1 Multiple Choice Solutions: Winter 2015 122 Exam 2

1.1 Problem 1

Here we want to use the equation relating potential energy of a point charge to a potential difference: U = q_0V. This is the potential energy in the cathode-screen-charge system before the charge has left the cathode. When the charge reaches the other side, the system has lost this potential energy but the overall energy is conserved and this potential energy was completely converted to the kinetic energy of the charge upon reaching the screen. Therefore, since the charge starts at rest and therefore 0 kinetic energy we have:

$$\Delta U + \Delta KE = 0 \tag{1}$$
$$-q_e \Delta V + (KE_{final} - KE_{initial}) = 0 \tag{2}$$
$$KE_{final} - 0 = (1.6 * 10^{-19}C) * (22 * 10^3 V) \tag{3}$$
$$KE_{final} = 3.52 * 10^{-15} J \tag{4}$$
$$\frac{1}{2}mv^2 = 3.52 * 10^{-15} J \tag{5}$$

Plug in mass and solve to get $v = 8.8 * 10^7 J$.

1.2 Problem 2

The definition of Electric Potential Difference is:

$$\Delta V = -\int_a^b \vec{E} \cdot \vec{dl} \tag{6}$$

Therefore anytime we choose a path with a component parallel to the electric field we will have a potential difference due to the dot product in the equation. In this problem we want to find the two points connected by a line that when we move along it, we will get no change in potential, $\Delta V = 0$. Therefore, we need to find two points that when we connect them have no parallel components to the electric field meaning this line must be perpendicular to the electric field. This leads us to the correct answer, 1 and 4. The line connecting these two points is perpendicular to the electric field so there is no electric potential difference between these two points.

1.3 Problem 3

For a system of point charges the electrostatic potential energy is $U = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$. For three charges, q_1, q_2, q_3, this works out to be:

$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_1}{r_{31}} \tag{7}$$

Plugging in values we get:

$$U = k*(\frac{(1*10^{-6}C*2*10^{-6}C)}{.3m}+\frac{(2*10^{-6}C*3*10^{-6}C)}{.3m}+\frac{(3*10^{-6}C*1*10^{-6}C)}{.3m}) \tag{8}$$

Calculating it out we get U = 0.330 J.

1.4 Problem 4

When we insert a dielectric into the parallel plate capacitor the capacitance between the plates increases as can be seen in the equation $C = \kappa C_0$. If we have the plates hooked up to a battery the voltage difference will be constant between the plates. Therefore, after inserting the dielectric we will have the equation $V = \frac{Q_{new}}{C_{new}} = \frac{Q_{new}}{\kappa C_0}$. In order to have the same V, Q_{new} must also increase to make up for the increased Capacitance. Thus, the answer is E.

1.5 Problem 5

Let's start with the integral equation for electric potential difference with our bounds running from b to 0:

$$V_b - V_0 = -\int_b^0 \vec{E} \cdot \vec{dl} \tag{9}$$

Since the electric field is 0 inside the conduction sphere this equation simplifies to: $-\int_b^a \vec{E} \cdot \vec{dl}$. Also remember that the electric field of a conducting sphere outside of the sphere is like that of point charge and points radially inward since Q_1 is negative. Our $\vec{dl}$ also points radially inward since we are traveling in a straight line from b to the center. Therefore, our dot product is equal to 1. Using all this we now have the problem set up as (remember we want the magnitude so take the absolute value):

$$|V_b - V_0| = \left| - \int_b^a (\frac{kQ_1}{r^2}) dr \right| = \left| \frac{kQ_1}{r} |_b^a \right| \tag{10}$$

$$|V_b - V_0| = \left| kQ_1 * (\frac{1}{.02m} - \frac{1}{.06m}) \right| = 12.0 * 10^5 V \tag{11}$$

1.6 Problem 6

If this happened we no longer would have $\vec{E} = 0$ inside the inner sphere. As a result, we would have to take into account the value of the integral $\int_a^0 \vec{E} \cdot \vec{dl}$ which was just 0 before. Inside this new insulating sphere the electric field would point inward towards the center so the dot product would once again be 1. In fact, all the signs would be the same as the integral between b and a meaning this integral would **increase** the magnitude of the potential difference between b and the center.

