In terms of the dipole moment $\vec{p}$, the electric field on the axis of the dipole at a point a great distance $|x|$ away is in the same direction as $\vec{p}$ and has magnitude

$$E = \frac{2kp}{|x|^3} \tag{21-10}$$

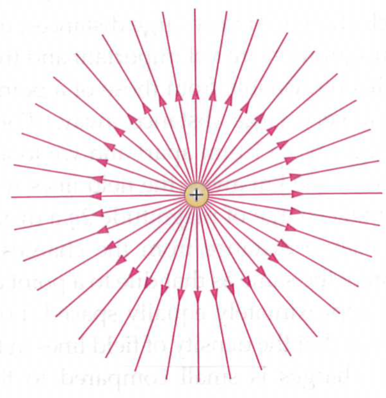
(see Example 21-9). At a point far from a dipole in any direction, the magnitude of the electric field is proportional to the magnitude of the dipole moment and decreases with the cube of the distance. If a system has a nonzero net charge, the electric field decreases as $1/r^2$ at large distances. In a system that has zero net charge, the electric field falls off more rapidly with distance. In the case of a dipole, the field falls off as $1/r^3$ in all directions.

21-5 ELECTRIC FIELD LINES

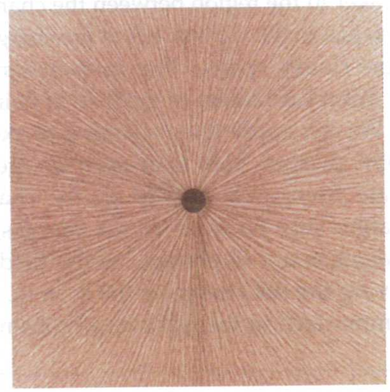
We can visualize the electric field by drawing a number of directed curved lines, called **electric field lines,** to indicate both the magnitude and the direction of the field. At any given point, the field vector $\vec{E}$ is tangent to the line through that point. (Electric field lines are also called *lines of force* because they show the direction of the electric force exerted on a positive test charge.) At points very near a positive point charge, the electric field $\vec{E}$ points directly away from the charge. Consequently, the electric field lines very near a positive charge also point directly away from the charge. Similarly, very near a negative point charge the electric field lines point directly toward the charge.

Figure 21-20 shows the electric field lines of a single positive point charge. The spacing of the lines is related to the strength of the electric field. As we move away from the charge, the field becomes weaker and the lines become farther apart. Consider an imaginary spherical surface of radius r that has its center at the charge. Its area is $4\pi r^2$. Thus, as r increases, the density of the field lines (the number of lines per unit area through a surface element normal to the field lines) decreases as $1/r^2$, the same rate of decrease as E. So, we adopt the convention of drawing a fixed number of lines from a point charge, the number being proportional to the charge q, and if we draw the lines equally spaced very near the point charge, the field strength is indicated by the density of the lines. The more closely spaced the lines, the stronger the electric field. The magnitude of the electric field is also called the **electric field strength.**

Figure 21-21 shows the electric field lines for two equal positive point charges q separated by a small distance. Near each point charge, the field is approximately that due to that charge alone. This is because the magnitude of the field of a single point charge is extremely large at points very close to the charge, and because the second charge is relatively far away. Consequently, the field lines near either charge are radial

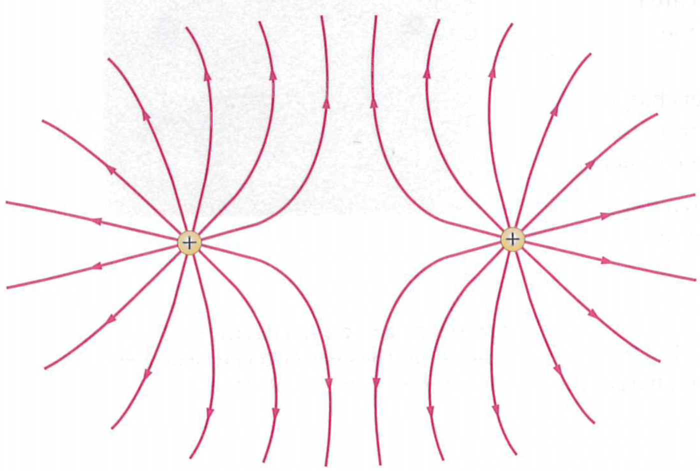


(a)

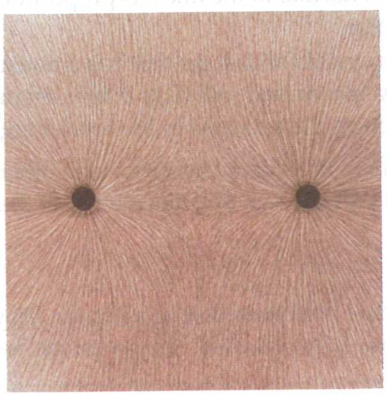


(b)

