1 Multiple Choice Solutions: Winter 2015 122 Exam 3

1.1 Problem 1

We know that the potential energy of particle at A from the electric potential between A and B is equal to $U = -q * V$ and as the particle accelerates through the potential it will convert all of that potential energy into the Kinetic Energy it will have upon entering the region where there is a magnetic field. Since U is directly proportional to V in the equation above, we can see that doubling V also doubles U. Therefore the particle will also have twice the Kinetic Energy when entering the magnetic field region. Since $KE = \frac{1}{2}mv^2$ doubling the kinetic energy will only increase the velocity by a factor of $\sqrt{2}$. Now, if we look at our equation sheet we know that the motion of a moving charged particle in a magnetic field follows the relationship: $qvB = \frac{mv^2}{r}$. Rewriting this to find r (which is the radius of the half circle that the particle's path traces), we get $r = \frac{mv}{qB}$. Since m, q, and B remain the same and only v changes by a factor of $\sqrt{2}$ as we found above then the radius r also increases by the same factor of $\sqrt{2}$ and thus since $d = 2r$ it also increases by a factor of $\sqrt{2}$ and we get our answer B.

1.2 Problem 2

Let's start by remembering that $velocity = \frac{\Delta x}{\Delta t}$ and so we can use this formula to find the time taken by the particle with the initial values. Remember that the circumference of a full circle is $\pi * diameter$ so in this case $\Delta t = \frac{\frac{\pi}{2}d}{v}$. Now using the formula we found in problem 1 for the radius of the semicircle we see that doubling the velocity doubles the radius and thus would double the distance travelled. So now we would have $\Delta t_{new} = \frac{\frac{\pi}{2}d_{new}}{v_{new}} = \frac{\frac{\pi}{2}2d}{2v} = \frac{\frac{\pi}{2}d}{v}$ which is the exact same as our initial Δt.

1.3 Problem 3

To solve this we need to find the magnetic force on the lower wire from the magnetic field of the upper wire. So we will have to start by finding the magnetic field of the upper wire with the equation: $B = \frac{\mu_0 I}{4\pi r} * (sin\theta_2 - sin\theta_1)$ where $\theta_2 = \frac{\pi}{2}$ and $\theta_1 = -\frac{\pi}{2}$ so that the term in parentheses becomes 2 and we get $B = \frac{\mu_0 I_{top}}{2\pi r}$. Now using this to solve for the force on the lower wire ($\vec{F} = I\vec{l} \times \vec{B}$) and keeping in mind that the magnetic field of the top wire is into the page using the right hand rule and thus is perpendicular to the current in the bottom wire:

$$F = I_{bottom} * l * B_{top} \tag{1}$$
$$\frac{F}{l} = I_{bottom} * \frac{\mu_0 I_{top}}{2\pi r} \tag{2}$$
$$\frac{F}{l} = (3.4A) * \frac{\mu_0 * 1.5A}{2\pi * 0.003m} \tag{3}$$
$$\frac{F}{l} = 3.4 * 10^{-4} N/m \tag{4}$$

1.4 Problem 4

Using the right hand rule for the magnetic field from a current carrying wire we know that the net magnetic field from the two wires in the plane of the page will point either into or out of the page. Furthermore, we know that the magnetic force on a charged particle is given by the equation $\vec{F} = q\vec{v} \times \vec{B}$ and in this case $\vec{v}$ for the charged particle is into the page. Since both the magnetic field and the velocity of the charged particle are either parallel or antiparallel (we don't even have to calculate the value!) the cross product between them is 0 and thus we have no force from the wires on the charged particle.

1.5 Problem 5

In order for the beam of electrons to not be deflected we need to have a region of space where the forces on the beam from magnetic and electric fields cancel out.

$$F_{magnetic} = F_{electric} \tag{5}$$
$$qvB = qE \tag{6}$$
$$v = \frac{E}{B} \tag{7}$$
$$v = \frac{3.4 * 10^4 V/m}{2.0 * 10^{-3} T} \tag{8}$$
$$v = 1.7 * 10^7 m/s \tag{9}$$

1.6 Problem 6

Induced emf is equal to $-\frac{d\Phi_B}{dt}$ where $\Phi_B = \vec{B} \cdot \vec{A}$ and $\vec{A}$ is the area vector of the loop. For loop 1 (keeping in mind that the area vector points outwards normal to the plane of the loop):

$$\Phi_B = \vec{B} \cdot \vec{A} = B(t) * (b_1 * a) * cos(60 degrees) = B(t) * (2 * b_2 * a * .5) = B(t) * (b_2 * a) \tag{10}$$

For loop 2:

$$\Phi_B = \vec{B} \cdot \vec{A} = B(t) * (b_2 * a) \tag{11}$$

Therefore we see that they have the same magnetic flux and since B(t) will be the same for both of them $-\frac{d\Phi_B}{dt}$ will be the same for both of them meaning the induced emf will be the same as well.

