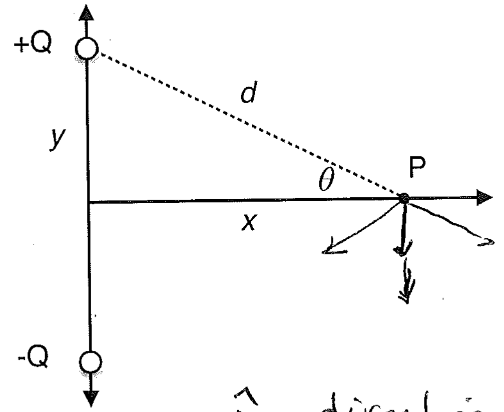
Name Gauss Carl F. (last, first) Student ID 3.1415926 Score 100

Part I [43 points]. Answer on your scantron sheet.

For the next **two questions**, consider the charge configuration shown to the right: Two equal and opposite charges are placed at equal distances ±*y* above the *x* axis.



1. [4 points] What is the **electric field** at point *P* on the *x* axis?

   A. $E = 0$

   B. $E = \frac{2kQ}{x^2}$, in +x direction

   C. $E = \frac{2kQ}{d^2}$, in -y direction

   D. $E = \frac{2kQ\cos\theta}{d^2}$, in +x direction

   E. $E = \frac{2kQ\sin\theta}{d^2}$, in -y direction

$-\hat{y}$ direction ✓

(I): $|E| = 2\times \frac{\sin\theta\, kQ}{d^2}$ ✓

(II) $\vec{E} = \frac{Qk(x\hat{x} - y\hat{y})}{d^3} - \frac{Qk(x\hat{x}+y\hat{y})}{d^3} = -\frac{2Qky}{d^3}\hat{y} = -\frac{2Qk\sin\theta}{d^2}$ ✓

2. [3 points] What is the **electric potential** at point P (relative to infinity)?

   A. $V = 0$

   B. $V = \frac{2kQ}{d}\cos\theta$

   C. $V = \frac{2kQ}{d}\sin\theta$

   D. $V = -\frac{2kQ}{d}$

   E. $V = \frac{2kQ}{d}$

$(+Q - Q)\frac{k}{d} = 0$

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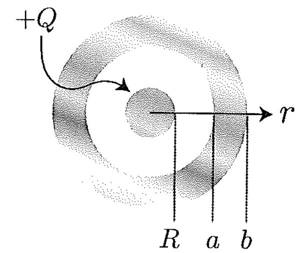
Consider the following charge configuration to answer the **next three questions**: A solid metal sphere of radius R has net charge $+Q$. It is surrounded by a thick concentric metal shell of inner radius a and outer radius b as shown. The shell has zero net charge. [Assume $V(\infty) = 0$.]



3.[4 points] Which choice best represents the electric potential on the outer surface of the shell $V(b)$?

A. $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ **B.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$ **C.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$ **D.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{Q}{a} + \frac{Q}{b}\right)$ **E.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{Q}{a} + \frac{Q}{b}\right)$

Charge enclosed + Gauss Law

4. [3 points] What is the potential on the sphere $V(R)$?

A. $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ **B.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$ **C.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$ **D.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{Q}{a} + \frac{Q}{b}\right)$ **E.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{Q}{a} + \frac{Q}{b}\right)$

$V(R)$

$V(R) - V(a)$

$V(a) - V(b) = 0$

5. [2 points] What is the potential at the center of the spherical conductor $V(0)$?

A. $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ **B.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$ **C.** $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$ **D.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{Q}{a} + \frac{Q}{b}\right)$ **E.** $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{Q}{a} + \frac{Q}{b}\right)$

$V(0) - V(R) = 0$

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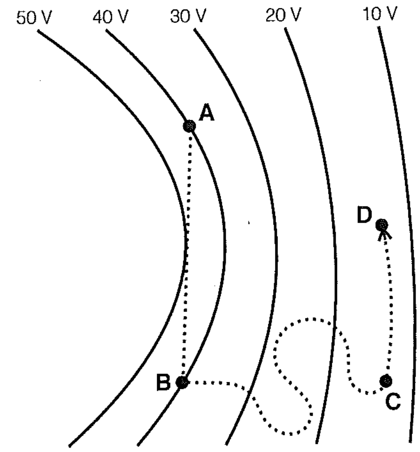
6. [3 points] A test charge is moved through a potential field, starting at point **A** and continuing to point **D** as shown at right. Which of the following statements are true about the potential difference between each segment of the move? [V_A is the potential at point **A**, and ΔV_{AB} is the potential from point **A** to point **B**.]



