Where is my clicker number?

Reading for next time

- 22-1
- 22-2

Other business…

- Number of midterms = 2
- Server down on Sunday due to power outage in physics
- Should be back up soon

FlipIt Physics

- Lots of questions about **flux**…
    - We will deal with this **next time**. I haven't forgotten the questions.
- We will talk today about **field lines** and **forces on charges**.

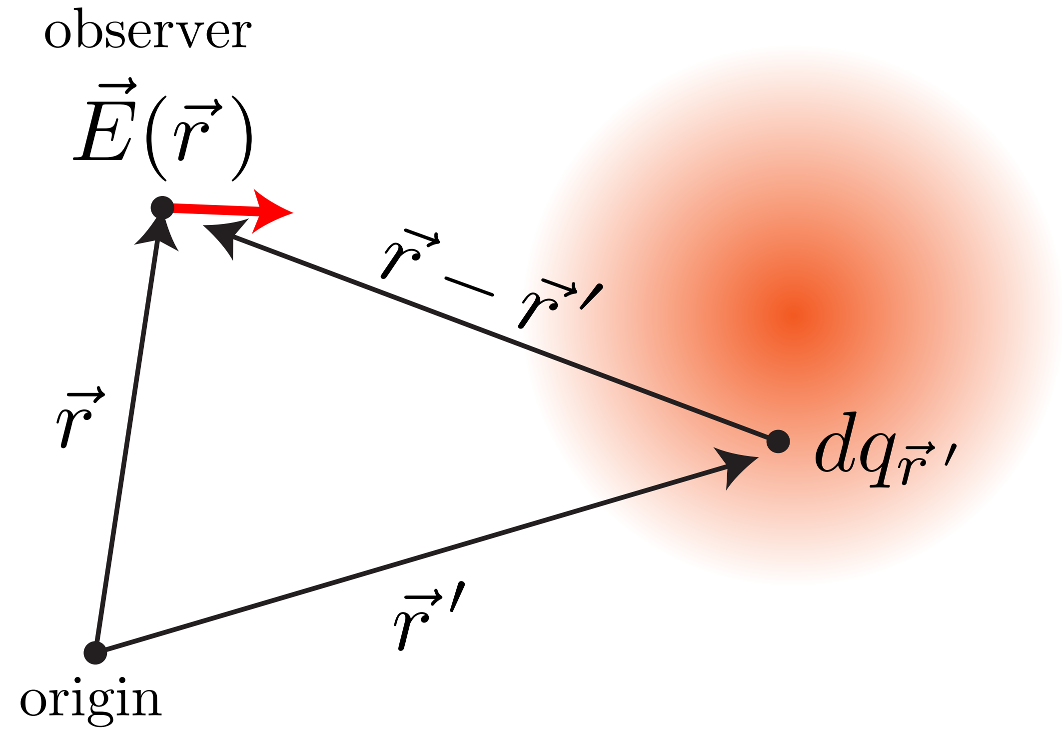
E field from a charge distribution

- **Electric Field** from a collection of charges:

$$\vec{E}(\vec{r}_0) = \sum_{j\neq 0} \frac{k\ q_j}{r_{j,0}^2}\hat{r}_{j,0}$$

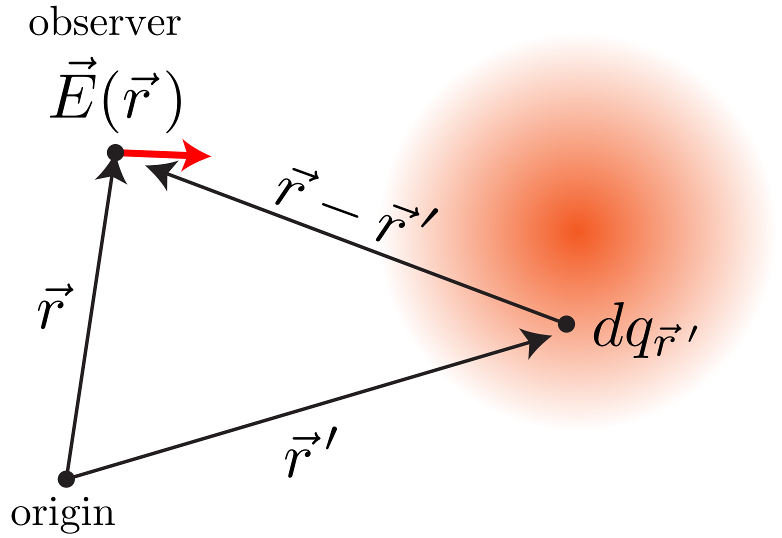
- **Electric Field** from a continuous charge distribution:

$$\vec{E}(\vec{r}) = k \int dq_{\vec{r}\,'}\ \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$



