Electric Flux & the Gauss Law

Lecture 5.

Announcements

- Homework due tonight!
- Next assignment has been posted
- Exam next week on Thursday
- Reading for Friday: 22.4-22.6

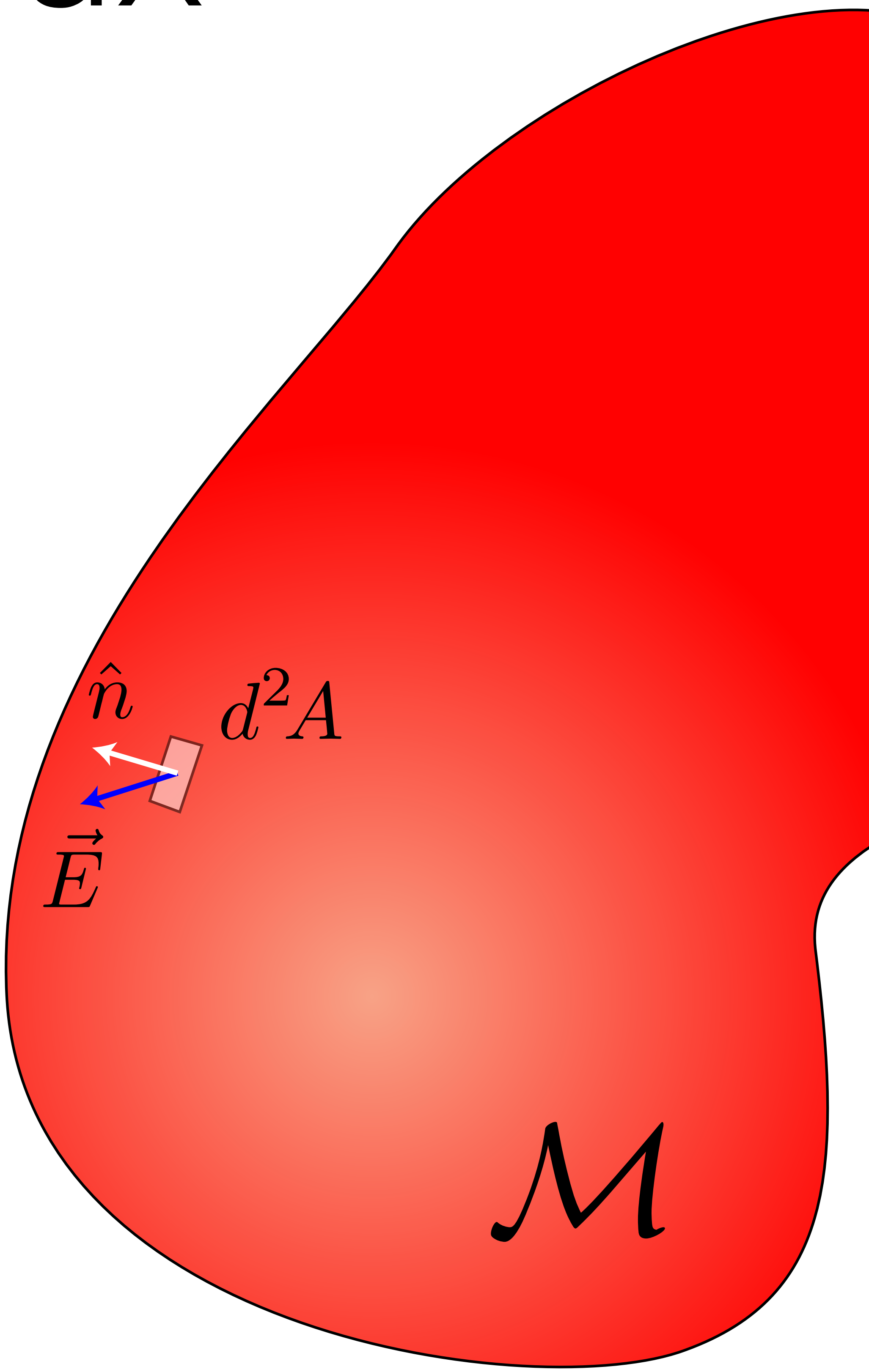
Electric Flux

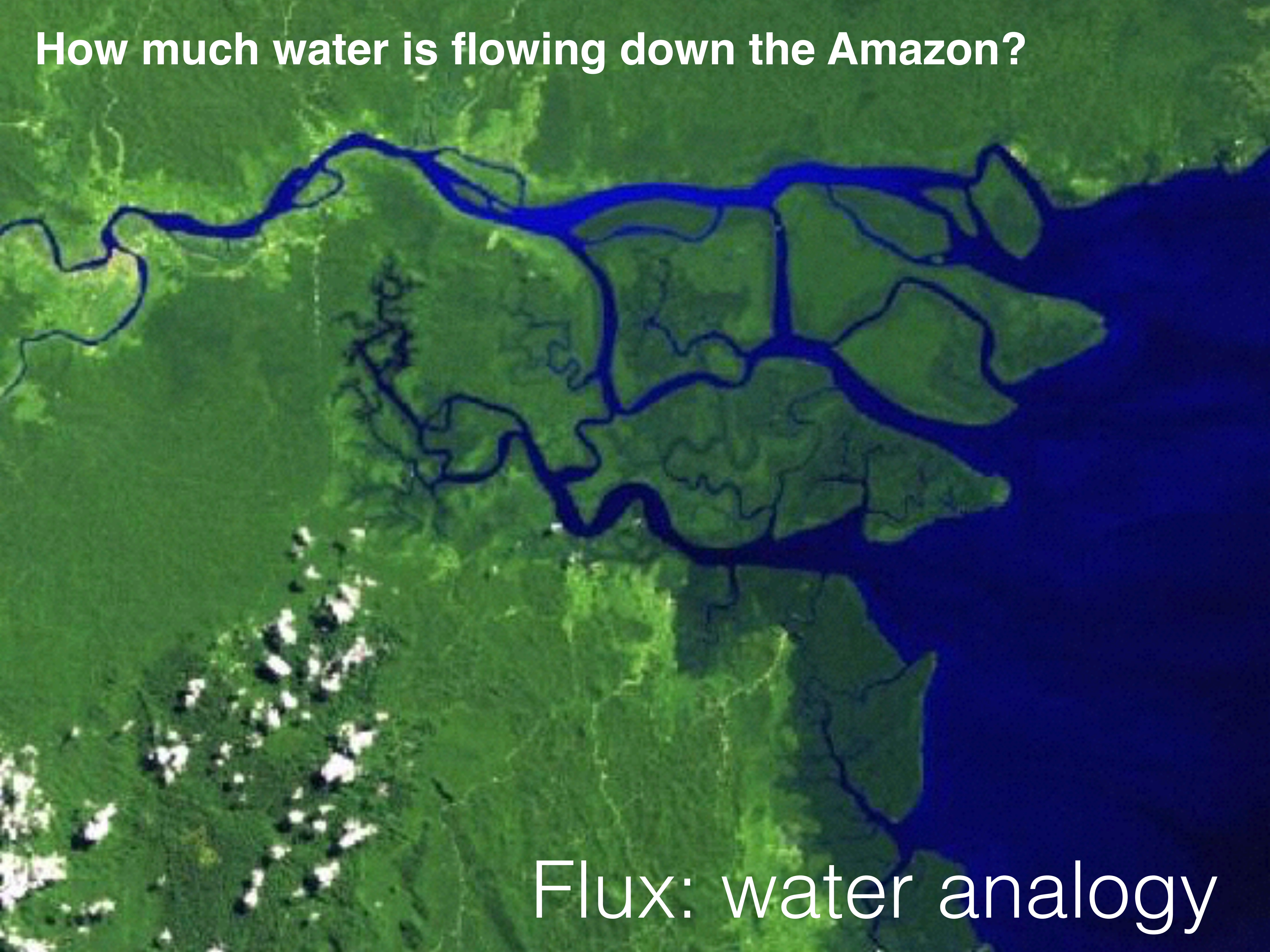
- Definition of Electric Flux:

$$\Phi_{\mathcal{M}} = \oint_{\mathcal{M}} d^2A \; \hat{n} \cdot \vec{E}$$

- Meaning of **flux**:
  ~ Number passing through
  a surface

- Flux measures *outgoingness* of vector field

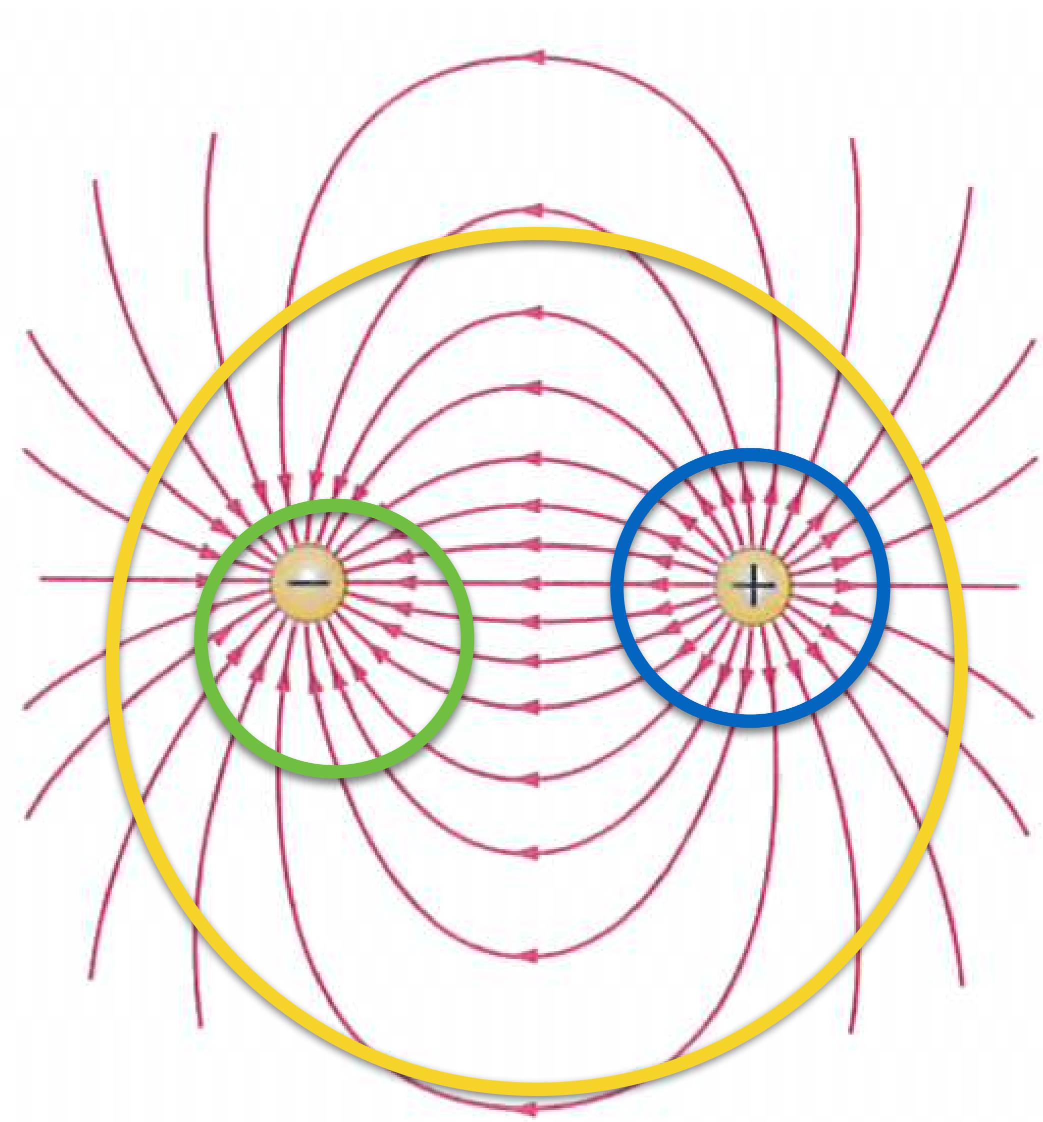



How much water is flowing down the Amazon?
Flux: water analogy

Clicker 4-7: Flux

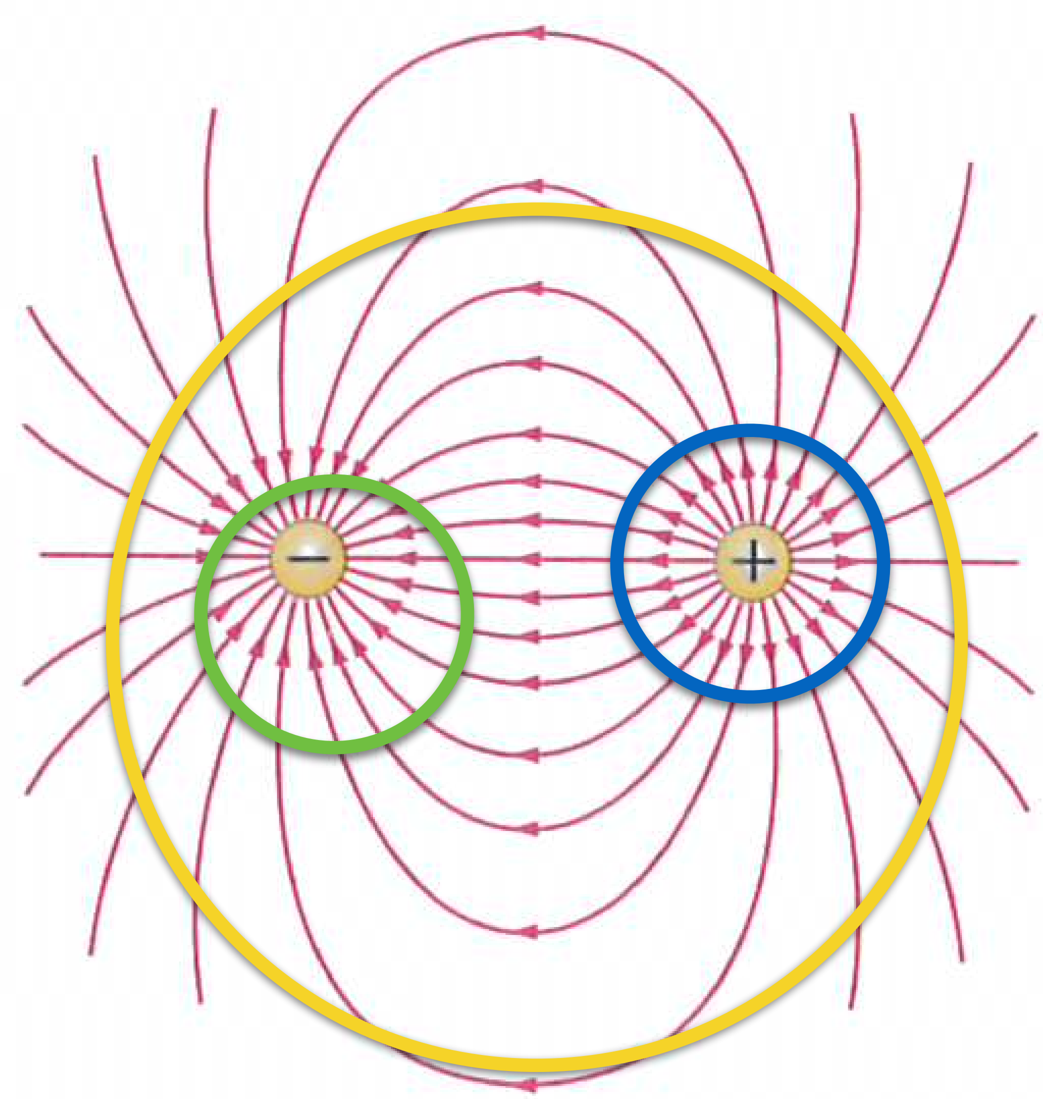
Relation between **Electric flux** and **E field lines**

- **Flux = # of field through surface**
- Blue sphere: 24 lines leaving
- Green sphere: 24 lines entering
- Yellow sphere: 0 net lines entering/leaving



Intuitive relation between **Electric flux** and **E field lines**

- **Flux "=" # of field through surface**
- Rule for field lines: # lines created "=" Q
- Flux "=" Q
- "Gauss Law"



Relation between **Gauss Law** & **Coulomb Law**

- **Gauss Law:** E Flux “=“ charge:

