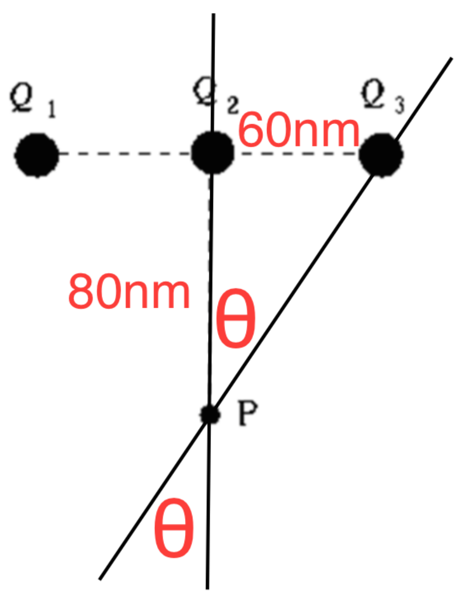
1 Multiple Choice Solutions: Winter 2015 122 Exam 1

1.1 Problem 1

First, let's find the Electric Field at P from Q_2. Using the formula $E = \frac{k*q}{r^2}$ we get 899000 N/C downward.

Now, let's find the Electric Field at point P from Q_1 and Q_3. Charges Q_1 and Q_3 are $\sqrt{60^2 + 80^2} = 100$ nm away from point P. And therefore their Electric Fields at P are equal in magnitude with a value of 575360 N/C but the directions are each away from each charge in a straight line. As a result the horizontal components from each of these Electric Fields cancels out and we merely need to find the vertical downward components of the Electric Field. We will do this by multiplying the magnitude of the respective E-Fields by the Cosine of the angle between their vectors and the vertical. This angle is shown below:



Looking at the figure and remembering that this hypotenuse of the triangle above it is the distance of 100 nm we previously calculated we see that this angle has a cosine of $80/100 = 0.8$. Therefore, the vertical component of the electric field from each of Q_1 and Q_3 is $\text{Cos}[\theta] * 575360 = 460288$ N/C.

So, our total Electric Field is then $E_{Q_2} + E_{Q_{1y}} + E_{Q_{3y}}$ = 899000 + 2*460288 = 1.8 * 10^6 N/C.

1.2 Problem 2

Since we know that the three charges are positive then we know that the Electric Field points radially outward from each point. Therefore, the field from the top left and top right charge point in opposite directions. Since we know that each charge is equal and it appears that point P is equidistant on a line between the two top charges these fields cancel out at this point. That leaves us with the Electric Field from the charge at the bottom left. Drawing a line radially outward from this charge gives us a field in the same direction as line 2.

1.3 Problem 3

Here we want to look at the density of field lines surrounding each point charge since the Electric Field strength will be proportional to these densities. Furthermore, we know from the equation for the Electric Field of a point charge: E = $\frac{k*q}{r^2}$ that the strength of the electric field at an equal distance away from each charge is directly proportional to the amount of charge. The last thing we need to remember is that Electric Field lines go out from positive charges and terminate on negative charges.

Using the above observations let's determine the strength of each charge. Looking at V we see that it has field lines connected to U. Since U is positive we know that these field lines must be directed outwards from U and therefore terminating on V. Thus, V must be negative. We also see that the density of field lines around V is $\frac{1}{2}$ that of U and conclude that V must have a charge of -1 * (.5) * U = -1C.

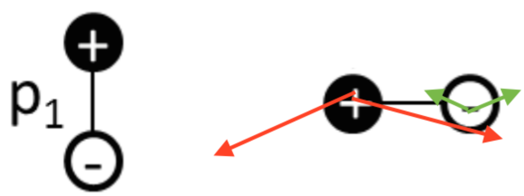
Next looking at W we notice that all our observations for V are the same here and conclude it also has a charge of -1C.

Now we look at Y and see that it has lines that connect from it to W and V. This tells us that it must have the opposite charge. Since W and V are negative Y must be positive. Now we compare the density of field lines around it and see that it has 3 times the amount of W and V. Therefore we calculate a charge of -1 * 3 * W = +3C.

Finally, looking at X we see that no lines go from V to X, but field lines connect from Y to X. Therefore X must be a negative charge based upon what we know above. Checking the density of field lines just around it we see the same amount of field lines coming out of X and V and W. This tells us that our last charge, X, has a charge of -1C.