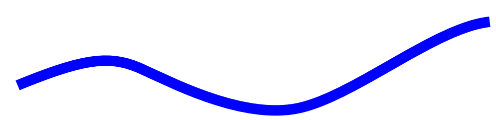
E field from a charge distribution

$$\vec{E}(\vec{r}) = k \int dq_{\vec{r}\,'} \, \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$



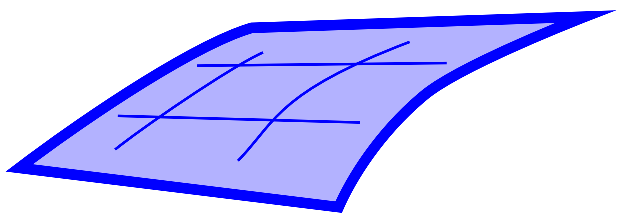
- **Electric Field** from a 1D source: [λ] = C/m^1

$$dq_{\vec{r}\,'} \to d\ell \, \lambda(\vec{r}\,')$$



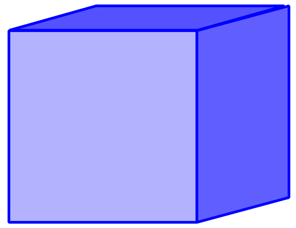
- **Electric Field** from a 2D source: [σ] = C/m^2

$$dq_{\vec{r}\,'} \to d^2x \, \sigma(\vec{r}\,')$$



- **Electric Field** from a 3D source: [ρ] = C/m^3

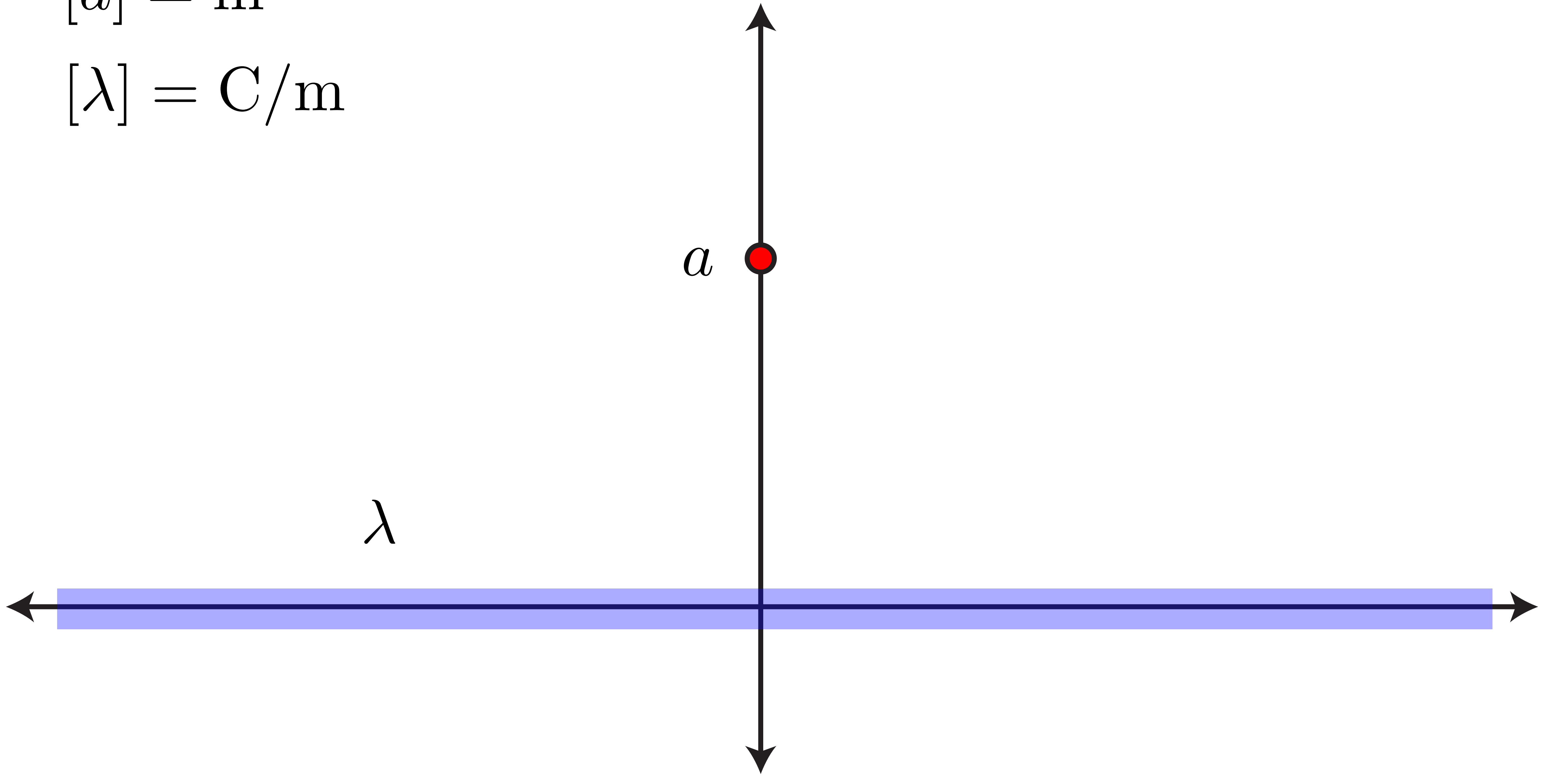
$$dq_{\vec{r}\,'} \to d^3x \, \rho(\vec{r}\,')$$