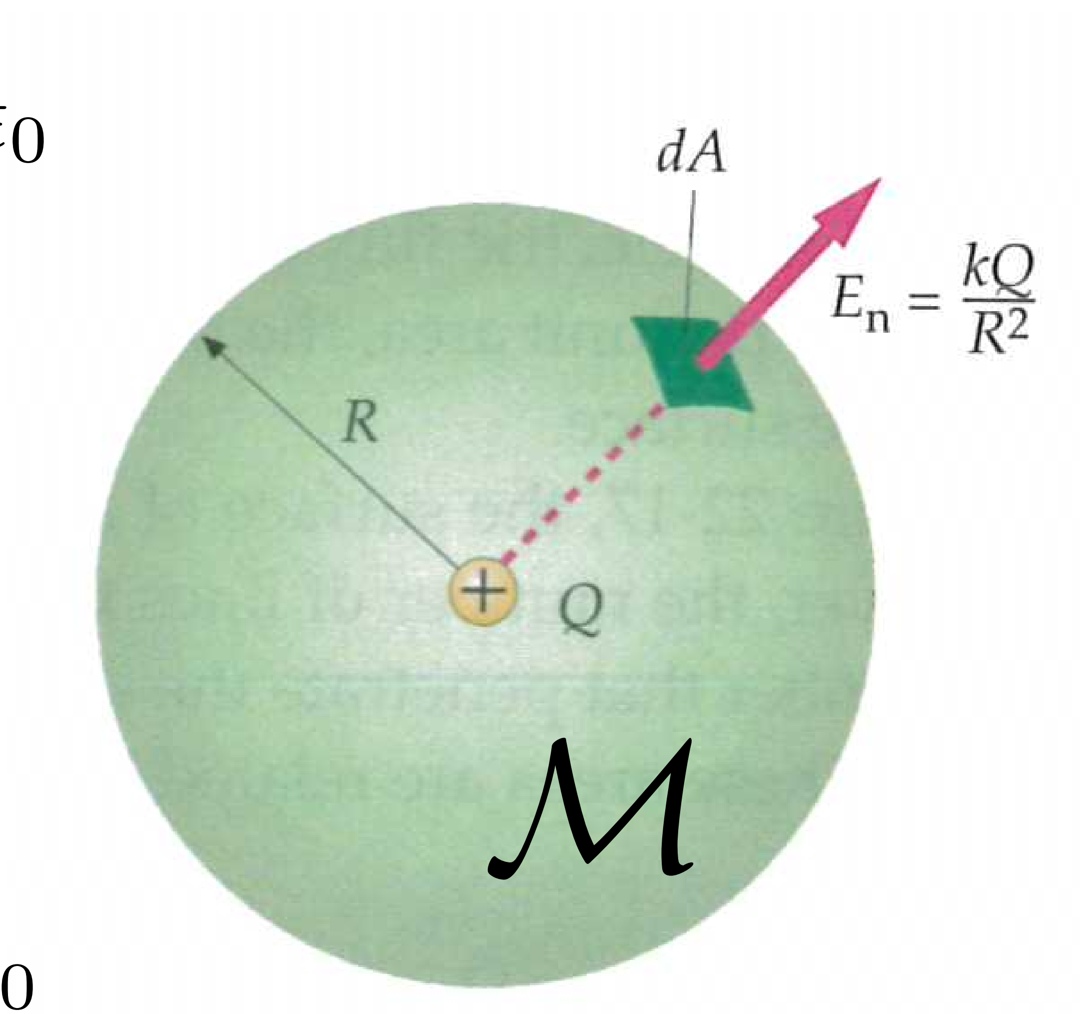
$$\Phi_{\mathcal{M}} = \oint_{\mathcal{M}} d^2A \; \hat{n} \cdot \vec{E} = Q_{\text{inside}}/\epsilon_0$$

ϵ_0 is the **Electric Constant**

- **Coulomb Law:**

$$\vec{E}(\vec{r}) = \frac{k \; q}{r^2}\hat{r}$$

- Work out the relation between ϵ_0 and k



Fundamental law of Electrostatics

- **Coulomb Law**: (Solution to DE) Force between two point charges:

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

or

Gauss Law: (DE) Relationship between charge of E field

Integral Equation:

$$\oint_{\mathcal{M}} d^2A \; \hat{n} \cdot \vec{E} = Q_{\text{inside}}/\epsilon_0$$

Differential Equation (DE):