1.7 Problem 7

For this we need to combine the capacitors according to the equations for adding capacitors in series and in parallel. First, let's combine the two capacitors on the left most branch of the circuit. Since these two capacitors are on the same branch, they are in series and add up according to a rule that follows the same form for resistors in parallel: $\frac{1}{C_{eq_1}} = \frac{1}{50\mu F} + \frac{1}{50\mu F}$ giving us $C_{eq} = 25\mu F$ for a single equivalent capacitor in that branch. Next, let's combine this new $25\mu F$ capacitor with the capacitor it is in parallel with on the branch that goes diagonally across the diagram. Combining these capacitors in parallel is like combining resistors in series so we just add: $C_{eq_2} = (50 + 25)\mu F = 75\mu F$. Now, our circuit diagram is just the very top capacitor in series with a single $75\mu F$ capacitor. Once again applying the parallel capacitor equation we get our final equation: $\frac{1}{C_{eq_{final}}} = \frac{1}{50\mu F} + \frac{1}{75\mu F} = \frac{5}{150\mu F}$. When we invert this we get our final answer: $C_{eq_{final}} = \frac{150}{5}\mu F = 30\mu F$.

1.8 Problem 8

First, let's combine the resistors into a single, equivalent resistance. Combining the two 6 Ω resistors in parallel on the left we get an equivalent resistance

$R_{Left} = 3\ \Omega$. Combining the three parallel resistors on the right we get R_{Right} $= 2\ \Omega$. Now we have two equivalent resistors in series so we can add them up $R_{Right} + R_{Left} = 5\ \Omega$. Finally, apply Ohm's Law, V = IR = 1.2 $A * 5\ \Omega$ = 6 V.

1.9 Problem 9

Since both objects are metal we know that they will both be conductors and charge will be able to freely move about in either object. When the metal ball reaches the bottom it comes into contact with the metal shell and charge can now travel from the ball into the shell and vice versa. However, the object's are now acting together as one large conductor and the charge within them is the sum of their individual charges before. In this case, that total charge is +Q since before the shell was uncharged and this was the charge on the metal ball. Since the two object's together are now acting as one giant conductor with charge +Q, the charge will move as far apart as possible and will configure itself so that everywhere inside the conductor is at an equipotential (the electric field inside will be 0). This happens when the entire +Q charge moves to the outside surface of the outer shell and the ball inside has zero charge.

1.10 Problem 10

Let's split C_2 into three vertical sections to start: the one on the left of the dielectric, the region where the dielectric is present and the right part of the capacitor on the right side of the dielectric. We can actually treat these three sections as three capacitors in parallel with one another. So, if we can find the capacitance of each section we can just add them up following the equation for adding parallel capacitors (see problem 7). The left section is identical to the right section so we can do one calculation to find the capacitance of each. The formula for capacitance is: $C = \frac{\epsilon_0 A}{d}$. For these pieces of the capacitor we have $\frac{1}{4}$ the area of C_1 but half the distance between plates and no dielectric. This means that each section has $\frac{1}{2}$ the capacitance of the original capacitor C_1 but when we add these two sections together because they are like capacitors in parallel we get a capacitance equivalent to C_1. For the middle section where the dielectric is we have $\frac{1}{2}$ the area and $\frac{1}{2}$ the distance between the plates so if there were no dielectric we would have a capacitance equal to C_1. But when we insert a dielectric our capacitance changes by the factor of κ corresponding to the dielectric ($C = \kappa C_0$). Since $\kappa = 3$ for our dielectric here, the capacitance of this middle region is $3C_1$. Adding in our capacitance from the left and right sections we get $C_2 = 3C_1 + C_1 = 4C_1$ for our final answer.

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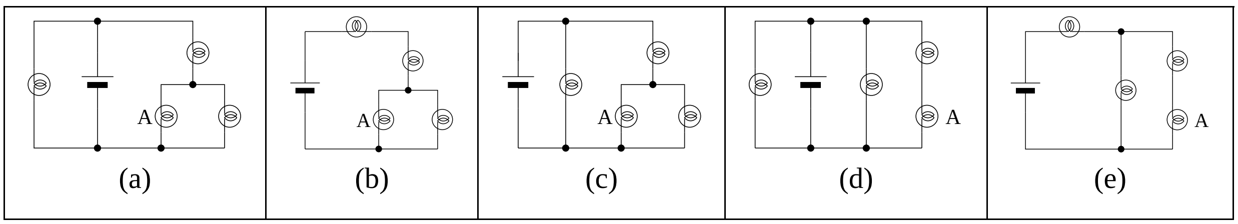
II. Lab questions [12 pts]

For problems 10–12, assume that the battery and ammeter are ideal and that all bulbs are identical.

11. [4 pts] In which circuit below is bulb A **brightest**?

In other words, in which circuit does Bulb A have the most current flowing through it? First, any branch directly connected across the battery that does NOT contain bulb A can be ignored, since the battery maintains a constant voltage. So in (a) and (c) we have 2-in-parallel in series with 1 more, in (b) we have 2-in-parallel in series with 2 more, in (d) we have 2-in-series only, and in (e) we have 2-in-series in parallel with 1 which is in series with 1 more. Remember what you saw in lab: when you connect a bulb in parallel with another that is already in series with a bulb, that is, make a change from a circuit of type (d) to types (a) or (c), you see the parallel bulbs become dimmer. Hence, bulb A must be brighter in (d) than in (a), (b) or (c). Finally, because the difference between (d) and (e) is the added bulb in series with the parallel network that is similar in both, less current must flow through bulb A in (e) than in (d). So the answer is (d).