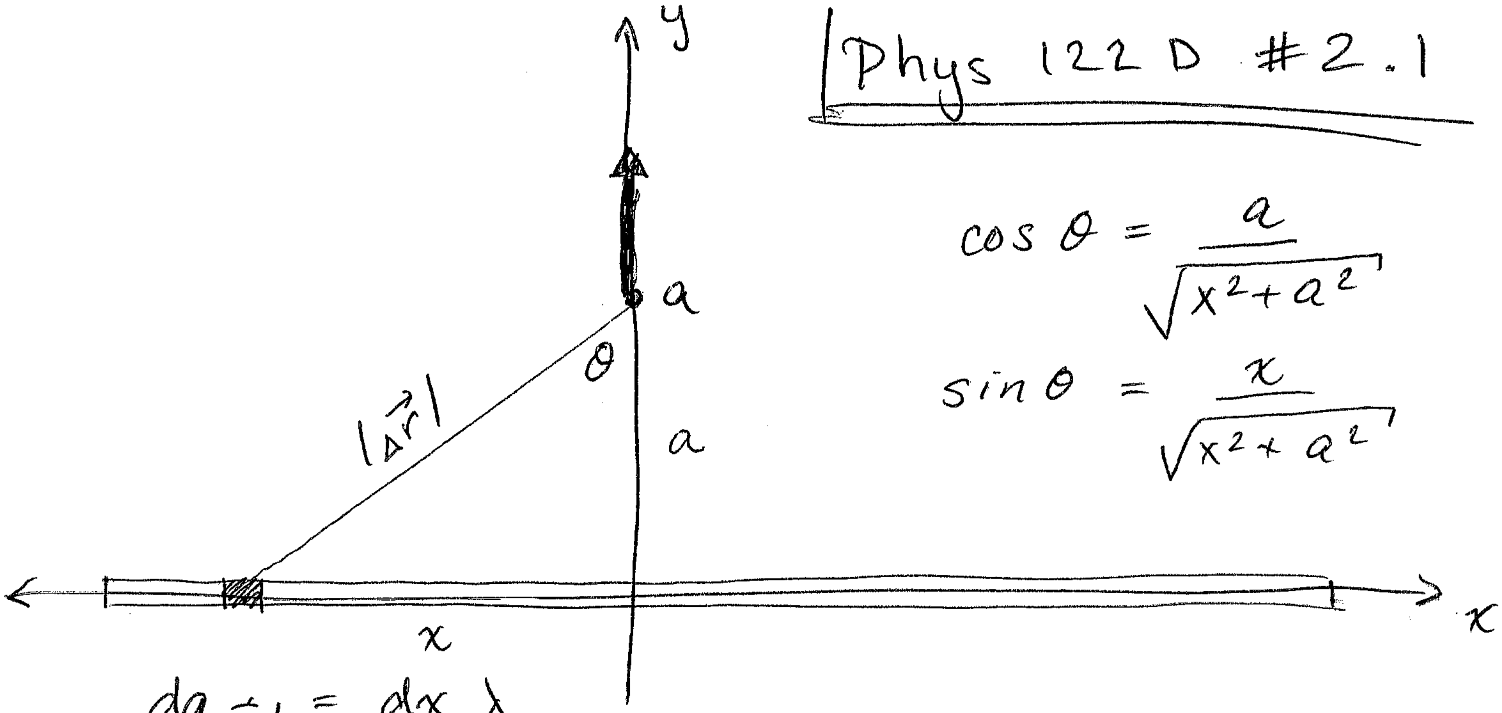
E field from a line charge

$[a] = \mathrm{m}$

$[\lambda] = \mathrm{C/m}$



Phys 122D #2.1



$$\cos\theta = \frac{a}{\sqrt{x^2+a^2}}$$

$$\sin\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$dq_{\vec{r}'} = dx\,\lambda$

↑ linear charge density

$[\lambda] = \frac{C}{m}$ Coulombs per meter

$$\vec{E} = \vec{E}(\underbrace{a\hat{j}}_{\vec{r}}) = k\int_{-\infty}^{\infty} \underbrace{dx\,\lambda}_{dq_{\vec{r}'}} \frac{\overbrace{a\hat{j}+0\hat{i}-x\hat{i}-0\hat{j}}^{\vec{r}-\vec{r}'}}{\underbrace{(a^2+x^2)^{3/2}}_{|\vec{r}-\vec{r}'|^3}}$$

$$= k\lambda\int_{-\infty}^{\infty} dx\,\frac{a\hat{j}-x\hat{i}}{(a^2+x^2)^{3/2}}$$

by symmetry: $\hat{i}$ x contribution must be 0.

Dimensional analysis:

get right units.

$$[\vec{E}] = \frac{N}{C} = [\lambda^\alpha a^\beta k^\gamma]$$

$$= \frac{C^\alpha}{m^\alpha} m^\beta \frac{N^\gamma m^{2\gamma}}{C^{2\gamma}}$$

$\gamma = 1$ to get N^1

$2\gamma + \beta - \alpha = 0 \Rightarrow \alpha - \beta = 2 \Rightarrow \beta = -1$

$\alpha - 2\gamma = -1 \Rightarrow \alpha = 1$

$$\vec{E} = \frac{k\lambda}{a} \cdot \text{const}$$

↑ doing the integral will give const. (order unity)