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

Intuitive meaning of the Gauss Law

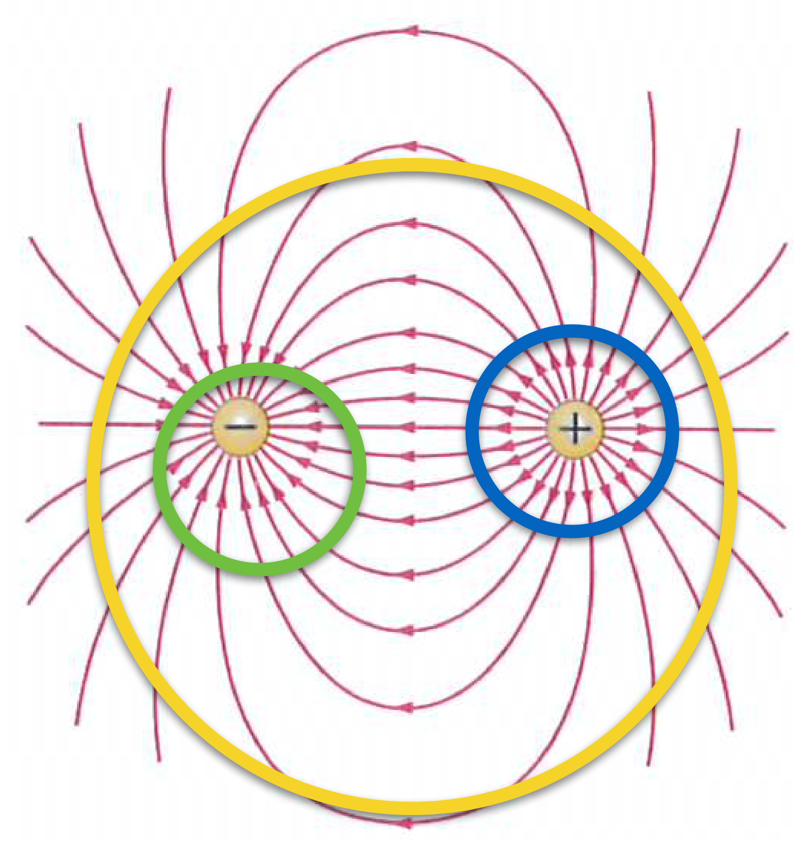
Integral Equation:

$$\oint_{\mathcal{M}} d^2A \; \hat{n} \cdot \vec{E} = Q_{\text{inside}}/\epsilon_0$$

Differential Equation:

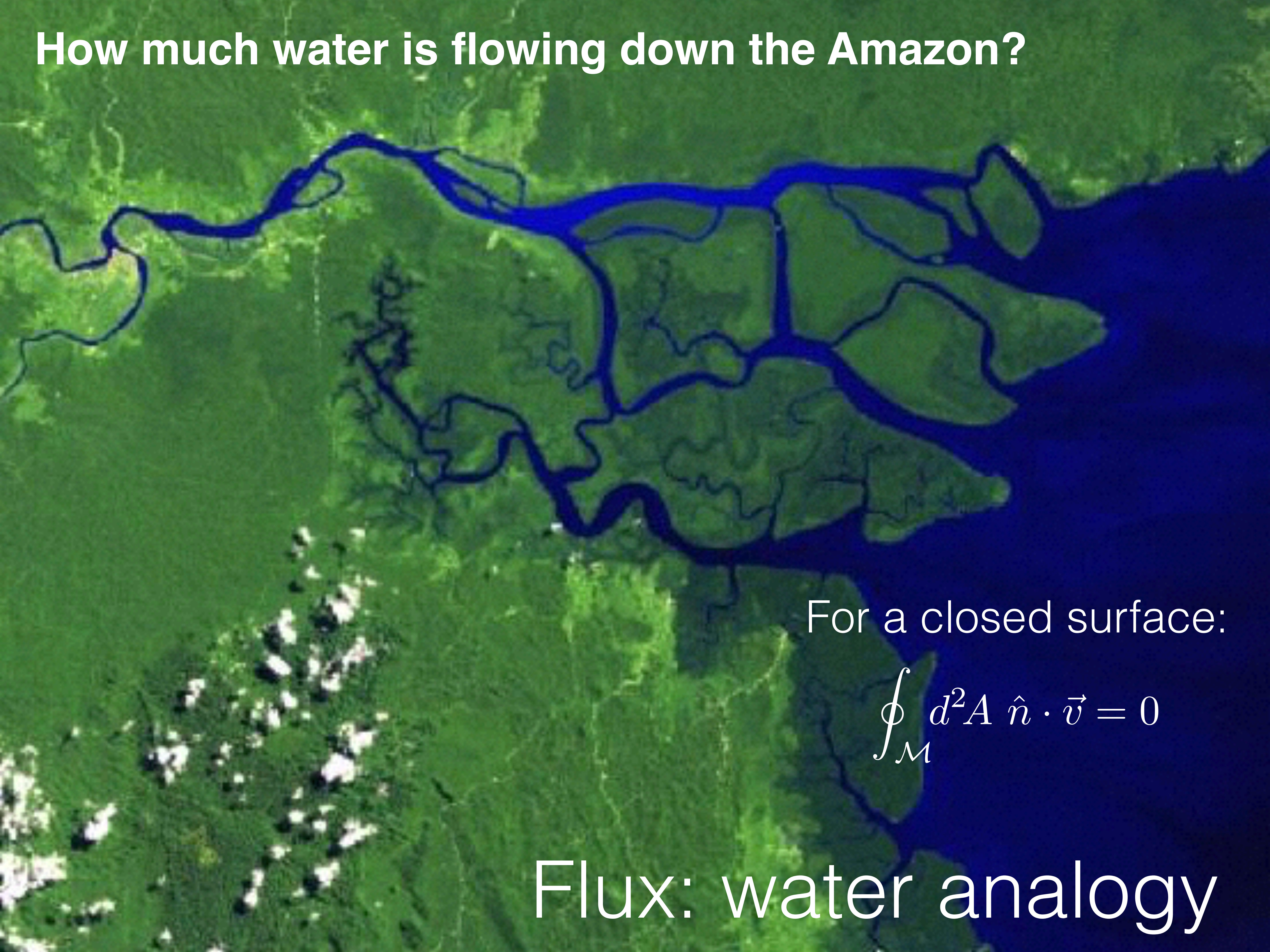
$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