1.4 Problem 4

The modified figure below has the Electric Fields from each charge in p_1 on each charge of p_2. We can see that the fields on the left are stronger than the fields on the right due to the closer distance. Also, we can see that the net field on the left is downwards while the net field on right is upwards. As a result, we will get a rotation in the direction of arrow 2 in the problem.



1.5 Problem 5

For this problem we need to look at three different areas.

1) The area outside the shell: If we draw a Gaussian sphere outside wall of the uncharged conduction shell, we know that the Electric Field at the surface of our Gaussian sphere will look like that of a point charge centered at the middle and with a charge equal to the charge enclosed by our Gaussian sphere. Since the shell has no charge, the only charge enclosed is the +5 μC inside. Therefore, our field lines will be radially outward from the center of the shell as they would be for a positive point charge located at the center.

2) The area inside the shell: Since the shell is a conductor we have an Electric Field of 0 inside so no lines are present here.

3) The area inside the shell: Due to our answer in area 1 we know that the outside surface of the conductor must have a positive charge density. But, the conductor as a whole must be uncharged so the inner surface has a negative charge density. Finally, since the charged solid sphere is closer to the upper left part of the conducting sphere we will not have a uniform

charge distribution on this inner surface. More of the negative charges will be attracted to the area closer to the sphere and as a result the electric field in this region will be stronger.

Taking into account this analysis of the three sections we see that B is the correct answer.

1.6 Problem 6

Take a Gaussian surface located at a radius inside the conducting shell. Since we are inside of a conductor we must have E = 0. Since our Gauss' Law equation is E = $\frac{Q_{enc}}{(Sphere'sSurfaceArea)*\epsilon_0}$ = 0 then the charge enclosed by our Gaussian sphere must be 0. Therefore, the total charge on the inside surface of the shell must be an amount that exactly cancels out the charge of the solid conducting sphere, -5μC.

1.7 Problem 7

We need to remember a bit of mechanics here and analyze the forces on the dust particle. We know that there will be an Electric Force pushing up since both the sheet and dust particle have the same charge and we know that the Weight Force will be pulling downwards. Since we are told that the dust particle floats we know it is at rest (a = 0) and use Newton's 2nd Law to conclude:

$$\Sigma\vec{F} = m\vec{a} \tag{1}$$

$$\vec{F}_E - \vec{W} = 0 \tag{2}$$

$$\vec{F}_E = \vec{W} \tag{3}$$

The Weight Force is just the mass of the particle times g. What is the Electric Force? Here we will use $\vec{F}_E = q * \vec{E}$ but still need to find $\vec{E}$. The field comes from an infinite plane of charge and thus we can look at the equation sheet and find that a positively charged sheet has an Electric Field pointing away in a direction perpendicular to the sheet with a magnitude of $E = \frac{\sigma}{2\epsilon_0}$. We know enter all of this information into our equation above:

$$q\vec{E} = mg \tag{4}$$

$$(12 * 10^{-12}C) * \frac{15 * 10^{-9}C/m^2}{2 * 8.85 * 10^{-12}C^2N^{-1}m^{-2}} = m * 9.81m/s^2 \tag{5}$$

$$1.04 * 10^{-9}kg = m \tag{6}$$

1.8 Problem 8

First of all, let's pretend the thick conducting metal plate is not in between the two infinite sheets and find the Electric Field in the space between the two sheets. As in the previous problem we need to use the formula from the equation sheet for the field from an infinitely charged sheet. Since sheet 1 has a positive charge density, σ_1 it's field will point directly away from the sheet and in the region between the two plates it points to the right. Sheet 2 also has a positive charge density, σ_2 and as a result it's field in between the two sheets points to the left. Therefore, in between the two sheets the field just due to the charge densities on the sheets is (positive x is to the right):

$$E_x = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} \tag{7}$$

$$E_x = \frac{\sigma_1 - \sigma_2}{2\epsilon_0} \tag{8}$$

$$E_x = \frac{7C/m^2}{2\epsilon_0} \tag{9}$$

So we have a field from just sheets 1 and 2 of $\frac{7C/m^2}{2\epsilon_0}$ to the right. Now we put the thick conducting metal plate back in. Inside the metal conducting plate our field must be equal to 0 since we are inside a conductor. As a result, the two side of the metal conducting plate act as two more infinitely charged sheets and must create a field inside the conductor that exactly opposes the field to the right caused by sheets 1 and 2. This means the two sides of the conductor which we'll call sheet L (for the left one) and sheet R (for the right one) must create a net Electric Field of $\frac{7C/m^2}{2\epsilon_0}$ to the left. Following our same procedure for finding the net Electric Field from sheets 1 and 2 we get that:

$$\frac{-7C/m^2}{2\epsilon_0} = \frac{\sigma_L - \sigma_R}{2\epsilon_0} \tag{10}$$