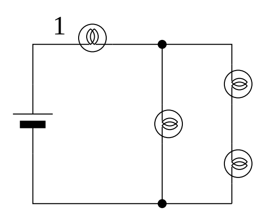


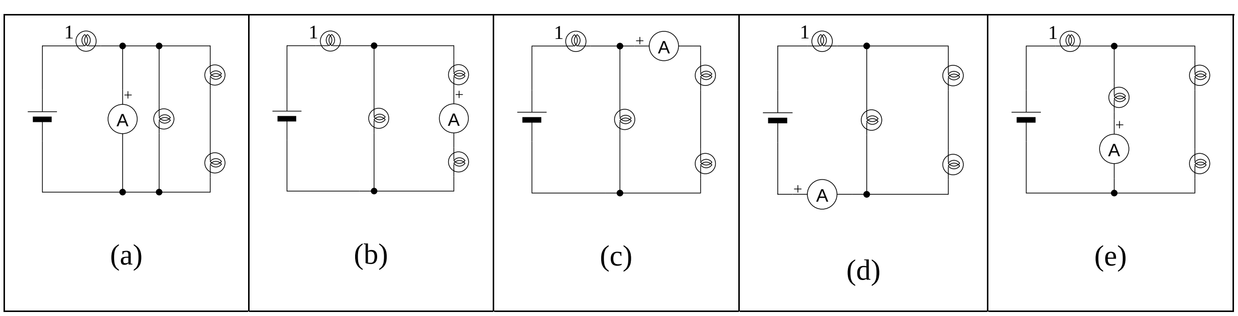
12. [4 pts] In which circuit **above** is the **power delivered by the battery** the **lowest**?

In other words, in which circuit does the battery deliver the least current (since its voltage is constant)? Look for the circuit with the overall greatest resistance. Circuit (b) has 2 bulbs in series and none in parallel across the battery. All others except (e) have at least 1 bulb connected across the battery, so their resistance is lower than (b)'s. The parallel network in (e) has a resistance lower than just 1 bulb, so its resistance is lower than (b)'s. Thus (b) has the highest overall resistance. The answer is (b).

13. [4 pts] An ammeter is to be **added to the circuit at right** in order to measure the current through the bulb labeled 1. Which placement of the ammeter will **correctly** measure the current through bulb 1?

To correctly measure the current, an ammeter must be placed in series with bulb 1, which is itself in series with the battery. So the circuit with the ammeter in series with the battery is the correct one: circuit (d).





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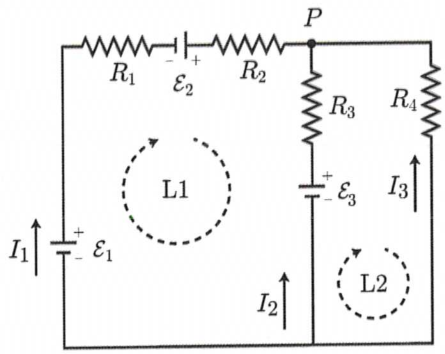
The next four problems are related.



Kirchhoff Laws. Study the circuit and answer the following questions.

X. (3 pts) Use the Kirchhoff Current Law to relate the three currents at **point P.**

P: $I_1 + I_2 + I_3 = 0$

X. (6 pts) Use the Kirchhoff Voltage Law to write equations for the sum of the **voltage drops** around loops L1 and L2. Express all equations in terms of the parameters defined in the figure above.

L1: $-\mathcal{E}_1 + R_1 I_1 - \mathcal{E}_2 + R_2 I_1 - R_3 I_2 + \mathcal{E}_3 = 0$

L2: $-\mathcal{E}_3 + R_3 I_2 - R_4 I_3 = 0$

X. (9 pts) Assume that all emf sources supply 5 V and all resistors have a resistance of 100 Ohms. I_1 is found to be 0.03 A. What are the remaining currents? (Show your work and put your final answers in the provided box.)