A. $\Delta V_{AB} > \Delta V_{CD}$; $V_A > V_D$

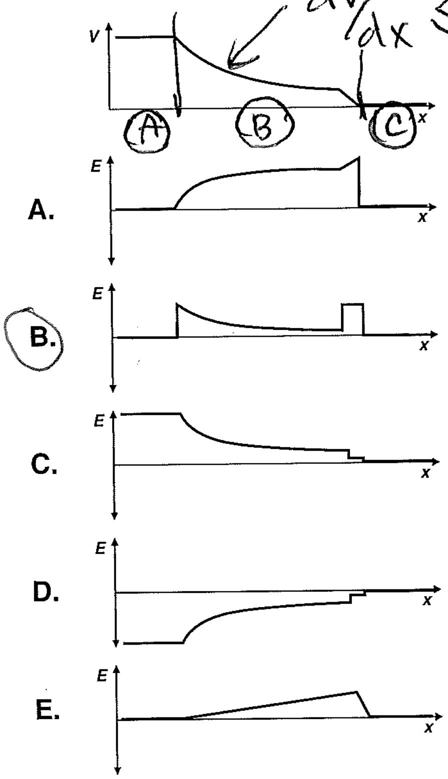
B. $\Delta V_{AB} = 0$; $V_B = V_D$

C. $\Delta V_{BC} = 0$; $V_A > V_C$

D. $V_A > V_B$; $V_C < V_D$

E. $\Delta V_{AD} = \Delta V_{BC}$; $V_A > V_D$

7. [3 points] A plot of the potential is shown in the first figure below. Choose which plot of the **Electric field** corresponds to this potential. (Hint: Where is the E field zero and where a maximum?)



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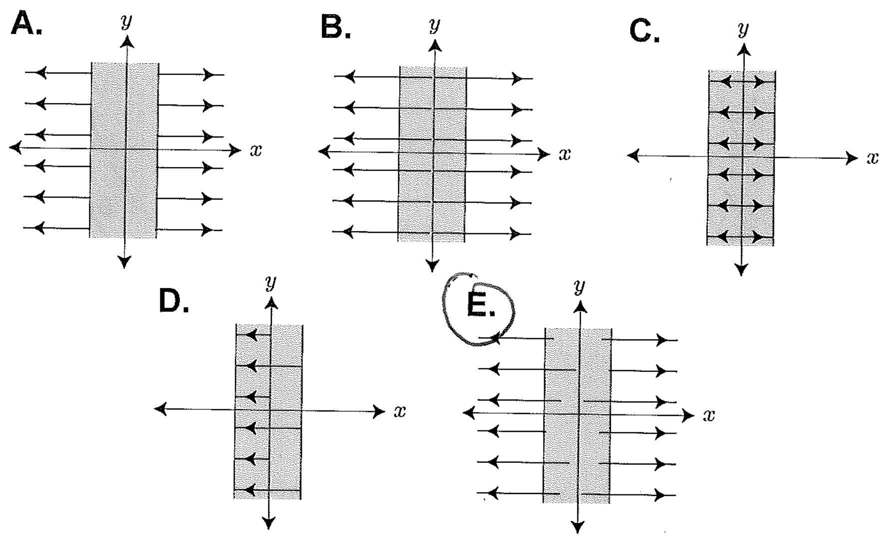
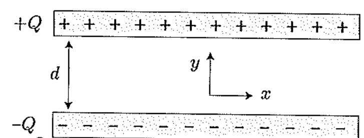
8. [4 points] Consider a large **insulating slab of uniform volume-charge density** ρ > 0, centered on the axis, with infinite extent in the y and z directions and finite width L in the x direction. Which **electric field line sketch** best approximates the field lines generated by the charge distribution?



9. [3 points] What is the **force** on the bottom plate of a large parallel-plate capacitor with capacitance C, area A, plate spacing d and charge Q? (Hint: Does the E field generated by the bottom plate generate a net force on itself?)