But since we also know the total charge on the conductor is -3.0 C/m^2 we have another equation: $\sigma_L + \sigma_R = -3.0C/m^2$. Since we want to find σ_R we can use this equation to replace σ_L in our equation above and get:

$$\frac{-7C/m^2}{2\epsilon_0} = \frac{-3.0C/m^2 - \sigma_R - \sigma_R}{2\epsilon_0} \tag{11}$$

$$-4.0C/m^2 = -2\sigma_R \tag{12}$$

$$2.0C/m^2 = \sigma_R \tag{13}$$

1.9 Problem 9

As we did in problem 6 we once again want to draw a Gaussian surface within the walls of a conducting shell. However, this time we wish it to be Gaussian cylinder with a radius r, such that a < r < b. Since the Electric Field must be 0 inside the walls of the conductor we can apply Gauss' Law setting E = 0. The only way we can get this in every direction within the conductor with our Gaussian surface is to have $Q_{enc} = 0$. To cancel out the charge from the +3μC/m charge density along the wire then the inner surface of the conductor must have charge density -3μC/m which is the overall charge density of the thick-walled conducting cylinder. As a result, there is no overall charge anywhere else in the conductor meaning the surface charge on the outside of the cylinder = 0.

1.10 Problem 10

We begin with Gauss' Law:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0} \tag{14}$$

and we will use a cylindrical Gaussian cylinder with a radius of 20cm. We know that a cylindrical line charge will have a radial electric field so the $\vec{E} \cdot d\vec{a}$ for the top and bottom of our Gaussian cylinder will be 0 since the area vector points normal to the surface and in each of these cases will point 90 degrees away from the $\hat{r}$ direction.

That just leaves us with the side of the cylinder where by convention we choose to point the area vector outwards from the surface and thus in the $\hat{r}$ direction. Therefore we get $\oint \vec{E} \cdot d\vec{a} = E \oint da$. Integrating the $\oint da$ gives us the surface area of the side of a cylinder which is $2\pi rh$. Rewriting Gauss' Law with our new information gives us:

$$E * 2\pi rh = \frac{\sigma h}{\epsilon_0} \tag{15}$$

where we remember that $Q_{enc} = \sigma *$ *length along the line charge* and in this case the length along the line charge is the height of our Gaussian cylinder. Canceling the h on either side of the equation and converting 20cm to .2m we get a value for the Electric Field:

$$E = \frac{6\mu C/m}{2\pi * .2m * \epsilon_0} = 0.54 * 10^6 N/C \tag{16}$$

1.11 Problem 11

We have 4 distinct regions in the plot that we will analyze individually.

1) r < 4cm (r < a): In this region if we draw our Gaussian cylinder our only charge enclosed is that from the infinite line of charge. We know that the Electric Field of an infinite line points out radially and looking at our equation sheet we see that it falls as 1/r in the radial direction.

2) 4cm < r < 5cm (a < r < b): In this region we are within the walls of the conductor and thus the Electric Field must be equal to 0.

3) 5cm < r < 10cm (b < r < c): If we draw our Gaussian cylinder in this region we are once again in the same situation as Problem 9. Since there is no outer surface charge density we have the same amount of charge enclosed, 0. Therefore, in this region the Electric Field is like the interior of the conductor and equal to 0.

4) r > 10cm (r > b): Here we can once again draw a Gaussian cylinder and see that our charge enclosed is now positive again due to the charge density located on the nonconducting cylindrical shell. Since we are still in a situation with cylindrical symmetry we have an Electric Field that drops in the same way a line charge would, 1/r in the radial direction.

Putting our analysis from these four regions together we see that the answer that correctly plots this Electric Field is (b).