FIGURE 21-20 (*a*) Electric field lines of a single positive point charge. If the charge were negative, the arrows would be reversed. (*b*) The same electric field lines shown by bits of thread suspended in oil. The electric field of the charged object in the center induces opposite charges on the ends of each bit of thread, causing the threads to align themselves parallel to the field. *(Harold M. Waage.)*



(a)



(b)

FIGURE 21-21 (*a*) Electric field lines due to two positive point charges. The arrows would be reversed if both charges were negative. (*b*) The same electric field lines shown by bits of thread in oil. *(Harold M. Waage.)*

and equally spaced. Because the charges are of equal magnitude, we draw an equal number of lines originating from each charge. At very large distances, the details of the charge configuration are not important and the electric field lines are indistinguishable from those of a point charge of magnitude $2q$ a very large distance away. (For example, if the two charges were 1 mm apart and we look at the field lines near a point 100 km away, the field lines would look like those of a single charge of magnitude $2q$ a distance 100 km away.) So at a large distance from the charges, the field is approximately the same as that due to a point charge $2q$ and the lines are approximately equally spaced. Looking at Figure 21-21, we see that the density of field lines in the region between the two charges is small compared to the density of lines in the region just to the left and just to the right of the charges. This indicates that the magnitude of the electric field is weaker in the region between the charges than it is in the region just to the right or left of the charges, where the lines are more closely spaced. This information can also be obtained by direct calculation of the field at points in these regions.

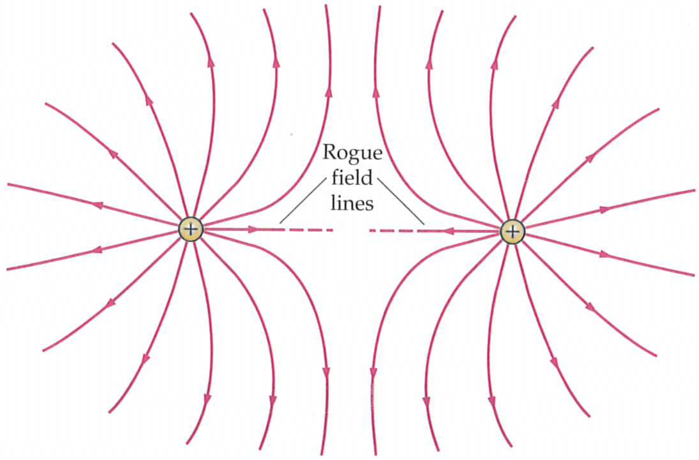


FIGURE 21-22 There are infinitely many field lines emanating from the two charges, two of which are rogue field lines. These rogue field lines terminate at the point midway between the two charges.

We can apply this reasoning to sketch the electric field lines for any system of point charges. Very near each charge, the field lines are equally spaced and emanate from or terminate on the charge radially, depending on the sign of the charge. Very far from all the charges, the detailed configuration of the system of charges is not important, so the field lines are like those of a single point charge having the net charge of the system. The rules for drawing electric field lines are summarized in the following Problem-Solving Strategy.

PROBLEM-SOLVING STRATEGY

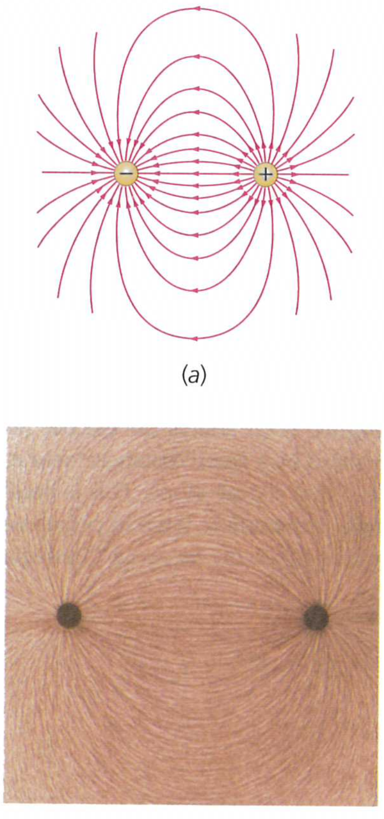
Drawing Field Lines

PICTURE Electric field lines emanate from positive charges and terminate on negative charges.*

SOLVE

1. The lines emanating from (or terminating on) an isolated point charge are drawn uniformly spaced as they emanate (or terminate).
2. The number of lines emanating from a positive charge (or terminating on a negative charge) is proportional to the magnitude of the charge.
3. The density of the lines at any point (the number of lines per unit area through a surface element normal to the lines) is proportional to the magnitude of the field there.
4. At large distances from a system of charges that has a nonzero net charge, the field lines are equally spaced and radial, as if they emanated from (or terminated on) a single point charge equal to the total charge of the system.