- E Field lines must start (+q) and end (-q) on charges
- Implies flux "conservation"
- Flux measures "outgoingness" of vector field



Flux: water analogy

How much water is flowing down the Amazon?



For a closed surface:

$$\oint_{\mathcal{M}} d^2A \; \hat{n} \cdot \vec{v} = 0$$

Solving Problems using Gauss Law

- Canonical approach (to date):

  1. Solve Differential Equation (GL) to get solution for a point charge (CL)

  2. Use superposition to solve for arbitrary charge distribution

- Solve problems directly from Gauss Law (new)

Gauss Law solution

- **Symmetry is required**
  (ie NOT generally applicable)

$$\oint_{\mathcal{M}} d^2A \ \hat{n} \cdot \vec{E} = Q_{\text{inside}}/\epsilon_0$$

- E is constant (or zero) on surfaces of $\mathcal{M}$ (the gaussian surface)

- $\vec{E}$ is either || or $\perp$ to the surface

(1) Infinite plane/plate

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

(2) Infinite line

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

(3) Point (or sphere)

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

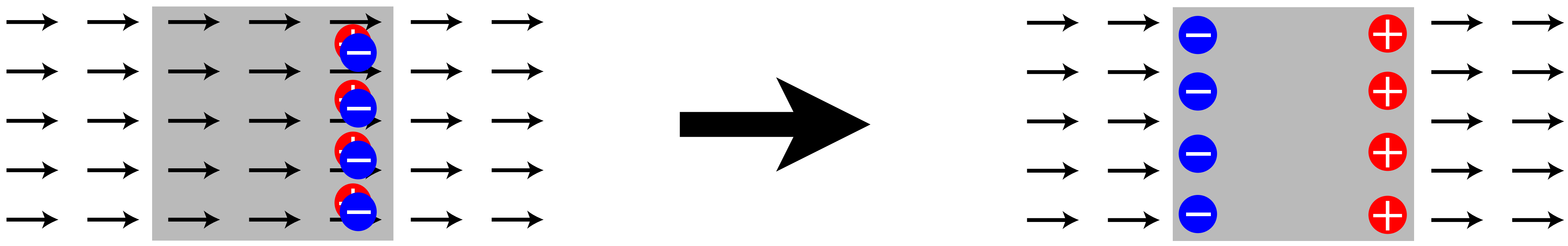
(3B) Charged sphere

1. Identify symmetry
2. Draw E field/field lines
3. Choose a gaussian surface
4. Compute E

Conductors vs insulators

- Conductor (Insulator): Charge is free (not free) to move to establish electrostatic equilibrium

- Conductor have free charge: A sufficient and equal number of + and - charges can be "created" to cancel any E field in the conductor



All free charge is at the surface of conductors

- Use Gauss Law…