Phys 122D #2.3

$$d(\cos\theta) = d\,\frac{a}{\sqrt{x^2+a^2}}$$

$$-\sin\theta\, d\theta = -\frac{1}{2}\,\frac{a\,2x\,dx}{(x^2+a^2)^{3/2}}$$

$$d\theta = \frac{a\,dx}{(x^2+a^2)}$$

$$\vec{E} = k\lambda \int_{-\infty}^{\infty} dx\, \frac{a\hat{j} - x\hat{i}}{(x^2+a^2)^{3/2}}$$

$$= \frac{k\lambda}{a} \int_{-\pi/2}^{\pi/2} d\theta\, \frac{\hat{j}\, a}{(x^2+a^2)^{1/2}}$$

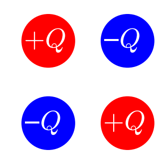
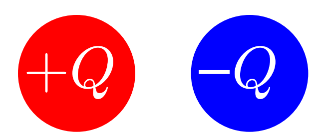
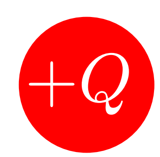
$$= \frac{k\lambda\hat{j}}{a} \int_{-\pi/2}^{\pi/2} d\theta\, \cos\theta$$

$$= \frac{k\lambda\hat{j}}{a} \sin\theta \Big]_{-\pi/2}^{\pi/2}$$

$$\boxed{\vec{E} = \frac{2k\lambda}{a}\hat{j}}$$

Clicker

Field lines more examples…

Charge configuration	**Symbol**	**Illustration**	**Asymptotic field**
quadrupole	Q_{ij}		$\propto r^{-4}$
dipole	p_i		$\propto r^{-3}$
point charge	q		$\propto r^{-2}$
line charge	λ		$\propto r^{-1}$
plane charge	σ		$\propto r^{0}$
…			

Work the integral…

Compute the force on a charge…

- Use the definition of the Electric field:

$$\vec{E} \equiv \frac{\vec{F}_{\mathrm{Net},0}}{q_0}$$

$$\vec{F}_{\mathrm{Net},0} = q_0 \vec{E}$$