1.12 Problem 12

Since the electroscope's vane was initially open we know that there must be some initial overall charge in the electroscope that is causing a repulsion between the ends of the vane and post. When the negatively charged teflon rod is brought closer to the electroscope disk it will repel more negative charges into the post and vane. Since this causes the repulsion to be even stronger (evidenced by the vane opening further), we can conclude that the charges are the same sign as the charges causing the initial repulsion. Therefore, the initial charge distribution must be negative and without the presence of the rod it should be evenly distributed giving us answer (d).

1.13 Problem 13

As we mentioned in the answer for 12 the presence of the rod repels like charges. In addition, without touching the electroscope and without any sparks jumping the overall charge on the electroscope must remain the same as it was before. Therefore, we should have the same amount and same type of charge that we had in our answer to 12. The answer choice that both conserves the same amount of charge and shows the like charges being repelled by the rod is answer choice (b).

1.14 Problem 14

Since the rod repels like charges, the negative charges are now repelled past the end of the electroscope and into the hand. Conversely, positive charges are attracted to the rod and move towards the electroscope disk and concentrate as close to the rod as possible. Therefore, we choose (a).

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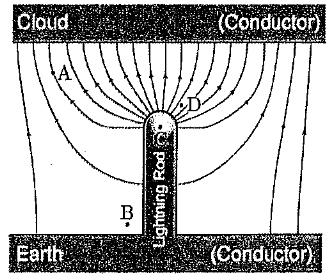
The next three problems are related.

A lightning rod in a storm. Study the field lines and answer the following questions.



1 X. (5 pts) **Rank** the **magnitude** of the **force** on a negative charge at points A, B, C and D from smallest to largest. (If the electric field at any two points is equal, state so explicitly.)

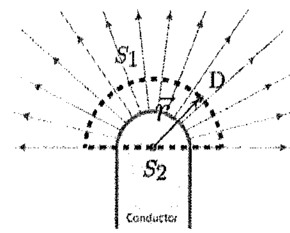
$F_C < F_B < F_A < F_D$

2 X. (5 pts) Why?

C: $\vec{E}=0$ in conductor $\Rightarrow \vec{F}=0$

B, A, D: Density of field lines. Density $\propto |\vec{E}| \propto |\vec{F}|$

Assume the E field outside the top cap of the lightning rod is well **approximated** by the field generated by a charged spherical conductor, as shown. Consider the **Gaussian surface** S constructed from the half sphere S_1 and circular end cap S_2, as shown.



3 X. (5 pts) If charge Q is enclosed by the **Gaussian surface** S, what is the **electric field vector** at point D in terms of Q, $\vec{r}$ and ε_0? Half Sphere Surface A.

$\vec{E}(S_2) \perp \hat{n}(S_2)$

$\vec{E}(S_1) \parallel \hat{n}(S_2)$

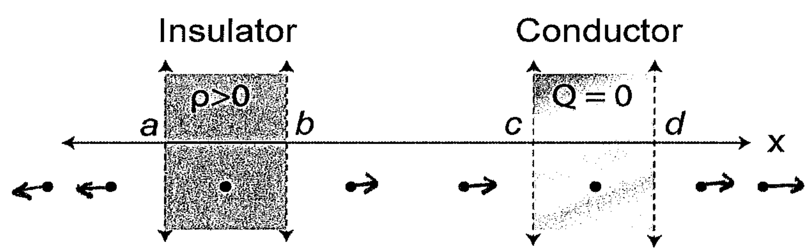
$\Rightarrow \oint d^2A\, \hat{n}\cdot\vec{E} = A_{S_1} E(S_1) + 0 = \frac{4\pi r^2}{2} E = \frac{Q}{\varepsilon_0}$

$\boxed{\vec{E} = \frac{Q}{2\pi\varepsilon_0 r^2}\hat{r}}$

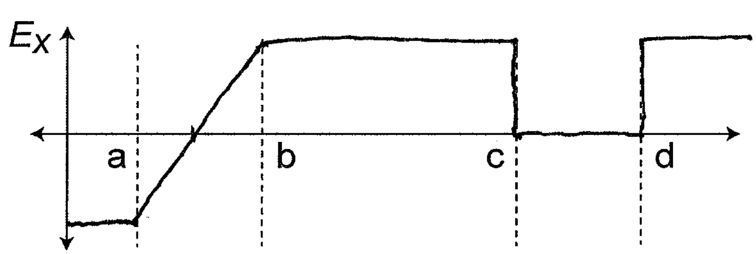
The next two problems are related.