CHECK Make sure that the field lines never intersect each other. (If two field lines intersected, that would indicate two directions for $\vec{E}$ at the point of intersection.)



(a)

(b)

FIGURE 21-23 (a) Electric field lines for a dipole. (b) The same field lines shown by bits of thread in oil. *(Harold M. Waage.)*

Figure 21-23 shows the electric field lines due to a dipole. Very near the positive charge, the lines are directed radially outward. Very near the negative charge, the

* Rogue field lines are field lines that do not follow this rule. An example of a rogue field line is a line that leaves one of the positive charges in Figure 21-22 and is directed toward the other charge. This field line terminates at the point midway between the two charges-as does a corresponding field line emanating from the second positive charge in the figure. For these two charges there are infinitely many field lines, two of which are rogue field lines.

lines are directed radially inward. Because the charges have equal magnitudes, the number of lines that begin at the positive charge equals the number that end at the negative charge. In this case, the field is strong in the region between the charges, as indicated by the high density of field lines in this region.

Figure 21-24*a* shows the electric field lines for a negative charge $-q$ at a small distance from a positive charge $+2q$. Twice as many lines emanate from the positive charge as terminate on the negative charge. Thus, half the lines emanating from the positive charge $+2q$ terminate on the negative charge $-q$; the other half of the lines emanating from the positive charge continue on indefinitely. Very far from the charges (Figure 21-24*b*), the lines are approximately symmetrically spaced and point radially away from a single point, just as they would for a single positive point charge $+q$.

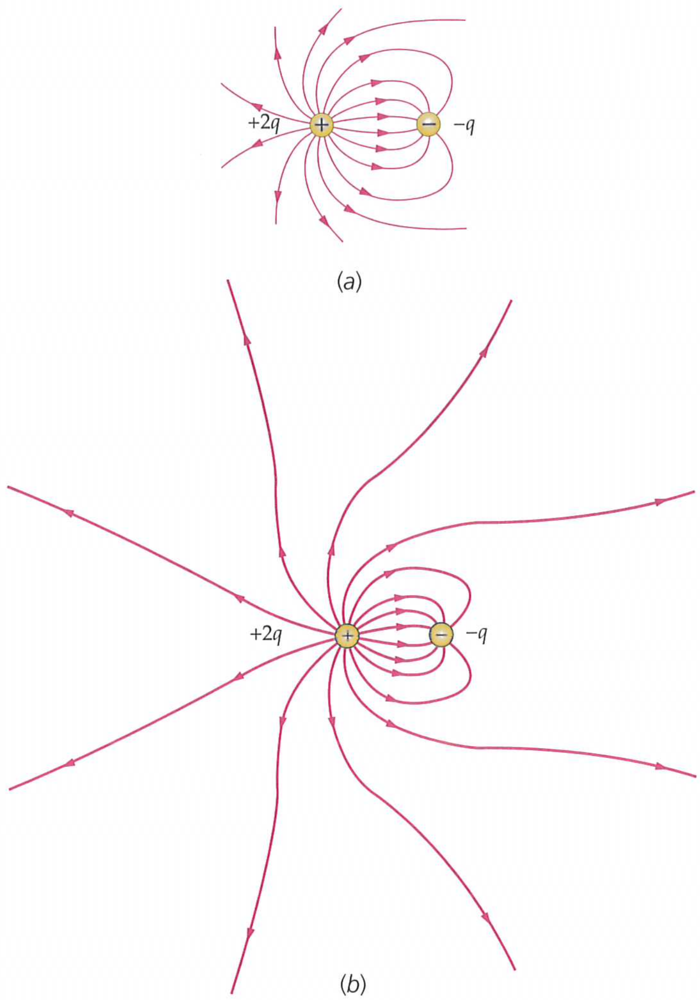


FIGURE 21-24 (*a*) Electric field lines for a point charge $+2q$ and a second point charge $-q$. (*b*) At great distances from the charges, the field lines approach those for a single point charge $+q$ located at the center of charge.

Example 21-10 Field Lines for Two Conducting Spheres *Conceptual*

The electric field lines for two conducting spheres are shown in Figure 21-25. What is the sign of the charge on each sphere, and what are the relative magnitudes of the charges on the spheres?