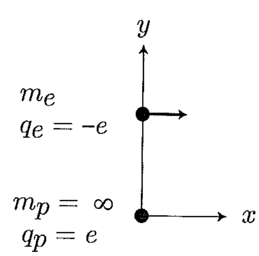
(I) $U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\epsilon_0 A}$ $F = \frac{dU}{dd} = \frac{Q^2}{2Cd}$ ✓

or

(II) $A\sigma = Q$ $\vec{F} = Q\vec{E}$ from top $= Q\frac{\sigma}{2\epsilon_0} \cdot \frac{A}{A} \cdot \frac{d}{d} = \frac{Q^2}{2Cd}$ ✓

A. $F = \frac{Q^2}{A\epsilon_0}$, **B.** $F = \frac{Q^2}{2Cd}$, **C.** $F = \frac{Q}{A\epsilon_0}$, **D.** $F = \frac{AQ}{d}$, **E.** $F = \frac{Q^2}{Cd}$,

10. [3 points] At $t = 0$ s, an electron at $\vec{r} = 10^{-10}\,\hat{y}$ m, has initial velocity $\vec{v} = 3\times10^6\,\hat{x}$ m/s. A proton sits at the origin. What is the magnitude of the **velocity** of the electron in the long time limit. (Hint: (i) Conservation of energy. (ii) Ignore the motion of the proton due to its large mass compared with the mass of the electron m_e. (iii) Formula sheet.)



A. $v = 3\times10^6$ m/s,
B. $v = 2\times10^6$ m/s,
C. $v = 6\times10^6$ m/s,
D. $v = 0$,
E. No well-defined limit since the electron is bound.

$K = \frac{1}{2}m_e v^2$

$U = -\frac{ke^2}{r}$

$E = K + U > 0$ (plugged in #'s)

not bound ⇒ goes to ∞

$U(\infty) = 0$

$\frac{1}{2}m_e v_f^2 = \frac{1}{2}m_e v^2 - \frac{ke^2}{r}$ $v_f = \left[v^2 - \frac{2ke^2}{m_e r}\right]$

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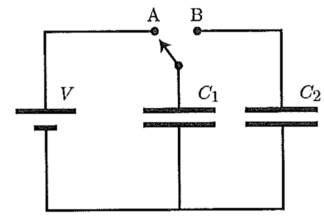
For the next **three questions**, consider the following experiment: All capacitors start uncharged. Capacitor 1 with capacitance C_1 is charged to V using a battery. Once capacitor 1 is charged, it is disconnected from the battery and then used to charge capacitor 2 with capacitance C_2. (I.e. the switch is connected first to terminal A and then to terminal B.)



11. [2 points] When the switch is initially in the A position, what is the charge on capacitor 1?

just def of C_1

A. $Q_1 = C_1V$, **B.** $Q_1 = C_2V$, **C.** $Q_1 = 0$, **D.** $Q_1 = (C_1 + C_2)V$, **E.** $Q_1 = \frac{C_1C_2}{C_1+C_2}V$,

12. [2 points] When the switch is in the B position, what is the total charge on both capacitors Q?

Charge has no where to go $\Rightarrow$ $Q_{TOT} = \text{const}$
$C_1V = Q_1 = Q_{TOT}$

A. $Q = C_1V$, **B.** $Q = C_2V$, **C.** $Q = 0$, **D.** $Q = (C_1 + C_2)V$, **E.** $Q = \frac{C_1C_2}{C_1+C_2}V$,

13. [3 points] When the switch is in the B position, which of the following is true?

since Cap's are ||.

A. $Q_1 = Q_2$ **B.** $V_1 = V_2$, **C.** $Q_1 = 0$, **D.** $Q_1 = (C_1 + C_2)V$, **E.** $Q_1 = \frac{C_1C_2}{C_1+C_2}V$,

14. [4 points] What is the **final charge** on capacitor 1?