E field from an infinite slab: Consider an infinite **insulating slab** with charge density ρ > 0 and an infinite **conducting slab** with no net charge (Q = 0) for the next two questions.

4 X. (5 pts) **Sketch** the E field **by drawing an E field vector at each black point**. If the E field is zero, leave the point blank.



5 X. (5 pts) **Sketch** the x component of the E field (E_x) as a function of the x position **on the plot to the right.**



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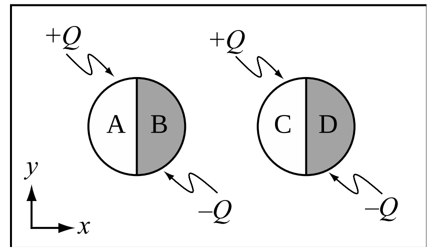
IV. [20 pts] The question consists of two independent parts, A and B.



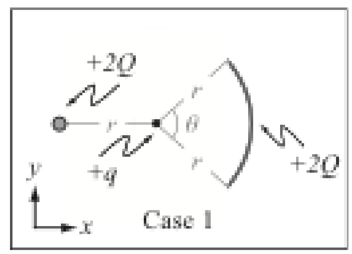
A. Two identical insulating spheres AB and CD consist of uniformly charged hemispheres A, B, C, and D as shown. Hemispheres A and C contain $+Q$ total charge each, and hemispheres B and D contain $-Q$ total charge each. Note the x- and y-directions defined in the figure.

i. [4 pts] In what direction is the net electric force exerted on hemisphere C by sphere AB alone (do not include any effects due to hemisphere D)? If the net electric force is zero, state so explicitly. Explain.

The direction of net electric force on C by AB is to the left. By Coulomb's Law, electric force is proportional to $1/r^2$ *so the attractive force between B and C is greater than the repulsive force between A and C. Summing the two forces gives a net force to the left.*

ii. [6 pts] In what direction is the net electric force exerted on sphere CD by sphere AB? If the net electric force is zero, state so explicitly. Explain.

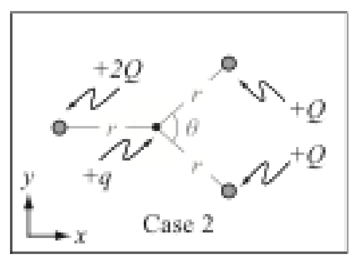
The direction of net electric force on CD by AB is to the left. Sphere AB is closer to C than it is to D. Thus the attractive force on C by AB is greater in magnitude than the repulsive force on D by AB by manipulations of Coulomb's Law. Summing the two forces gives a net force to the left.



B. In case 1 (shown at right), a small point charge of $+q$ is placed to the left of a charged arc at its center. The arc is uniformly charged and contains $+2Q$ total charge, has a radius of r, and an angle of θ that is less than 90°. A point charge of $+2Q$ is placed to the left of the small charge at a distance r away. Note the x- and y-directions defined in the figure.

i. [5 pts] In what direction is the net electric force exerted on the small charge $+q$? If the net electric force is zero, state so explicitly. Explain.

The direction of net electric force on the small charge +q is to the right. In the charged arc, charge is distributed such that the y-components of the force contribution cancel, resulting in a smaller sum in the x-component. Thus the repulsive force by the point charge +2Q will be stronger than the repulsive force of the charged arc, so the net force will be to the right.



ii. [5 pts] In case 2 (shown at right), the charged arc from case 1 is replaced with two point charges of $+Q$ each, located at the ends of where the charged arc used to be.

Is the magnitude of the net electric force exerted on the small charge $+q$ in case 2 *greater, less than,* or *equal to* the magnitude of the net electric force exerted on the small charge $+q$ in case 1? If there is not enough information, state so explicitly. Explain.

The magnitude of the net electric force on the small charge +q will increase. By replacing the charged arc with two point charges, the charge is now localized such that more of the force contribution has y-components that cancel, resulting in a smaller sum in the x-component. Since the part of the electric force pointing to the left is decreasing, the magnitude of net electric force is increasing to the right.