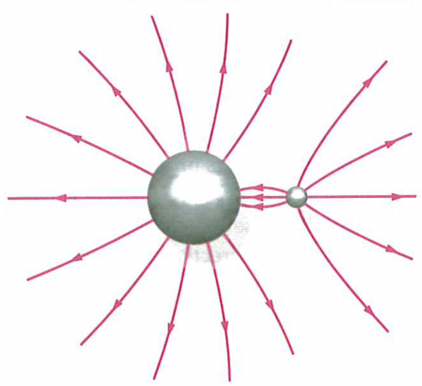


FIGURE 21-25

PICTURE The charge on an object is positive if more field lines emanate from it than terminate on it, and negative if more terminate on it than emanate from it. The ratio of the magnitudes of the charges equals the ratio of the net number of lines emanating from or terminating on the spheres.

SOLVE

1. By counting field lines, determine the net number of field lines emanating from the larger sphere: Because 11 electric field lines emanate from the larger sphere and 3 lines terminate on it, the net number of lines emanating from it is 8.

2. By counting field lines, determine the net number of field lines emanating from the smaller sphere: Because 8 electric field lines emanate from the smaller sphere and no lines terminate on it, the net number of lines emanating from it is 8.

3. Determine the sign of the charge on each sphere: Because both spheres have more field lines emanating from than terminating on them, **both spheres are positively charged.**

4. Determine the relative magnitudes of the charges on the two spheres: Because both spheres have the same net number of lines emanating from them, the **charges on them are equal in magnitude.**

The convention relating the electric field strength to the density of the electric field lines works only because the electric field varies inversely as the square of the distance from a point charge. Because the gravitational field of a point mass also varies inversely as the square of the distance, field-line drawings are also useful for picturing gravitational fields. Near a point mass, the gravitational field lines terminate on the mass just as electric field lines terminate on a negative charge. However, unlike electric field lines near a positive charge, there are no points in space from which gravitational field lines emanate. That is because the gravitational force between two masses is never repulsive.

21-6 ACTION OF THE ELECTRIC FIELD ON CHARGES

A uniform electric field can exert a force on a single charged particle and can exert both a torque and a net force on an electric dipole.

MOTION OF POINT CHARGES IN ELECTRIC FIELDS

When a particle that has a charge q is placed in an electric field $\vec{E}$, it experiences a force $q\vec{E}$. If the electric force is the only force acting on the particle, the particle has acceleration

$$\vec{a} = \frac{\Sigma\vec{F}}{m} = \frac{q}{m}\vec{E}$$

where m is the mass of the particle. (If the particle is an electron, its speed in an electric field is often a significant fraction of the speed of light. In such cases, Newton's laws of motion must be modified by Einstein's special theory of relativity.) If the electric field is known, the charge-to-mass ratio of the particle can be determined from the measured acceleration. J. J. Thomson used the deflection of electrons in a uniform electric field in 1897 to demonstrate the existence of electrons and to measure their charge-to-mass ratio. Familiar examples of devices that rely on the motion of electrons in electric fields are oscilloscopes, computer monitors, and television sets that use cathode-ray-tube displays.

A cathode-ray-tube display used for color television. The beams of electrons from the electron gun on the left activate phosphors on the screen at the right, giving rise to bright spots whose colors depend on the relative intensity of each beam. Electric fields between deflection plates in the gun (or magnetic fields from coils surrounding the gun) deflect the beams. The beams sweep across the screen in a horizontal line, are deflected downward, then sweep across again. The entire screen is covered in this way 30 times per second. *(Science & Society Picture Library/Contributor/ Getty Images.)*

Example 21-11 Electron Moving Parallel to a Uniform Electric Field

An electron is projected into a uniform electric field $\vec{E} = (1000\ \text{N/C})\hat{i}$ with an initial velocity $\vec{v}_0 = (2.00 \times 10^6\ \text{m/s})\hat{i}$ in the direction of the field (Figure 21-26). How far does the electron travel before it is brought momentarily to rest?

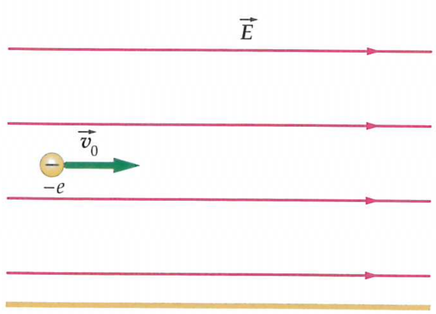


FIGURE 21-26

PICTURE Because the charge of the electron is negative, the force $\vec{F} = -e\vec{E}$ acting on the electron is in the direction opposite that of the field. Because $\vec{E}$ is constant, the force is constant and we can use constant acceleration formulas from Chapter 2. We choose the field to be in the $+x$ direction.