1.7 Problem 7

To find the induced emf we will use all the equations we already used above in problem 6, but this time we will insert values from the problem.

$$\Phi_{B_2} = \vec{B} \cdot \vec{A_2} = B(t) * (b_2 * a) \tag{12}$$

$$\Phi_{B_2} = (2mT) * exp(-t/3\mu s) * (.01m * .025m) \tag{13}$$

$$\epsilon(t = 6\mu s) = -\frac{d\Phi_{B_2}}{dt} = (-2mT) * (-\frac{1}{3\mu s}) * exp(-6\mu s/3\mu s) * (.00025m^2) \tag{14}$$

$$\epsilon(t = 6\mu s) = 22.56mV \tag{15}$$

We now have the induced emf, but we want the induced current so we merely divide ϵ by our 2Ω resistance to get 11.28mA.

1.8 Problem 8

As the magnet falls with its north end pointing downward, the magnetic field is directed downward through the loop and increasing the closer the magnet gets to the loop. That means that there will be an induced magnetic field opposing the increase in the magnetic field through the loop due to the magnet and thus the induced field is in the upward direction. To connect this to the current, we use the right hand rule for a current loop and get that the induced current will be positive with the arrows we are given. Now, after the magnet passes halfway through the loop, the southern end of the magnet is closest to the loop and as it falls away the magnetic field lines are directed downward again, but decreasing in strength. In order to make up for this the induced magnetic field is also in the downward direction this time. Once again using the right hand rule for a current loop we see that this requires and induced current in the negative direction with the arrows shown.

1.9 Problem 9

The potential energy of the current loop is given by $U = -\vec{\mu} \cdot \vec{B}$. Remembering to take into account the negative sign, we can deduce that the potential energy is the most negative (and thus the lowest) when $\vec{B}$ and $\vec{\mu}$ are in the same direction. Using the right hand rule for the current loop we see that when $\alpha = 0$ we get the desired situation where $\vec{\mu}$ of the loop points up in the same direction as $\vec{B}$.

1.10 Problem 10

Remember from Physics 121 that $Work_{net\ external} = \Delta E$. In this case if we rotate the coil from rest at $\alpha = 30$ degrees to $\alpha = 0$ degrees we are changing the energy of the coil by changing the potential energy of the coil and the work that you do is the external work. Using the equation from Problem 9 and the work equation we get

$$U_{30} = -(35A * m^2) * (0.2T) * cos(30degrees) = 6.06J \tag{16}$$

$$U_0 = -35(A * m^2) * (0.2T) = 7 \tag{17}$$

$$W_{net\ ext} = \Delta E = \Delta U = U_{30} - U_0 = (6.06 - 7)J \tag{18}$$

$$W_{net\ ext} = -0.94J \tag{19}$$

1.11 Problem 11

Since we know that the voltage drop across parallel branches are the same then we know that the voltage drop across the middle branch with the ?? and the voltage drop across the right branch with three resistors will be the same. We are told that the voltage drop across one of the resistors in the right branch is 1V. Since the resistors are in series we know that the same current passes through each of them. We also know that each resistor is identical. Thus, using Ohm's Law, V=IR, we know that the voltage drop across each of the resistors must also be identical and since they are in series we can add up the voltage drops to find the total voltage drop across that branch. This gives us 3V. Since the middle branch is parallel to the right branch it must have the same voltage drop of 3V.

1.12 Problem 12

We want to find the voltage and current through X and only X at any given time. Therefore, we need to remember how an ideal ammeter and an ideal voltmeter act. An ammeter acts as if it has 0 resistance and we place it in series with the object whose current we are trying to measure. In the circuits pictured the ammeter is always in series with object X. Therefore, we need to look at the placement of the voltmeter. We want to place the voltmeter in parallel with the voltage drop we are trying to measure. As a result, we can look at the circuits and immediately rule out 1 and 3 since the voltmeter is not in parallel with only the voltage drop across X. Looking at 2 we need to remember that the ideal ammeter acts as if it has 0 resistance and thus no voltage drop across it. This means that 2 is an acceptable circuit for our purposes. Now, looking at 4 it's easier to see that the voltmeter is only measuring the voltage drop across X and thus is also compatible with our needs. The answer is then D, 2 or 4.

1.13 Problem 13

After a long time the capacitor is fully charged and has a voltage equal to that of the battery. Thus, current does not flow through the right branch and bulb B is initially off. Current does flow through the middle branch though and bulb A is initially on. When the switch is flipped to position 2 the capacitor immediately starts discharging with the charges traveling along the path of least resistance. This means that the left branch shorts the middle branch and no current flows through bulb A meaning it goes out immediately. Current is now flowing through the right branch as the capacitor discharges meaning bulb B lights up, but as the capacitor discharges over time, bulb B dims. Looking for these characteristics before and after the switch flips guides us to choose graph E.

