


…approximations…

Clicker questions

All free charge is at the surface of conductors

- Use Gauss Law…



- Charge at the surface:

$$E_{\perp} = \frac{\sigma}{\epsilon_0}$$

$$\vec{E}_{||} = 0$$ (Prove this soon!)

Induced surface charge density is typically non-uniform in conductors.

Draw field lines…

Clicker questions

Electric Potential Energy

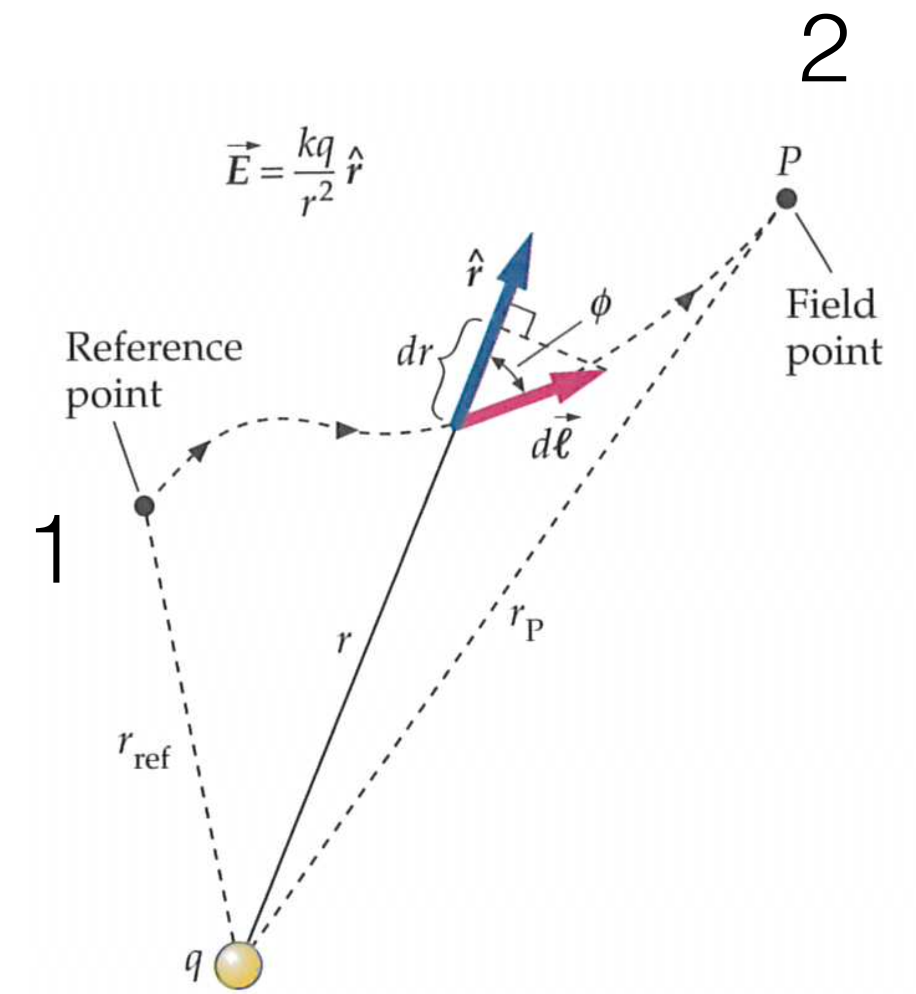
- Electric forces is **conservative**:
- The **work** performed by the electric field on a charge is **path independent.**
- Proof:
  - Show for a point charge.
  - Use superposition of point charges to represent arbitrary charge distribution.

Electric Potential Energy

- Show for a point charge…
- Work done by field on charge:

$$dW = \vec{F} \cdot d\vec{\ell}$$

- (… on board …)
- Result:



$$\Delta W = W_{1\to 2} \equiv W_2 - W_1 = -\frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Electric Potential Energy

- Define the change is Electric Potential Energy as

$$\Delta U = U_{1\to 2} \equiv U_2 - U_1 = -W_{1\to 2}$$

- Electric Potential Energy difference:

$$U_{1\to 2} \equiv U_2 - U_1 = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Electric Potential Energy

- Define Electric Potential as follows:

$$U_1 \equiv U_1 - U_\infty = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{\infty}\right] = \frac{q_1 q_2}{4\pi\epsilon_0 r_1}$$

  relative to a point at ∞.