SOLVE

1. The displacement Δx is related to the initial and final velocities: $\quad v_x^2 = v_{0x}^2 + 2a_x\Delta x$

2. The acceleration is obtained from Newton's second law: $\quad a_x = \frac{F_x}{m} = \frac{-eE_x}{m}$

3. When $v_x = 0$, the displacement is:

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - v_{0x}^2}{2(-eE_x/m)} = \frac{mv_0^2}{2eE} = \frac{(9.11 \times 10^{-31}\ \text{kg})(2.00 \times 10^6\ \text{m/s})^2}{2(1.60 \times 10^{-19}\ \text{C})(1000\ \text{N/C})}$$

$$= 1.14 \times 10^{-2}\ \text{m} = \boxed{1.14\ \text{cm}}$$

CHECK The displacement Δx is positive, as is expected for something moving in the $+x$ direction.

Example 21-12 Electron Moving Perpendicular to a Uniform Electric Field

An electron enters a uniform electric field $\vec{E} = (-2.0\ \text{kN/C})\hat{j}$ with an initial velocity $\vec{v}_0 = (1.0 \times 10^6\ \text{m/s})\hat{i}$ perpendicular to the field (Figure 21-27). (*a*) Compare the gravitational force acting on the electron to the electric force acting on it. (*b*) By how much has the electron been deflected after it has traveled 1.0 cm in the x direction?

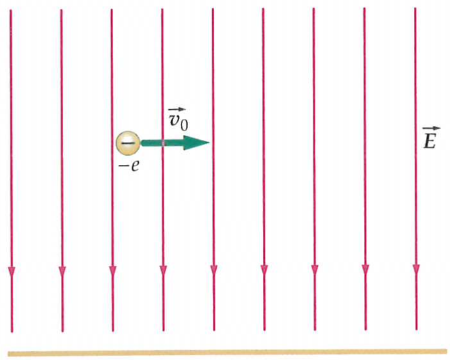


FIGURE 21-27

PICTURE (*a*) Calculate the ratio of the magnitude of the electric force $|q|E = eE$ to that of the gravitational force mg. (*b*) Because mg is, by comparison, negligible, the net force on the electron is equal to the vertically upward electric force. The electron thus moves with constant horizontal velocity v_x and is deflected upward by an amount $\Delta y = \frac{1}{2}at^2$, where t is the time to travel 1.0 cm in the x direction.

SOLVE

(*a*) 1. Calculate the ratio of the magnitude of the electric force, F_e, to the magnitude of the gravitational force, F_g:

$$\frac{F_e}{F_g} = \frac{eE}{mg} = \frac{(1.60 \times 10^{-19}\ \text{C})(2000\ \text{N/C})}{(9.11 \times 10^{-31}\ \text{kg})(9.81\ \text{N/kg})} = \boxed{3.6 \times 10^{13}}$$

(*b*) 1. Express the vertical deflection in terms of the acceleration a and time t:

$$\Delta y = \frac{1}{2}a_y t^2$$

2. Express the time required for the electron to travel a horizontal distance Δx with constant horizontal velocity v_0:

$$t = \frac{\Delta x}{v_0}$$

3. Use this result for t and eE/m for a_y to calculate Δy:

$$\Delta y = \frac{1}{2}\frac{eE}{m}\left(\frac{\Delta x}{v_0}\right)^2 = \frac{1}{2}\frac{(1.6 \times 10^{-19}\ \text{C})(2000\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\left(\frac{0.010\ \text{m}}{10^6\ \text{m/s}}\right)^2$$

$$= \boxed{1.8\ \text{cm}}$$

CHECK The step-4 result is positive (upward), as is expected for an object accelerating upward that was initially moving horizontally.

TAKING IT FURTHER (*a*) As is usually the case, the electric force is huge compared with the gravitational force. Thus, it is not necessary to consider gravity when designing a cathode-ray tube, for example, or when calculating the deflection in the problem above. In fact, a television picture tube works equally well upside down and right side up, as if gravity were not even present. (*b*) The path of an electron moving in a uniform electric field is a parabola, the same as the path of a neutral particle moving in a uniform gravitational field.

