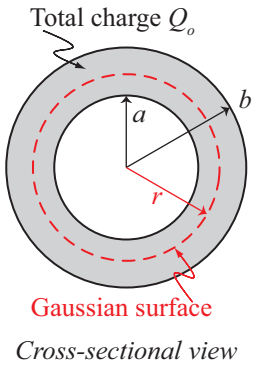
Name _____ **Student ID** _____ **Score**_____
last *first*

III. [25 points total] A non-conducting, thick spherical shell with inner radius a and outer radius b has a total charge Q_o spread uniformly throughout its volume.



Cross-sectional view

A. In terms of variables given and fundamental constants, determine the **magnitude of the electric field** in the following regions:

i. [4 pts] For $0 < r < a$. Show your work.

Owing to the spherical symmetry, we know the electric field must point inward or outward, and it must have constant magnitude over a sphere with the same origin as the charge distribution.

A spherical Gaussian surface concentric with the shell, and with a radius $r < a$, has no charge enclosed. Owing to the symmetry discussed above, ***the electric field is zero everywhere $r < a$.***

ii. [8 pts] For $a < r < b$. Show your work.

Choose the same Gaussian surface, now with $a < r < b$ as shown on the diagram.

The enclosed charge is the total charge Q_o times the ratio of (1) the volume of charged material inside the Gaussian surface and (2) the total volume of charged material: $Q_{\text{enc}} = \frac{(r^3-a^3)Q_o}{(b^3-a^3)}$.

The electric field and the area vector both point outwards everywhere on our surface, so the flux integral in Gauss' law reduces: $\oint \vec{E} \cdot d\vec{A} = \int E dA$. *The electric field must be constant over our spherical surface, so E can be taken out of the integral:* $\int E dA = E \int dA = 4\pi r^2 E$.

Putting this all together in Gauss' law, we can now solve for E:

$$4\pi r^2 E = \frac{(r^3 - a^3)Q_o}{\epsilon_o(b^3 - a^3)}$$

$$\boldsymbol{E = \frac{(r^3 - a^3)Q_o}{4\pi\epsilon_o(b^3 - a^3)r^2}}$$

iii. [5 pts] For $b < r$. Show your work.

Choose the same Gaussian surface, now with $r > b$, we enclose the entire charge of the shell: $Q_{enc} = Q_o$. *The flux integral reduces for the same reasons as part ii:* $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$.

Inserting these expressions into Gauss' law:

$$4\pi r^2 E = \frac{Q_o}{\epsilon_o}$$

$$\boldsymbol{E = \frac{Q_o}{4\pi\epsilon_o r^2}}$$

B. [8 pts] In the space below, sketch a qualitatively correct plot of the magnitude of the electric field for the charged sphere.