$V_1 = V_2$

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$\frac{Q_1}{C_1} = \frac{Q - Q_1}{C_2}$

$\frac{C_2 Q_1}{C_1} = C_1V - Q_1$

$\frac{(C_2 + C_1)}{C_1} Q_1 = C_1V$

$Q_1 = \frac{C_1^2}{C_1 + C_2} V$ ✓

A. $Q_1 = \frac{(C_2)^2}{C_1+C_2}V$, **B.** $Q_1 = \frac{C_1+C_2}{C_1C_2}V$, **C.** $Q_1 = \frac{(C_1)^2}{C_1+C_2}V$, **D.** $Q_1 = 0$, **E.** $Q_1 = \frac{C_1C_2}{C_1+C_2}V$,

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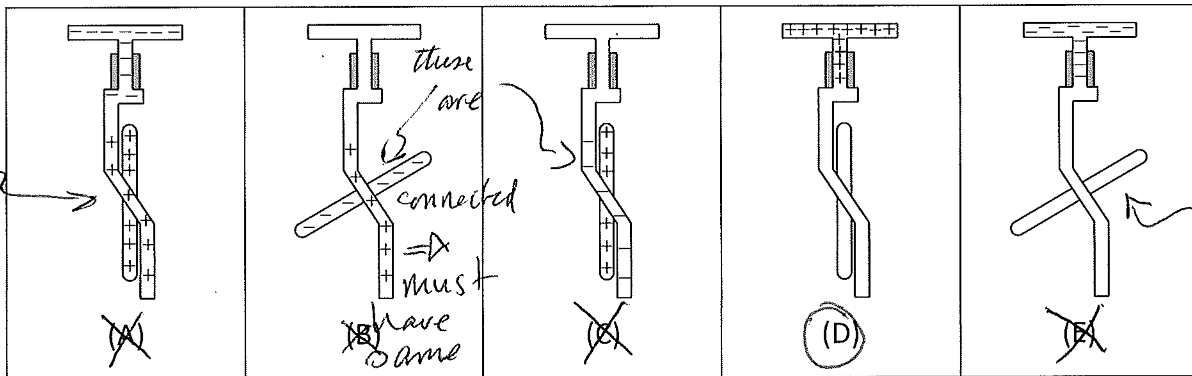
II. Lab questions [12 pts] ANSWER THESE ON YOUR SCANTRON SHEET.

6. [6 pts] Below are diagrams showing an electroscope (with possibly non-zero charge) along with charge distributions drawn on it. ***Other charged objects may be near the electroscope but are not shown.*** Only one diagram shows a **correct** vane position plus charge distribution. Which one? Look carefully, the differences are subtle.



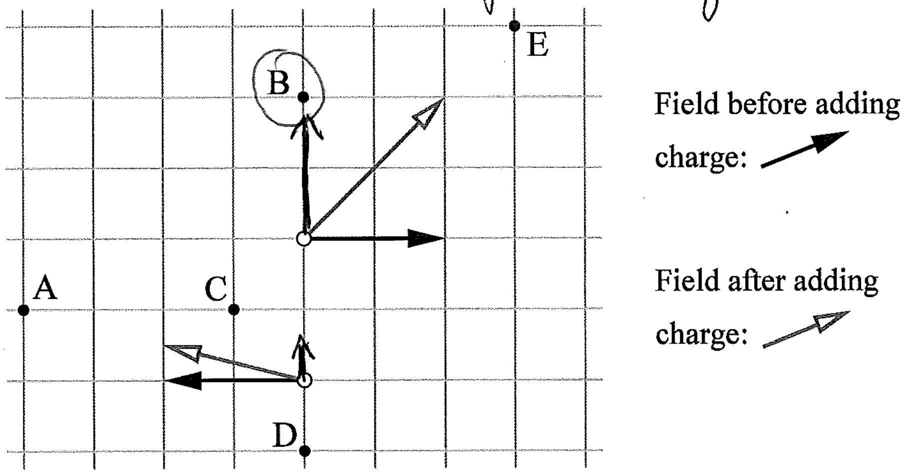
These would repel

these are connected ⇒ must have same charge density

This would not repel

7. [6 pts] Some charge **which is not shown** is put in the plane below, and the electric field is measured at the two points designated by the open circles. The field at these points is shown by the black vectors. **Then one more charge is added** (without changing the original charge distribution) at **one** of the labeled points, and the field is measured again as shown by the gray vectors. At which point is the extra charge added?

Direction + magnitude of $\vec{E}$ ⇒ $q < 0$ on B.



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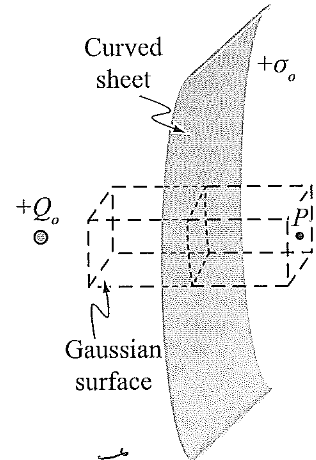
IV. [20 points total] A Gaussian surface in the shape of a rectangular prism is centered in the middle of a very large curved sheet as shown at right. The curved sheet has a uniform charge density $+\sigma_o$. Only a portion of the sheet is shown. Point P is at the center of the right end-cap of the Gaussian surface.



