Equations for Physics 122 Midterm 1:

Constants:

$$k = \tfrac{1}{4\pi\epsilon_0},$$
$$k = 8.99 \times 10^9 \ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}},$$
$$\epsilon_0 = \tfrac{1}{4\pi k},$$
$$\epsilon_0 = 8.85 \times 10^{-12} \tfrac{\mathrm{C^2}}{\mathrm{N\cdot m^2}},$$
$$e = 1.60 \times 10^{-19}\ \mathrm{C},$$
$$m_e = 9.1 \times 10^{-31}\ \mathrm{kg}$$

Charge densities & dipole moment:

$$\sigma = \tfrac{Q}{A},$$
$$\lambda = \tfrac{Q}{L},$$
$$\rho = \tfrac{Q}{v},$$
$$\vec{p} = q\vec{L}$$

Force and torque:

$$\vec{F} = q\vec{E},$$
$$\vec{\tau} = \vec{p} \times \vec{E},$$
$$\vec{F} = \vec{p} \cdot \vec{\nabla}\vec{E},$$

Electric Field:

$$\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}),$$
$$\vec{E}(\vec{r}) = \frac{q\hat{r}}{4\pi\epsilon_0 r^2},$$
$$\vec{E}(\vec{r}) = \sum_i \frac{q_i \hat{r}_i}{4\pi\epsilon_0 r_i^2},$$
$$\vec{E}(\vec{r}) = \int dq_{\vec{r}'} \frac{\vec{r} - \vec{r}'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3},$$
$$\vec{E}(\vec{r}) = \frac{\lambda\,\hat{r}}{2\pi\epsilon_0 r},$$
$$\vec{E}(\vec{r}) = \frac{\sigma\,\hat{n}}{2\epsilon_0},$$
$$\vec{E}(\vec{r}) = \frac{\sigma\,\hat{n}}{\epsilon_0},$$
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\hat{r}\cdot\vec{p})\hat{r} - \vec{p}\right],$$

Electric flux and Gauss Law:

$$\Phi_{\mathcal{M}} \equiv \oint_{\mathcal{M}} d^2A\,\hat{n}\cdot\vec{E}(\vec{r}),$$
$$= \frac{Q_{\mathrm{ins}}}{\epsilon_0},$$
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0},$$

Electric Potential:

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \mathrm{d}\vec{\ell}'\cdot\vec{E}(\vec{r}'),$$
$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0 r},$$
$$V(\vec{r}) = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i},$$
$$V(\vec{r}) = \int dq_{\vec{r}'} \frac{1}{4\pi\epsilon_0|\vec{r} - \vec{r}'|},$$
$$V(\vec{r}) = -\frac{\lambda}{2\pi\epsilon_0}\log r,$$
$$V(\vec{r}) = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$$

Energy and work:

$$\mathrm{d}U = \mathrm{d}Q\,V,$$
$$dW = \mathrm{d}\vec{\ell}\cdot\vec{F},$$
$$W = -\Delta U,$$
$$U = \tfrac{1}{2}\sum_{i\neq j} \tfrac{q_i q_j}{4\pi\epsilon_0 r_{ij}},$$
$$U = -\vec{p}\cdot\vec{E},$$
$$K = \tfrac{1}{2}mv^2,$$

Capacitance:

$$Q = CV,$$
$$C = \tfrac{\epsilon_0 A}{\ell},$$
$$C = \tfrac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1},$$
$$C = \tfrac{2\pi\epsilon_0 L}{\log\frac{R_2}{R1}},$$
$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \tfrac{1}{2C}Q^2,$$
$$C_{\mathrm{eq}} = C_1 + C_2,$$
$$C_{\mathrm{eq}}^{-1} = C_1^{-1} + C_2^{-1},$$

Energy density:

$$u = \tfrac{1}{2}\epsilon_0 E^2,$$

Differential geometry:

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z},$$
$$\equiv \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial y} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi},$$
$$\equiv \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z},$$

$$\mathrm{d}\vec{\ell} = \hat{x}\,\mathrm{d}x + \hat{y}\,\mathrm{d}y + \hat{z}\,\mathrm{d}z,$$
$$\mathrm{d}\vec{\ell} = \hat{r}\,\mathrm{d}r + \hat{\theta}\,r\mathrm{d}\theta + \hat{\phi}\,r\,\sin\theta\,\mathrm{d}\phi,$$
$$\mathrm{d}\vec{\ell} = \hat{r}\,\mathrm{d}r + \hat{\phi}\,r\mathrm{d}\phi + \hat{z}\,\mathrm{d}z,$$
$$\mathrm{d}^2A = r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi,$$
$$\mathrm{d}^2A = r\,\mathrm{d}\theta\,\mathrm{d}z,$$
$$\mathrm{d}^3v = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi,$$
$$\mathrm{d}^3v = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$$

Geometry:

$$A = 4\pi R^2,$$
$$v = \tfrac{4}{3}\pi R^3,$$
$$A = 2\pi RL,$$
$$v = \pi R^2 L$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$