Example 21-13 The Electric Field in an Ink-Jet Printer

Context-Rich

You have just finished printing out a long essay for your English professor, and you wonder about how the ink-jet printer knows where to place the ink. You search the Internet and find a picture (Figure 21-28) showing that the ink drops are given a charge and pass between a pair of oppositely charged metal plates that provide a uniform electric field in the region between the plates. Because you have been studying the electric field in physics class, you wonder if you can determine how large a field is used in this type of printer. You search further and find that the 40.0-μm-diameter ink drops have an initial velocity of 40.0 m/s, and that a drop that has a 2.00-nC charge is deflected upward a distance of 3.00 mm as the drop travels through the 1.00-cm-long region between the plates. Find the magnitude of the electric field. (Neglect any effects of gravity on the motion of the drops.)

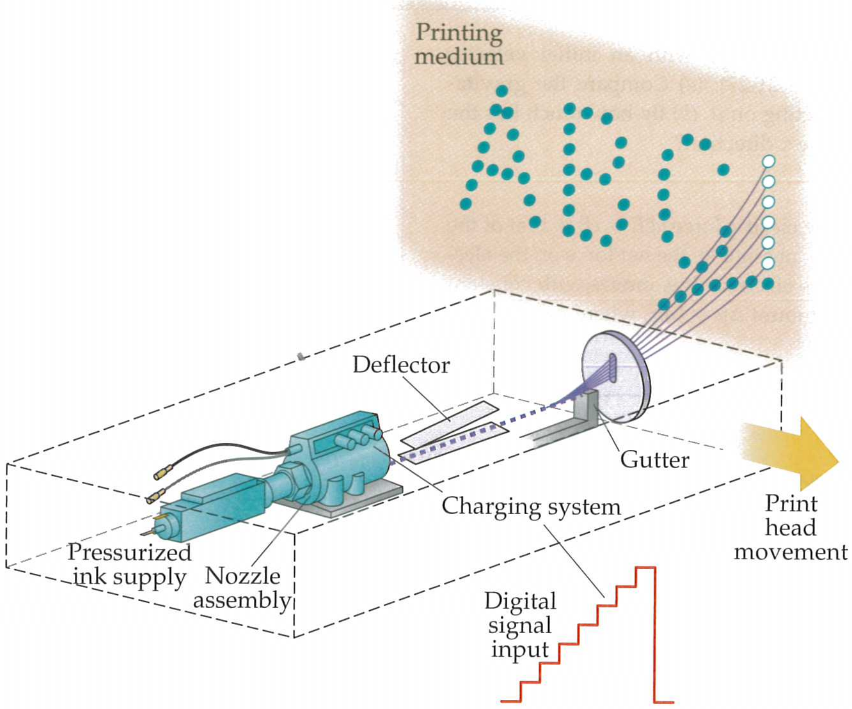


FIGURE 21-28 An ink-jet used for printing. The ink exits the nozzle in discrete droplets. Any droplet destined to form a dot on the image is given a charge. The deflector consists of a pair of oppositely charged plates. The greater the charge a drop receives, the higher the drop is deflected as it passes between the deflector plates. Drops that do not receive a charge are not deflected upward. These drops end up in the gutter, and the ink is returned to the ink reservoir. *(Courtesy of Videojet Systems International.)*

PICTURE The electric field $\vec{E}$ exerts a constant electric force $\vec{F}$ on the drop as it passes between the two plates, where $\vec{F} = q\vec{E}$. We are looking for E. We can get the force $\vec{F}$ by determining the mass and acceleration $\vec{F} = m\vec{a}$. The acceleration can be found from kinematics and mass can be found using the radius. Assume the density ρ of ink is 1000 kg/m³ (the same as the density of water).

SOLVE

1. The electric field strength equals the force to charge ratio: $E = \dfrac{F}{q}$

2. The force, which is in the $+y$ direction (upward), equals the mass multiplied by the acceleration: $F = ma_y$

3. The vertical displacement is obtained using a constant-acceleration kinematic formula with $v_{0y} = 0$: $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$

4. The time is how long it takes for the drop to travel the $\Delta x = 1.00$ cm at $v_0 = 40.0$ m/s: $\Delta x = v_{0x}t = v_0 t \quad \text{so} \quad t = \Delta x / v_0$

5. Solving for a_y gives: $a_y = \dfrac{2\Delta y}{t^2} = \dfrac{2\Delta y}{(\Delta x/v_0)^2} = \dfrac{2v_0^2\Delta y}{(\Delta x)^2}$

6. The mass equals the density multiplied by the volume: $m = \rho V = \rho\frac{4}{3}\pi r^3$

7. Solve for E:

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{\rho\frac{4}{3}\pi r^3}{q}\frac{2v_0^2\Delta y}{(\Delta x)^2} = \frac{8\pi}{3}\frac{\rho r^3 v_0^2\Delta y}{q(\Delta x)^2}$$

$$= \frac{8\pi}{3}\frac{(1000\ \text{kg/m}^3)(20.0\times10^{-6}\ \text{m})^3(40.0\ \text{m/s})^2(3.00\times10^{-3}\ \text{m})}{(2.00\times10^{-9}\ \text{C})(0.0100\ \text{m})^2} = \boxed{1.61\ \text{kN/C}}$$

CHECK The units in last line of step 7 are kg·m/(C·s²). The units work out because 1 N = 1 kg·m/s².