Solve for I_2 first:

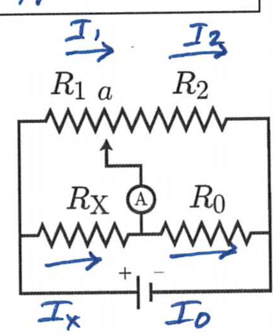
$-5V + \frac{3}{100} A \cdot 100\Omega - 5V + \frac{3}{100} A \, 100\Omega - 100\Omega I_2 + 5V = 0$

$1V = 100\Omega I_2 \Rightarrow I_2 = 0.01 A$

Now use $I_1 + I_2 + I_3 = 0$

$I_3 = -0.04 A$

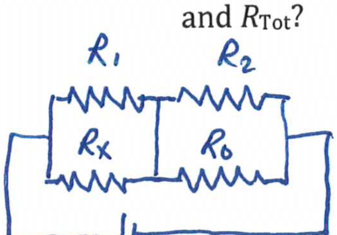
	I_2	I_3
(Amps)	0.01 A	-0.04 A



X. (7 pts) **Wheatstone Bridge: Measuring the resistance.** The variable resistor is adjusted by moving the contact position *a*. *a* is the position relative to the total length of the resistor such that the resistance from the LHS of the resistor to the contact point is $R_1 = a R_{Tot}$ and resistance from the contact point to the RHS of the resistor is $R_2 = (1-a) R_{Tot}$.

The contact position *a* is varied until there is **no current** flowing through the ammeter (A). What is the resistance of R_X as a function of R_0, *a*, and R_{Tot}?

As discussed in class:



$\frac{I_1}{I_x} = \frac{R_x}{R_1}$ & $\frac{I_2}{I_0} = \frac{R_0}{R_2}$ since voltage drop over Resistors in // is =.

No current through A $\Rightarrow \frac{I_1}{I_x} = \frac{I_2}{I_0}$

$R_x = \frac{R_1 R_0}{R_2} = \frac{a}{1-a} R_0$

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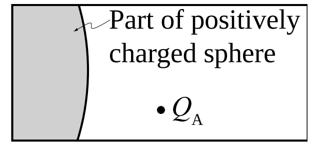
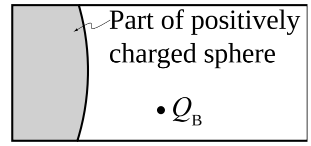
V. [20 points total] Two experiments are conducted with two identical positively charged spheres and two **different** test charges. Q_A = –1.5 nC, Q_B = +3 nC.



Q_A is released at point P.



Q_B is moved from point P to the sphere.

Experiment A: Q_A is released from rest at point P, and moves toward the sphere. When it reaches the surface of the sphere it has 8 J of kinetic energy.

Experiment B: A hand moves Q_B from rest at point P to rest at the surface of the sphere.

In each experiment, consider the system of the sphere and test charge.

A. [4 pts] In Experiment A, does the potential energy of the system *increase,* ***decrease****,* or *remain the same* as Q_A moves toward the sphere? Explain.

There are no external forces acting on the system, so no external work is done, and thus the total energy does not change. Kinetic energy increases, so potential energy decreases.

B. Let ΔU_A and ΔU_B represent the changes in **electric potential energy** in Experiments A and B, respectively.

i. [3 pts] Is the magnitude of ΔU_A *greater than,* ***less than****,* or *equal to* the magnitude of ΔU_B? Explain.

In experiment B there is no change in kinetic energy, so the work done by the hand is equal to the change in potential energy. The magnitude of the electric force on Q_B is twice that on Q_A, and the hand's force equals the electric force to keep Q_B from gaining kinetic energy. Since the force on Q_A is half that on Q_B and both charges move the same distance, ΔU_A is less.

ii. [3 pts] Is the sign of ΔU_A *the same as* or ***different from*** the sign of ΔU_B? Explain.

The work done by the hand on Q_B is positive since the force is in the same direction as the displacement. Thus ΔU_B is positive, so it is the opposite sign as ΔU_A.

C. [4 pts] If the reference point for the **electric potential** is at the surface of the sphere, is the electric potential at point P in Experiment A *positive,* ***negative****,* or *zero?* Explain.

Q_A loses potential energy as it moves from point P to the sphere, so if it moved the other way it would gain potential energy. $\Delta V = \Delta U/q_{test}$, and Q_A is negative so the potential difference is negative. Thus if the reference point is at the sphere, the potential at point A is negative.

D. Let ΔV_A and ΔV_B represent the **electric potential differences** from point P to the surface of the sphere in Experiments A and B, respectively.

i. [3 pts] Is the magnitude of ΔV_A *greater than, less than,* or ***equal to*** the magnitude of ΔV_B? Explain.

From part B we know that ΔU_A is half that of ΔU_B, and the opposite sign. The same is true for Q_A compared to Q_B, so therefore the magnitude of ΔV is the same for both charges.

ii. [3 pts] Is the sign of ΔV_A ***the same as*** or *different from* the sign of ΔV_B? Explain.

As in the answer above, the sign of Q and ΔU both change from A to B, so ΔV is the same sign.