A positive point charge $+Q_o$ is placed on the left side of the sheet as shown.

A. [5 pts] When $+Q_o$ is added, does the magnitude of the net electric field at point P *increase, decrease,* or *remain the same?* Explain.

Principle of super position: $\hat{r} \approx \hat{x}$

$$\Delta \vec{E} = \vec{E}_{Q_0} = \frac{Q_0}{4\pi\epsilon_0 r^2}\hat{r}$$

(E from Q_0)

original field $\vec{E}_0 \approx \frac{\sigma}{2\epsilon_0}\hat{x}$ since fields $\vec{E}_0$ & $\Delta\vec{E}$ are in same direction, magnitude increases//.

B. [5 pts] When $+Q_o$ is added, does the electric flux through the right end-cap of the Gaussian surface *increase, decrease,* or *remain the same?* Explain your reasoning.

increases. $\vec{E}_0 + \vec{E}_{Net}$ are roughly constant on end-cap and $\hat{n} \approx \hat{x}$ ⇒ $\Phi_{End\ Cap} > 0$ & $\Delta\Phi_{End\ cap} > 0$ since $\Delta\Phi_{End\ Cap} \propto |\Delta\vec{E}|$

($\hat{n}$: surface normal)

C. [6 pts] Is it possible to use Gauss' law to find the magnitude of the net electric field at point P? If so, describe the steps required to find the magnitude of the net electric field. If not, explain why it is not possible.

No. No symmetry. $\vec{E}\cdot\hat{n}$ is neither 0 nor constant on all surfaces.

$$\Rightarrow \Phi_M \neq A|\vec{E}_P|$$

(A: end cap area.)

D. [4 pts] Suppose the electric field at point P were measured using a positive test charge $+q_{test}$, then measured again using a negative test charge $-2q_{test}$.

Would the magnitude of the net electric field acting on $-2q_{test}$ be *greater than, less than,* or *equal to* that acting on $+q_{test}$? Explain your reasoning. If the direction of the electric field is affected, describe how it is affected.

Electric field does not depend on the charge of the test charge. ie $\vec{E}_M \equiv \frac{\vec{F}}{q_0} = \frac{q_0\vec{E}}{q_0}$ ("measured"; q_0 cancels.)

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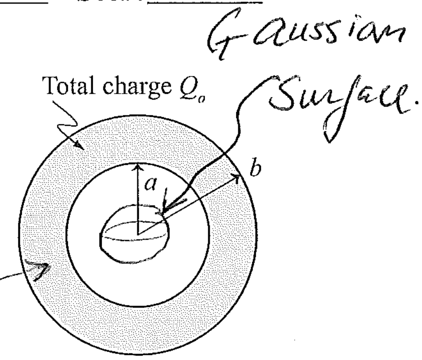
III. [25 points total] A non-conducting, thick spherical shell with inner radius a and outer radius b has a total charge Q_o spread uniformly throughout its volume.



Cross-sectional view

A. In terms of variables given and fundamental constants, determine the **magnitude of the electric field** in the following regions:

i. [4 pts] For $0 < r < a$. Show your work.

$$\rho = \frac{3Q}{4\pi(b^3-a^3)}$$

Gauss Law:

$$\Phi = 4\pi r^2 E = \frac{Q_{ins}}{\epsilon_0} = 0 \Rightarrow E(r) = 0, \quad 0<r<a$$

ii. [8 pts] For $a < r < b$. Show your work.

Gauss Law:

$$4\pi r^2 E = \frac{Q_{ins}}{\epsilon_0} \Rightarrow E = \frac{Q_{ins}}{4\pi\epsilon_0 r^2} = \frac{\rho V(r)}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r^2}\left(\frac{r^3-a^3}{b^3-a^3}\right)$$

(volume)

iii. [5 pts] For $b < r$. Show your work.

Gauss Law:

$$E = \frac{Q_{ins}}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$$

B. [8 pts] In the space below, sketch a qualitatively correct plot of the magnitude of the electric field for the charged sphere.