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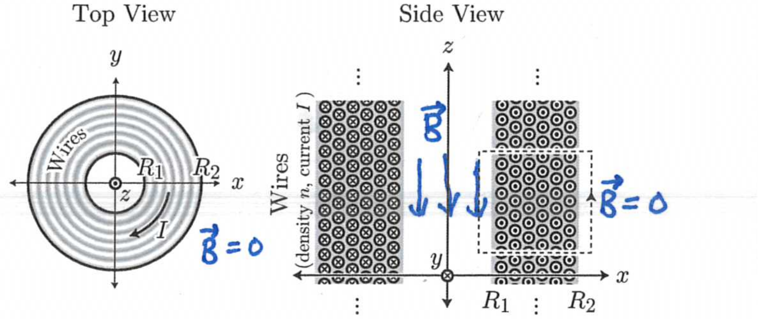
The next two problems are related. An infinitely long solenoid is oriented along the z-axis as shown. The solenoid is wrapped with multiple concentric layers of wire with a density of n **wires per unit cross-sectional area**. The inner radius of the solenoid is R_1 and the outer radius is R_2. The direction of the current I is shown in the figure.



X. (8 pts) Use the **Ampère's Law** to **derive** an expression for the magnetic field vector as a function of the displacement r from the z-axis as a function of r, R_1, R_2, n, I, and μ_0. (Hint: Consider the Ampèrian Loop drawn in the side view.) Loop height L. Keep RHS outside coil.

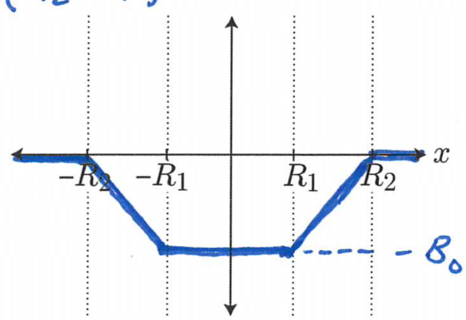
$\oint d\vec{l}\cdot\vec{B} = \mu_0 I_{enc} =$ areal density

$-\vec{B}(r)\cdot L\hat{k} = \mu_0 I N = \mu_0 I n A = \mu_0 I n L \begin{cases} 0, & R_2 < r \\ R_2 - r, & R_1 \le r < R_2 \\ R_2 - R_1, & r < R_1 \end{cases}$

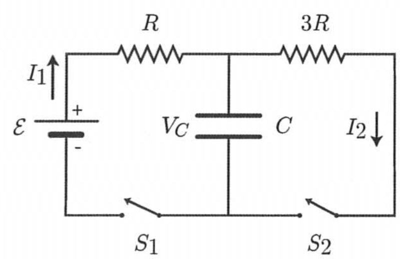
(N: # coils; A: area)

$\vec{B}$ for $r < R_1$	$-\mu_0 I n (R_2 - R_1)\,\hat{k}$
$\vec{B}$ for $R_1 < r < R_2$	$-\mu_0 I n (R_2 - r)\,\hat{k}$
$\vec{B}$ for $R_2 < r$	0

X. (5 pts) **Sketch** the z component of the B field in the figure to the right. $B_0 = \mu_0 I n (R_2 - R_1)$



The next four problems are related. Consider the RC circuit shown to the right. Before time zero, both switches are open and the capacitor is uncharged. At $t = 0$ switch S_1 is closed. Switch S_1 is opened at $t = RC$. Switch S_2 is then closed at $t = 2RC$.



X. (3 pts) **Sketch** the voltage across the capacitor V_C.

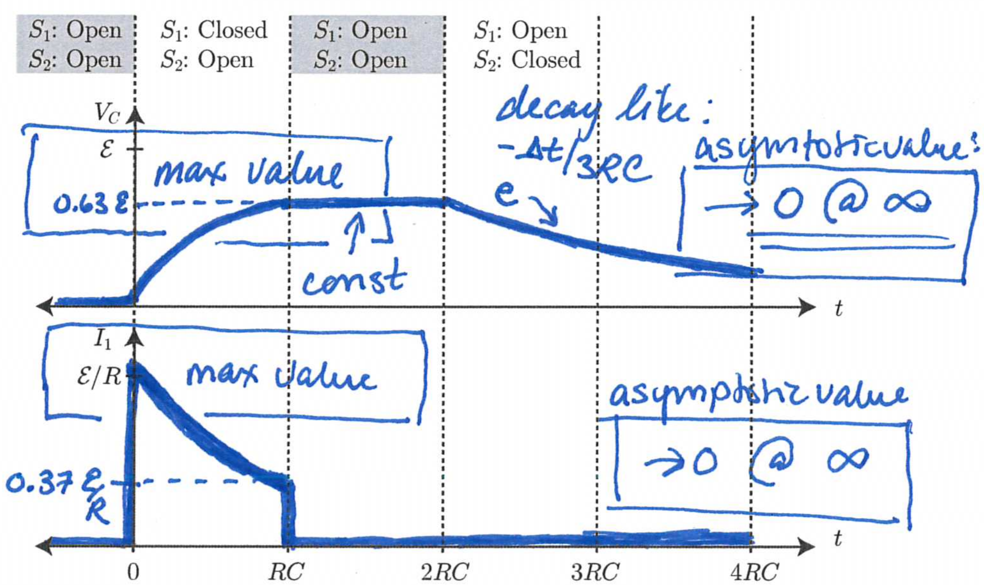
X. (3 pts) **Label max value** and **asymptotic value** ($t \to \infty$).