TAKING IT FURTHER The ink-jet in this example is called a multiple-deflection continuous ink-jet. It is used in some industrial printers. The low-cost ink-jet printers sold for home use do not use charged droplets deflected by an electric field.

DIPOLES IN ELECTRIC FIELDS

In Example 21-9 we found the electric field produced by a dipole, a system of two equal and opposite point charges that are close together. Here we consider the behavior of a dipole in an external electric field. Some molecules have permanent dipole moments due to a nonuniform distribution of charge within the molecule. Such molecules are called **polar molecules.** An example is HCl, which is essentially a positive hydrogen ion of charge $+e$ combined with a negative chloride ion of charge $-e$. The center of charge of the positive ion does not coincide with the center of charge for the negative ion, so the molecule has a permanent dipole moment. Another example is water (Figure 21-29).

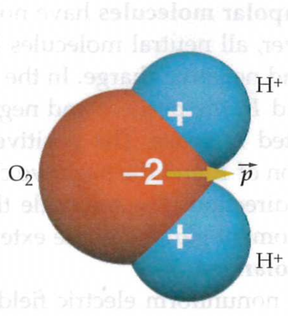


FIGURE 21-29 An H_2O molecule has a permanent dipole moment that points in the direction from the center of negative charge to the center of positive charge.

A uniform external electric field exerts no net force on a dipole, but it does exert a torque that tends to rotate the dipole so as to align it with the direction of the external field. We see in Figure 21-30 that the torque $\vec{\tau}$ calculated about the position of either charge has the magnitude $F_1 L \sin\theta = qEL \sin\theta = pE \sin\theta$.* The direction of the torque vector is into the paper such that it tends to rotate the dipole moment vector $\vec{p}$ so it aligns with the direction of $\vec{E}$. The torque can be expressed most concisely as the cross product:

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad 21\text{-}11$$

If the dipole rotates through angle $d\theta$, the electric field does work:

$$dW = -\tau d\theta = -pE \sin\theta \, d\theta$$

(The minus sign arises because the torque opposes any increase in θ.) Setting the negative of this work value equal to the change in potential energy, we have

$$dU = -dW = +pE \sin\theta \, d\theta$$

Integrating, we obtain

$$U = -pE \cos\theta + U_0$$

If we choose the potential energy U to be zero when $\theta = 90°$, then $U_0 = 0$ and the potential energy of the dipole is

$$U = -pE \cos\theta = -\vec{p} \cdot \vec{E} \qquad 21\text{-}12$$

POTENTIAL ENERGY OF A DIPOLE IN AN ELECTRIC FIELD

Microwave ovens take advantage of the dipole moment of water molecules to cook food. Like other electromagnetic waves, microwaves have oscillating electric fields that exert torques on dipoles, torques that cause the water molecules to rotate with significant rotational kinetic energy. In this manner, energy is transferred from the microwave radiation to the water molecules at a high rate, accounting for the rapid cooking times that make microwave ovens so convenient.

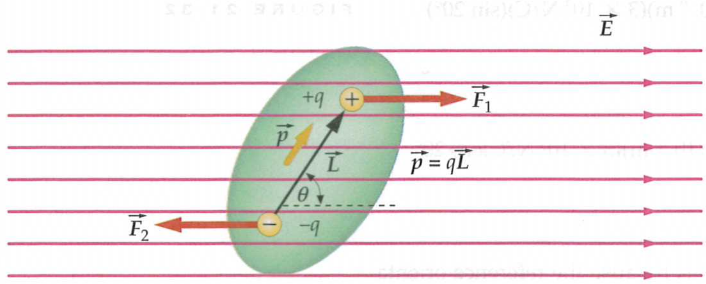


FIGURE 21-30 A dipole in a uniform electric field experiences equal and opposite forces that tend to rotate the dipole so that its dipole moment $\vec{p}$ is aligned with the electric field $\vec{E}$.

* The torque produced by two equal and opposite forces (an arrangement called a couple) is the same about any point in space.

