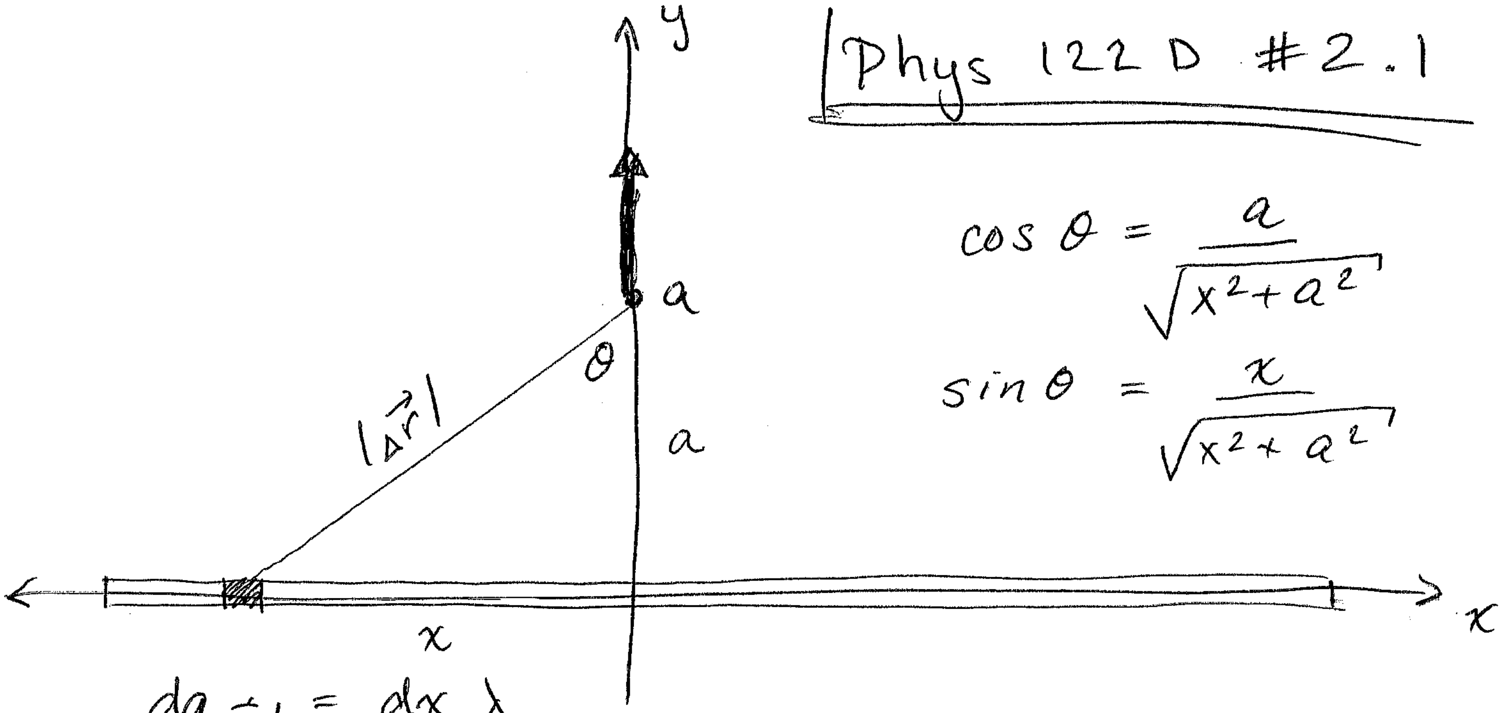
Phys 122 D #2.1



$$\cos\theta = \frac{a}{\sqrt{x^2+a^2}}$$

$$\sin\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$dq_{\vec{r}'} = dx\ \lambda$

↑

linear charge density

$[\lambda] = \frac{C}{m}$ Coulombs per meter

$$\vec{E} = \vec{E}(\underbrace{a\hat{j}}_{\vec{r}}) = k\int_{-\infty}^{\infty} \underbrace{dx\ \lambda}_{dq_{\vec{r}'}} \frac{\overbrace{a\hat{j}+0\hat{i}-x\hat{i}-0\hat{j}}^{\vec{r}-\vec{r}'}}{\underbrace{(a^2+x^2)^{3/2}}_{|\vec{r}-\vec{r}'|^3}}$$

$$= k\lambda\int_{-\infty}^{\infty} dx\ \frac{a\hat{j}-x\hat{i}}{(a^2+x^2)^{3/2}}$$

by symmetry : $\hat{\imath}$ x contribution must be 0.

Dimensional analysis :

get right units.

$$[\vec{E}] = \frac{N}{C} = [\lambda^{\alpha} a^{\beta} k^{\gamma}]$$

$$= \frac{C^{\alpha}}{m^{\alpha}} m^{\beta} \frac{N^{\gamma} m^{2\gamma}}{C^{2\gamma}}$$

$\gamma = 1$ to get N^1

$2\gamma + \beta - \alpha = 0 \Rightarrow \alpha - \beta = 2 \Rightarrow \beta = -1$

$\alpha - 2\gamma = -1 \Rightarrow \alpha = 1$

$$\vec{E} = \frac{k\lambda}{a} \cdot \text{const}$$

doing the integral will give const. (order unity)

Phys 122D #2.3

$$d(\cos\theta) = d\,\frac{a}{\sqrt{x^2+a^2}}$$

$$-\sin\theta\, d\theta = -\frac{\cancel{1}}{\cancel{2}}\,\frac{a\,\cancel{2}x\;dx}{(x^2+a^2)^{3/2}}$$

$$d\theta = \frac{a\,dx}{(x^2+a^2)}$$

$$\vec{E} = k\lambda \int_{-\infty}^{\infty} dx\,\frac{a\hat{j} - \cancelto{0}{x\hat{i}}}{(x^2+a^2)^{3/2}}$$

$$= \frac{k\lambda}{a}\int_{-\pi/2}^{\pi/2} d\theta\,\frac{\hat{j}\,a}{(x^2+a^2)^{1/2}}$$

$$= \frac{k\lambda\hat{j}}{a}\int_{-\pi/2}^{\pi/2} d\theta\,\cos\theta$$

$$= \frac{k\lambda}{a}\hat{j}\,\sin\theta\Big]_{-\pi/2}^{\pi/2}$$

$$\boxed{\vec{E} = \frac{2k\lambda}{a}\hat{j}}$$