$1 - e^{-1} \cong 0.63$

X. (3 pts) **Sketch** current I_1.

X. (3 pts) **Label max value** and **asymptotic value** ($t \to \infty$).

$e^{-1} \cong 0.37$



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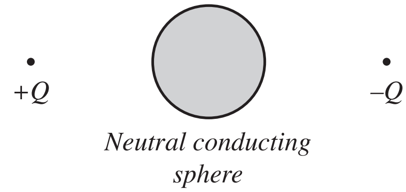
V. [20 points total] Parts A and B of this problem are independent.

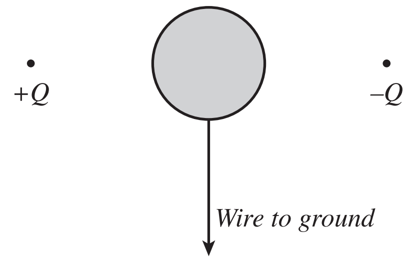
A. A neutral conducting sphere is placed halfway between two point charges, with charge $+Q$ and $-Q$ respectively.



i. [2 pts] On the diagram, sketch the induced charge distribution on the sphere.

The sphere will polarize, with negatives on the left and an equal number of positives on the right.

ii. [4 pts] The sphere is now connected to ground through a wire, as shown.



When the sphere is grounded, do any charges flow to or from ground? If **no charges flow** or if there is not enough information, state so explicitly. Explain.

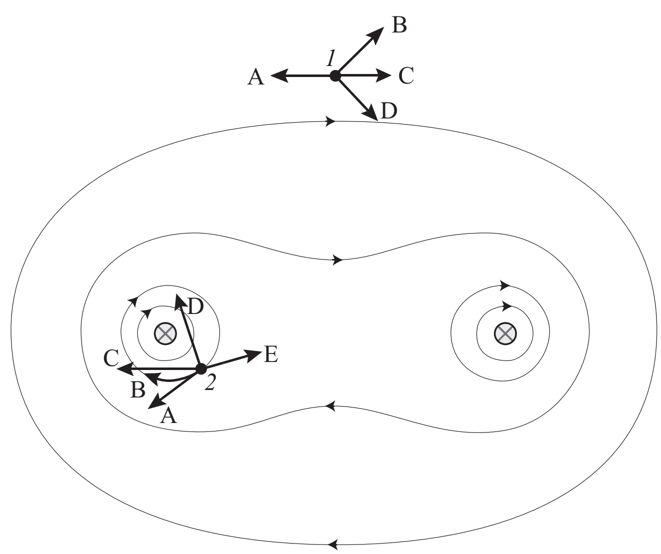
Charges flow in a conductor if there is an electric field pushing them in some direction. In the wire, the vertical components of the field due to the external charges and the induced distribution cancel by symmetry. There is still a horizontal component to the field in the wire, but charges in the wire can't leave the wire, so the horizontal component doesn't move charges either. Thus no charges move through the wire.

iii. [4 pts] The wire connecting the sphere to ground is now removed.

Is the electric potential difference from the sphere to ground *positive, negative,* or ***zero**?* Explain.

When the sphere is connected to ground, the potential difference between the sphere and ground becomes zero (since the potential difference across a conducting wire is zero). When the wire is removed, no charges move so no potential differences change. The potential difference to the ground is still zero.

B. The magnetic field lines due to two wires with equal currents into the page are shown. Point *1* is equidistant from the wires. Which vector best represents the magnetic field at point *1* and at point *2?*



i. [5 pts] Point *1:*
(Circle the correct answer below.)

A B **C** D Zero

Explain.

The magnetic field at a point is given by the tangent to the field line, in the direction of the field line, so C is the correct vector.

ii. [5 pts] Point *2*:
(Circle the correct answer below.)

A B C D E

Explain.

Similarly, the field must be tangent to the field lines, in the direction of the field line, so A is the correct vector.