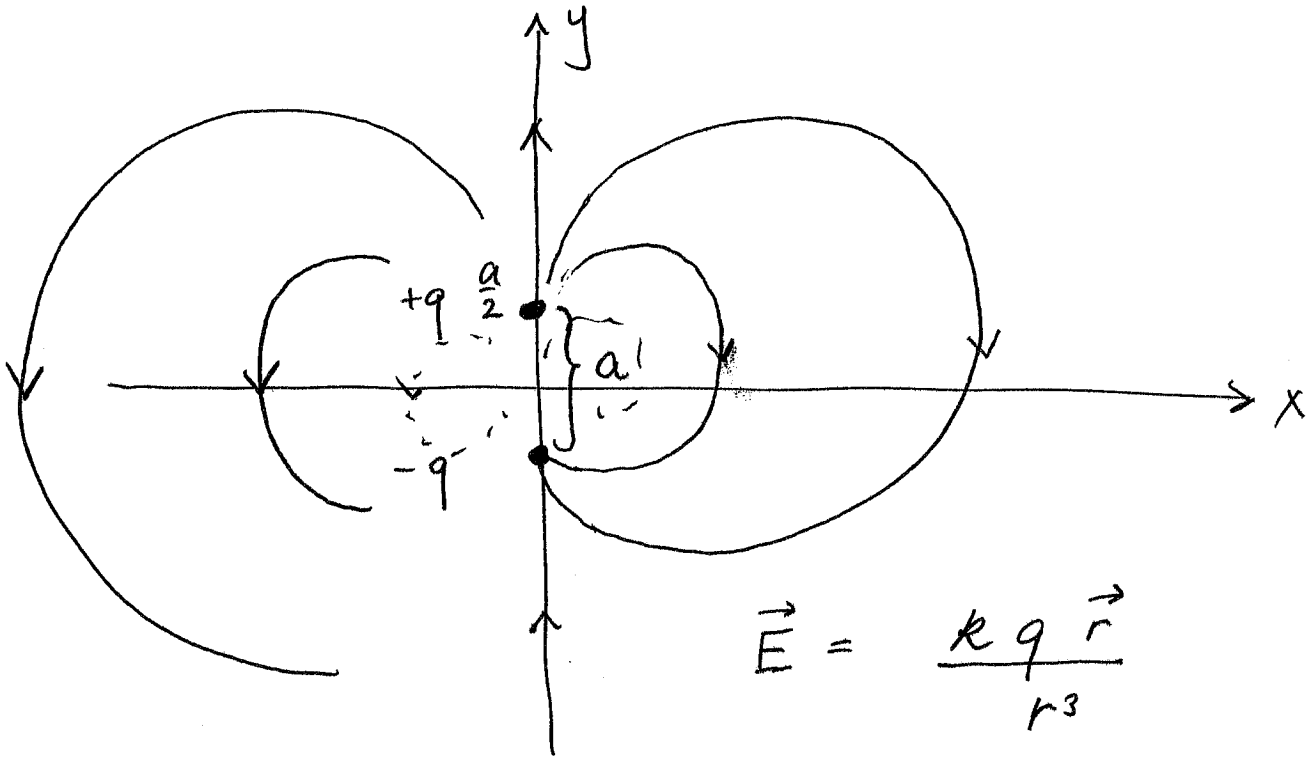
Phys 122 D #3.1

Field from a dipole:



$$\vec{E} = \frac{kq\,\vec{r}}{r^3}$$

$$\vec{E}_{TOT} = kq\left[\frac{\vec{r} - \frac{a}{2}\hat{j}}{\left|\vec{r} - \frac{a}{2}\hat{j}\right|^3} - \frac{\vec{r} + \frac{a}{2}\hat{j}}{\left|\vec{r} + \frac{a}{2}\hat{j}\right|^3}\right]$$

Consider small $a \to 0$:

$$= kq\left[\frac{\vec{r} - \frac{a}{2}\hat{j}}{\left[r^2 - a\hat{j}\cdot\vec{r} + \cdots\right]^{3/2}} - \cdots\right]$$

$$= \frac{kq}{r^3}\left[\frac{\vec{r} - \frac{a}{2}\hat{j}}{\left[1 - a\frac{\hat{j}\cdot\vec{r}}{r^2} + \cdots\right]^{3/2}} - \cdots\right]$$

$$= \frac{kq}{r^3}\left[\left(\vec{r} - \frac{a}{2}\hat{j}\right)\left(1 + \frac{3}{2}\frac{a\hat{j}\cdot\vec{r}}{r^2}\right) - \cdots\right]$$

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$$= \frac{kq}{r^3}\left[\cancel{\vec{r}} - \frac{a}{2}\hat{j} + \frac{3}{2}\frac{a\hat{j}\cdot\vec{r}}{r^2}\vec{r} - \cancel{\vec{r}} - \frac{a}{2}\hat{j} + \frac{3}{2}a\hat{j}\cdot\frac{\vec{r}}{r^2}\vec{r} + \cdots \right]$$

$$= \frac{kqa\hat{j}}{r^3}\cdot\left[-\mathbb{1} + 3\frac{\vec{r}\otimes\vec{r}}{r^2} + \cdots \right]$$

$$= \frac{k}{r^3}\left[3\hat{r}\otimes\hat{r} - \mathbb{1} \right]\cdot\vec{p}$$

$$= \frac{k}{r^3}\left(3\hat{r}(\hat{r}\cdot\vec{p}) - \vec{p} \right)$$