Nonpolar molecules have no permanent dipole moment. However, all neutral molecules have equal amounts of positive and negative charge. In the presence of an external electric field $\vec{E}$, the positive and negative charge centers become separated in space. The positive charges are pushed in the direction of $\vec{E}$ and the negative charges are pushed in the opposite direction. The molecule thus acquires an induced dipole moment parallel to the external electric field and is said to be **polarized.**

In a nonuniform electric field, a dipole experiences a net force because the electric field has different magnitudes at the positive and negative charge centers. Figure 21-31 shows how a positive point charge polarizes a nonpolar molecule and then attracts it. A familiar example is the attraction that holds an electrostatically charged balloon against a wall. The nonuniform field produced by the charge on the balloon polarizes molecules in the wall and attracts them. An equal and opposite force is exerted by the wall molecules on the balloon.

The diameter of an atom or molecule is of the order of 10^{-12} m = 1 pm (one picometer). A convenient unit for the dipole moment of atoms and molecules is the fundamental charge e multiplied by the distance 1 pm. For example, the dipole moment of H_2O in these units has a magnitude of about 40 $e \cdot$pm.

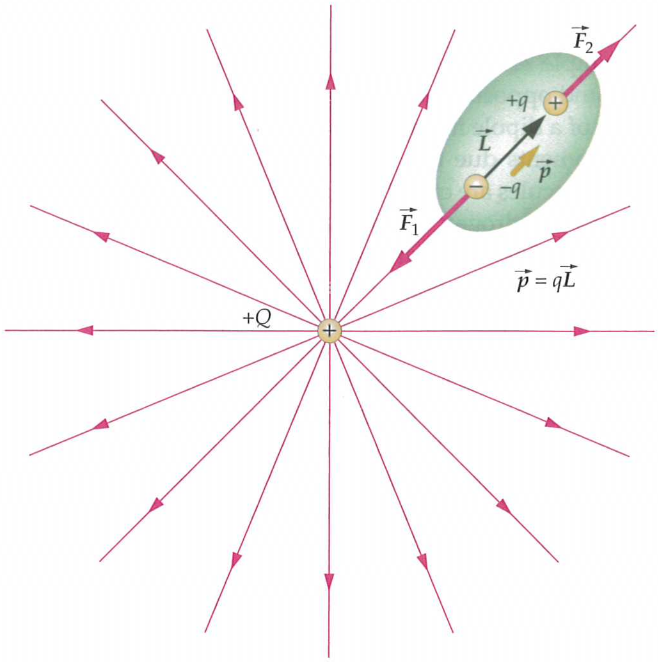


FIGURE 21-31 A nonpolar molecule in the nonuniform electric field of a positive point charge $+Q$. The point charge attracts the negative charges (the electrons) in the molecule and repels the positive charges (the protons). As a result the center of negative charge $-q$ is closer to $+Q$ than is the center of positive charge $+q$ and the induced dipole moment $\vec{p}$ is parallel to the field of the point charge. Because $-q$ is closer to $+Q$ than is $+q$, F_1 is greater than F_2 and the molecule is attracted to the point charge. In addition, if the point charge were negative, the induced dipole moment would be reversed, and the molecule would again be attracted to the point charge.

Example 21-14 Torque and Potential Energy

A polar molecule has a dipole moment of magnitude 20 $e \cdot$pm that makes an angle of 20° with a uniform electric field of magnitude 3.0×10^3 N/C (Figure 21-32). Find (*a*) the magnitude of the torque on the dipole, and (*b*) the potential energy of the system.

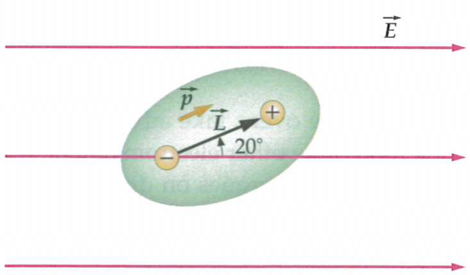


FIGURE 21-32

PICTURE The torque is found from $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy is found from $U = -\vec{p} \cdot \vec{E}$.

SOLVE

1. Calculate the magnitude of the torque:

$$\tau = |\vec{p} \times \vec{E}| = pE\sin\theta = (20\, e\cdot\text{pm})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= (0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= \boxed{3.3 \times 10^{-27}\ \text{N}\cdot\text{m}}$$

2. Calculate the potential energy:

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$
$$= -(0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})\cos 20°$$
$$= \boxed{-9.0 \times 10^{-27}\ \text{J}}$$

CHECK The sign of the potential energy is negative. That is because the reference orientation of the potential energy function $U = -\vec{p} \cdot \vec{E}$ is $U = 0$ for $\theta = 90°$. For $\theta = 20°$ the potential energy is less than zero. The system has more potential energy if $\theta = 20°$ than it does if $\theta = 90°$.