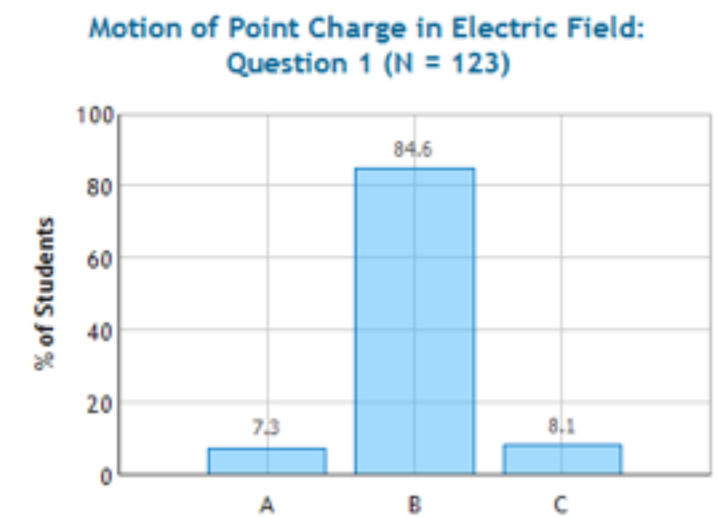
- Superposition applies

- Potential energy of a charge distribution
  = energy required to assemble from ∞.

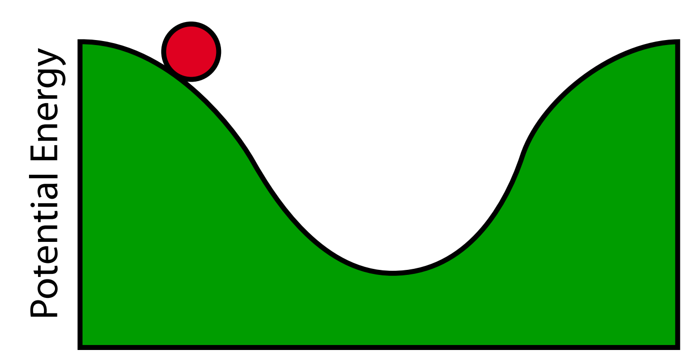
Electric Potential Energy

- Plot potential $U(\vec{r}) = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$

- (… on board …)



- Heuristic for remembering signs: "Physics is always going down hill."

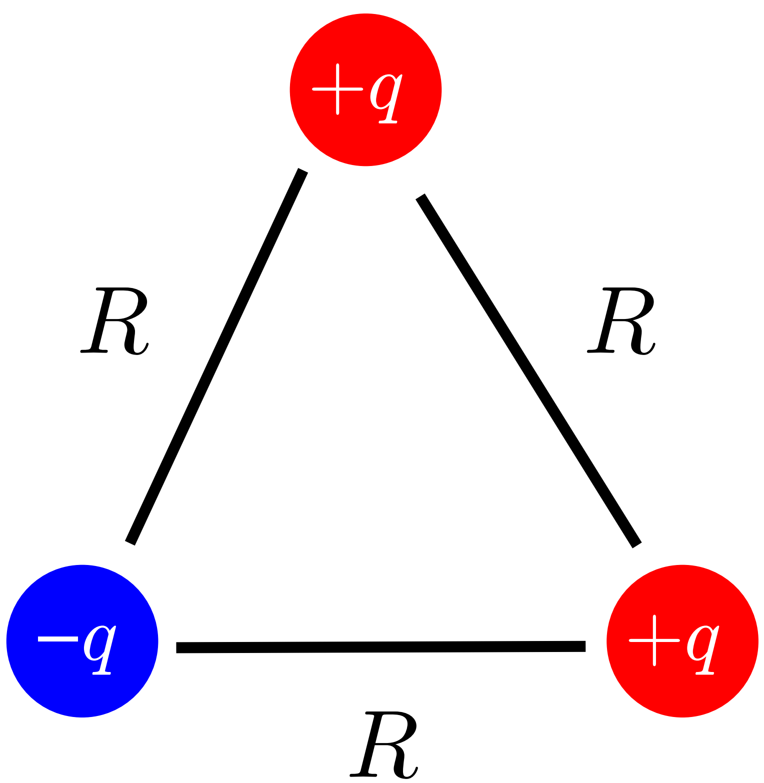


Question from smartPhysics:

Is there a connection between
field lines and **electric potential**?

(Soon!)

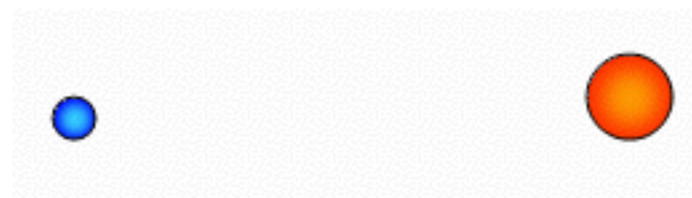
Electric Potential Energy of a charge distribution



$$U_{\mathrm{Tot}} = \sum_{\mathrm{pairs}} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{1}{2}\sum_{i\neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

From the checkpoint…

Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is twice as large as the first.



Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?

(A) YES, as long as the third charge is positive
(B) YES, as long as the third charge is negative
(C) YES, no matter what the sign of the third charge
(D) NO